Calculus of Variation, HW #1

變分學(2010,春季)第一次作業

繳交時間:四月14日(星期三)上課前

(Time Due: April 14 (Wed.) before class)

Find the extremals of the functionals:

(1) $J[y] = \int_{x_0}^{x_1} \frac{\sqrt{1+y'^2}}{y} dx;$

(2) $J[y] = \int_{x_0}^{x_1} \sqrt{y(1+y'^2)} dx;$

(3) $J[y] = \int_{x_0}^{x_1} y'(1+x^2 y') dx;$

(4) $J[y] = \int_{x_0}^{x_1} (y^2 + y'^2 - 2y \sin x) dx;$

(5) $J[y] = \int_{x_0}^{x_1} [16y^2 - (y'')^2 + x^2] dx;$

(6) $J[y] = \int_{x_0}^{x_1} [y^2 + 2y'^2 + (y'')^2] dx;$

(7) $J[y] = \int_1^2 \frac{x^3}{y'^2} dx, \quad y(1) = 1, \quad y(2) = 4;$

(8) $J[y] = \int_1^2 (12xy + y'^2) dx, \quad y(1) = 0, \quad y(3) = 26.$

(9) Find extremals for an isoperimetric problem

$$J[y] = \int_0^1 (y'^2 + x^2) dx, \quad y(0) = 0, \quad y(1) = 0$$

with the condition

$$\int_0^1 y^2 dx = 2$$

(10) Find the Euler-Lagrange equations corresponding to the following functionals

(a) $J[u] = \iint_\Omega (x^2 u_x^2 + y^2 u_y^2) dx dy, \quad u = u(x, y)$

(b) $J[u] = \iint_\Omega (u_t^2 - c^2 u_x^2) dt dx, \quad u = u(x, t), \quad c = \text{constant}$