Outline:

Part 1. 光的电磁理論

L1. Waves

L2. EM waves and optics

L3. Reflection and Refraction

L4. Dispersion, Absorption, Scattering

Part 2. 光的干涉

L5. Interference

L6. Thin-Film interference

L7. Coherence

Part 3. 繞射与成像

L8. Diffraction

L9. Optical Imaging

L10. Diffraction & Imaging

Part 4. 晶体光学

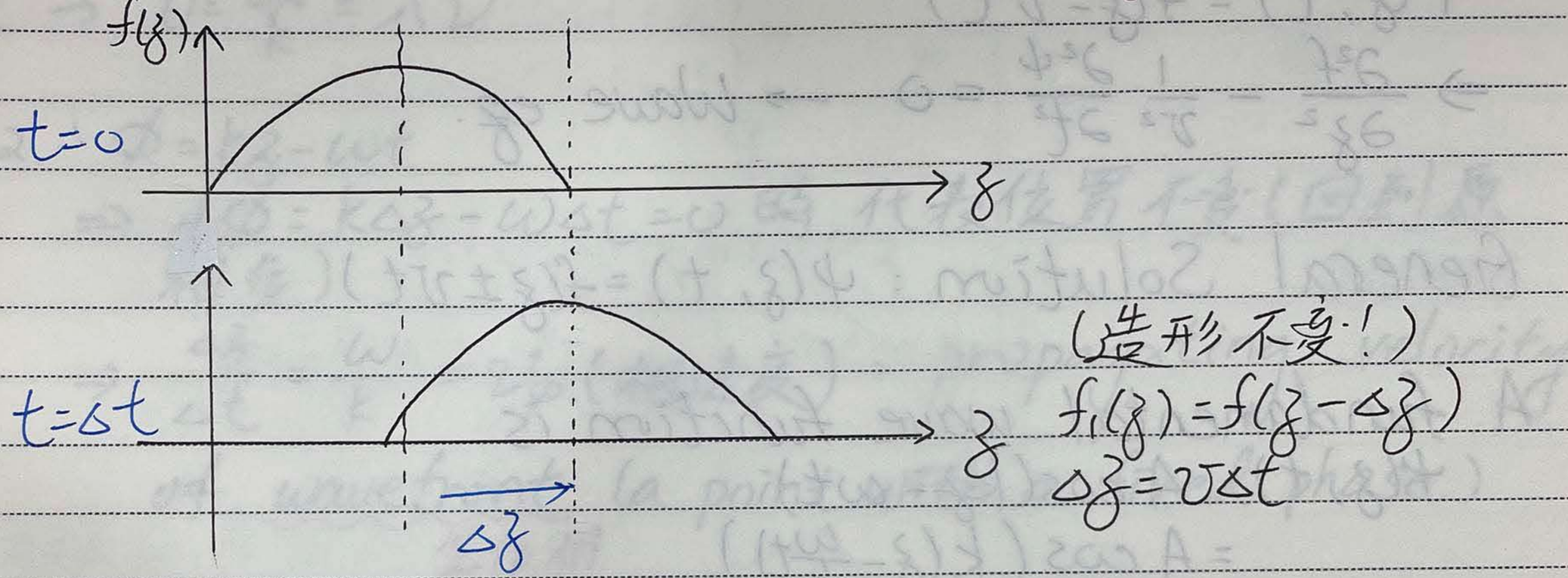
L11. Polarization

Lecture 1. Waves

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§1. Waves. (1-1)

Waves motions: A time varying phenomena of spatial distribution of some physical quantities



(造形不变!)

$f_1(\xi) = f(\xi - \Delta\xi)$

$\Delta\xi = v\Delta t$

Wave function: $\psi(\xi, t) = f(\xi - vt)$

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§2 Wave Equations (1-2 ~ 1-4)

Dynamic equation for wave motions $f(z-vt)$
Let $z' = z - vt$, 求 $\frac{\partial f}{\partial z}, \frac{\partial^2 f}{\partial z^2}, \frac{\partial f}{\partial t}, \frac{\partial^2 f}{\partial t^2}$ for

$$\Psi(z,t) = f(z-vt)$$

$$\Rightarrow \frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} = 0 \rightarrow \text{Wave eq.}$$

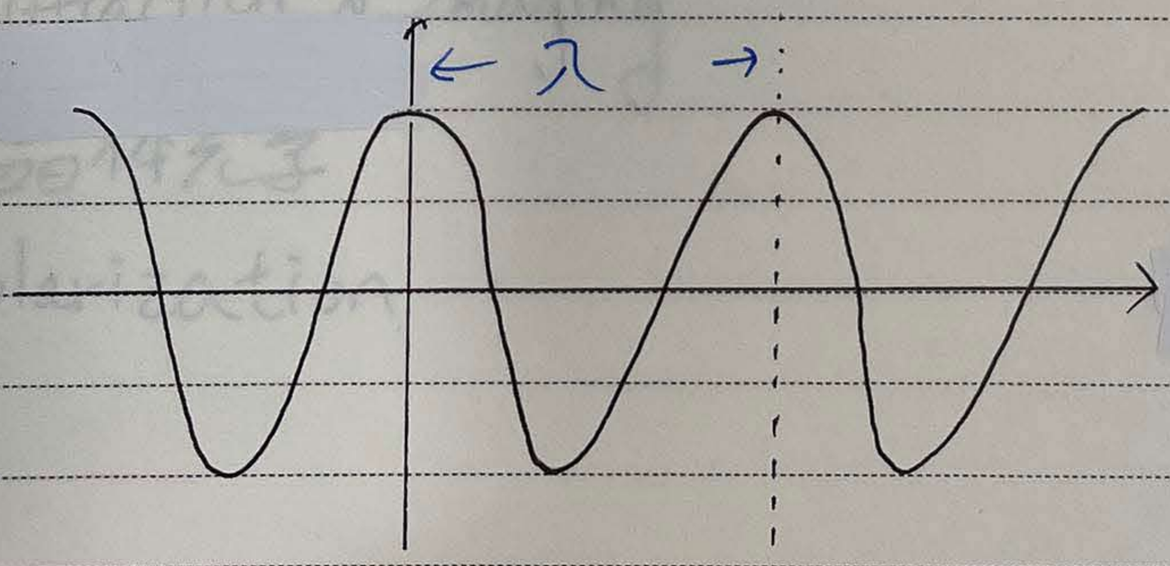
General Solution: $\Psi(z,t) = f(z \pm vt)$

A fundamental wave function is

$$\Psi(z,t) = A\cos(kz - \omega t)$$
$$= A\cos\left(k\left(z - \frac{\omega}{k}t\right)\right)$$

由 Fourier Series, 此 $\Psi(z,t)$ 可疊加出各种波形

$$\begin{cases} A: \text{amplitude} \\ \phi = kz - \omega t: \text{phase} \\ v = \frac{\omega}{k} \end{cases}$$



1. $\begin{cases} \text{spatial} \rightarrow \text{period } \lambda \rightarrow \Psi(z+\lambda, t) = \Psi(z,t) \\ \text{temporal} \rightarrow \text{period } \tau \rightarrow \Psi(z, t+\tau) = \Psi(z,t) \end{cases}$

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→ $k = \frac{2\pi}{\lambda}$ = 一波長內有多少個 π (rad/m)

(propagation number)

→ $\omega = \frac{2\pi}{T} = 2\pi\nu$, ν: frequency (s^{-1}, Hz)

→ $v = \frac{\omega}{k} = \lambda\nu$

2. $\phi = kz - \omega t$

⇒ $d\phi = k\Delta z - \omega\Delta t = 0$ 時，代表位置不變（回到原相位）

⇒ $\frac{\Delta z}{\Delta t} = \frac{\omega}{k} = v_\phi$（相速度）: propagating velocity of wavefront (a point with a fixed phase)

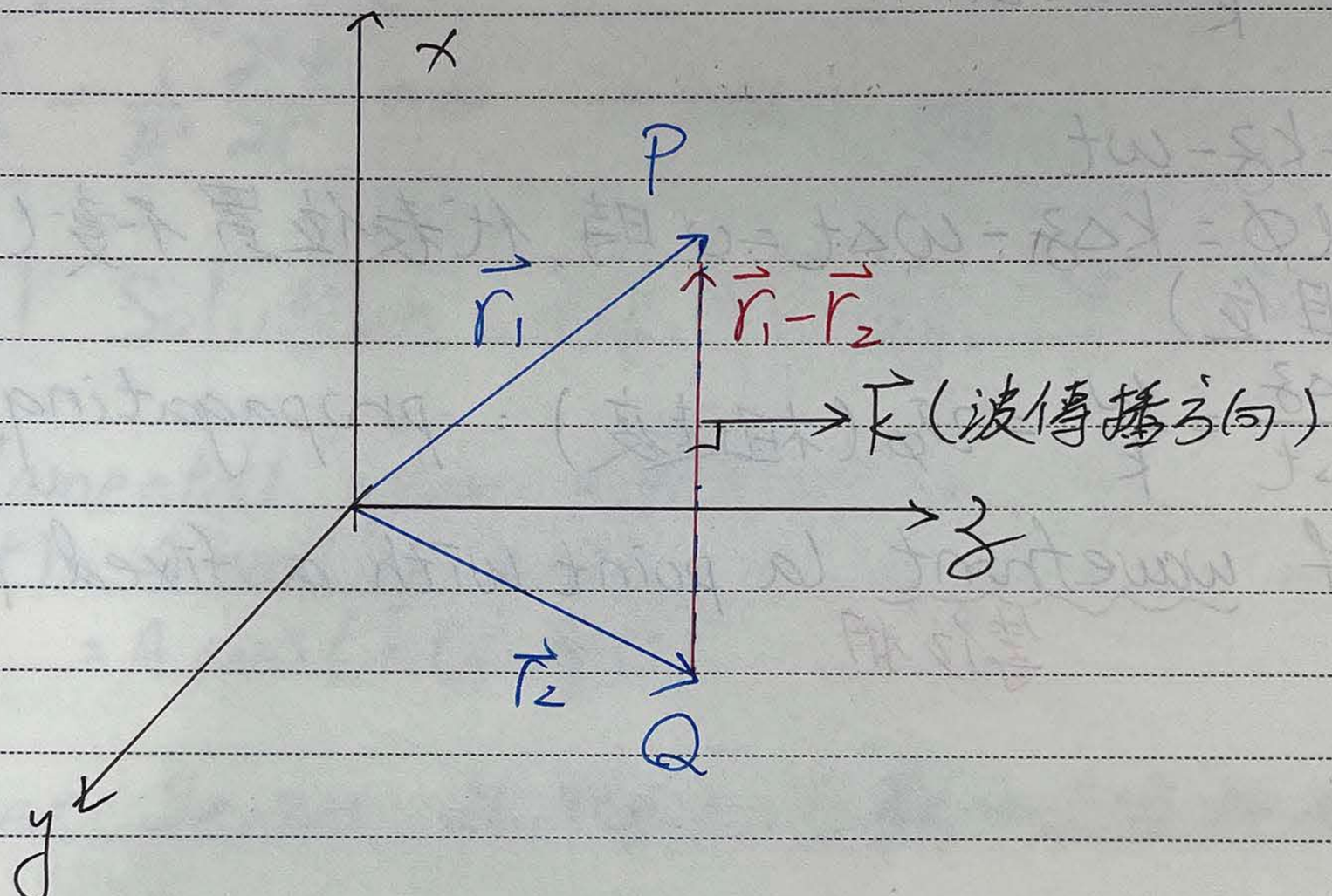
等位相

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§3. The wave eq. in 3D (1-7)

Define $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\nabla^2 \psi - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0$.

Let $\psi(\vec{r}, t) = A\cos(\vec{k}\cdot\vec{r} - \omega t)$



求此波之 wavefront (等相位面) = ?

⇒ 設 P, Q 在等位相面上, 則 $\phi_P = \phi_Q$

⇒ $\vec{k}\cdot\vec{r}_1 - \omega t = \vec{k}\cdot\vec{r}_2 - \omega t$

⇒ $\vec{k}\cdot(\vec{r}_1 - \vec{r}_2) = 0 \leftrightarrow \vec{k} \perp \vec{r}_1 - \vec{r}_2$

即: wavefront 為垂直於 $\vec{k}$ 之集合。以此而來說, wavefront 為一平面, 故平面波。

§4. Complex Representation

Date. (1-6) Page.

1. $e^{i\theta} = \cos\theta + i\sin\theta$

$\Rightarrow \cos\theta = Re\{e^{i\theta}\}$

$\Rightarrow \psi(\vec{r},t) = A\cos(\vec{k}\cdot\vec{r} - \omega t)$

$= Re\{Ae^{i(\vec{k}\cdot\vec{r}-\omega t)}\}$

ex. $a(t) = |A|\cos(\omega t+\alpha)$

$b(t) = |B|\cos(\omega t+\beta)$

$$\frac{da(t)}{dt} = -|A|\omega\sin(\omega t+\alpha)$$

$$= Re\left\{\frac{d}{dt}(|A|e^{i(\omega t+\alpha)})\right\}$$

→方便起見用複數形式

* $a(t)b(t) = \frac{1}{2}|A||B|(\cos(2\omega t+\alpha+\beta) + \cos(\alpha+\beta))$

In complex form: $a(t)b(t) = |A||B|e^{i(2\omega t+\alpha+\beta)}$, but

$Re\{a(t)b(t)\} = |A||B|\cos(2\omega t+\alpha+\beta)$

⇒ complex form不能用於乘、、高次項！

In general, $Re\{a\}Re\{b\} \neq Re\{ab\}$

2. Observation detection: $T >> \tau$

比如: visible light $\lambda = 500\,nm$, $\nu \sim 10^{14}\,Hz$,

$\tau = 10^{-14}\,s$, 遠小於人眼反應時間

⇒偵測其實是看到一段時間之平均

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$\Rightarrow \langle a(t)b(t)\rangle_T$ = The real detection

$$\triangleq \frac{1}{T}\int_0^T a(t)b(t)dt$$

$$= \frac{1}{2}|A||B|\cos(\alpha+\beta)$$

$$= \mathrm{Re}\left\{\frac{1}{2}|A|e^{i\alpha}|B|e^{-i\beta}\right\}$$

$$= \frac{1}{2}\mathrm{Re}\{a(t)b^*(t)\}$$

→ 用 complex form 就不用真的積分!

Lecture 2. EM waves & Optics

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§1 Maxwell's eq. (2-1, 2-2)

$$\begin{cases} \nabla \cdot \vec{D} = \rho & -① \\ \nabla \cdot \vec{B} = 0 & -② \\ \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 & -③ \\ \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} & -④ \end{cases}$$

Material equations:

$$\begin{cases} \vec{D} = \varepsilon \vec{E} = \varepsilon_0 \vec{E} + \vec{P} \\ \vec{B} = \mu \vec{H} = \mu_0 \vec{H} + \mu_0 \vec{M} \end{cases}$$

$\vec{P}$: polarization $= \chi_e \varepsilon_0 \vec{E}$

$\vec{M}$: magnetic polarization $= \chi_m \vec{H}$

If $\rho = 0$, $\vec{J} = 0$. By ③, take $\nabla \times$ and $+ ④$

$$\Rightarrow \begin{cases} \nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ v = \frac{1}{\sqrt{\mu\varepsilon}} \\ \nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \end{cases}$$

→ 不用电流、电荷，$\vec{E}$ 跟 $\vec{B}$ 可以自己传播

∵ $\mu\varepsilon$ 之值与 c 算出来一样

∴ Maxwell 推测光是一种电磁波（由 Hertz 做实验证明）

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真空中 $\begin{cases} \mu \triangleq \mu_0 = 4\pi \times 10^{-7} \ H/m \\ \varepsilon = \varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} \ F/m \\ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \ m/s \end{cases}$

$n \triangleq \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$ index of refraction

(for dielectric material such as glass, fibers)

§2. Characteristics of EM Waves

Date. Page. (2-3)

Consider $\begin{cases} \vec{E} = \hat{u}_1 E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \\ \vec{H} = \hat{u}_2 H_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \end{cases}$

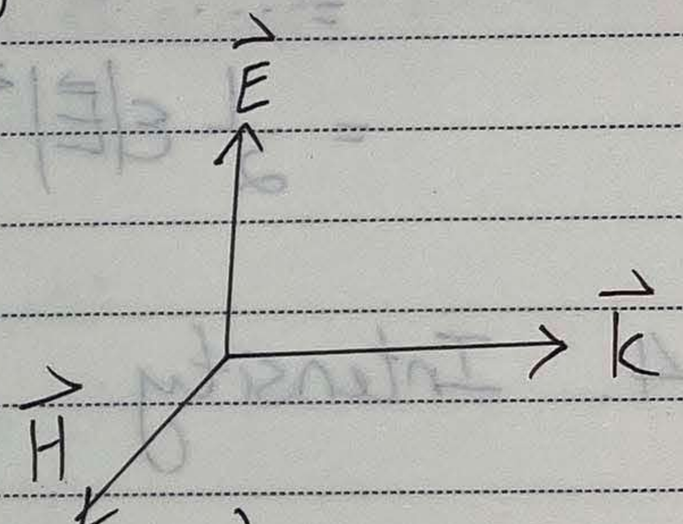
1. $\because \begin{cases} \nabla\cdot\vec{D}=0 \\ \nabla\cdot\vec{E}=0 \end{cases}$, where

$$\nabla = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)$$

$$\therefore (\hat{u}_1\cdot\hat{k})\, e^{i(\vec{k}\cdot\vec{r}-\omega t)} = 0$$

$\Rightarrow \hat{u}_1 \perp \vec{k}$

$\Rightarrow \vec{E} \perp k$



同理, $\hat{u}_2 \perp \vec{k}$ 故 $\vec{H} \perp \vec{k}$

此种波称平面波 (transverse wave)

2. $\vec{E}$、$\vec{H}$ 代入③

$$\nabla \times F \triangleq \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\Rightarrow i E_0 (\vec{k}\times\hat{u}_1) = (i\omega\mu H_0)\hat{u}_2$$

① $\vec{k}\times\hat{u}_1 = \hat{u}_2$

$\Rightarrow \vec{H}, \vec{E}, \vec{k}$ mutually orthogonal (满足右手定则)

② $E_0 k = \omega\mu H_0$

$$\Rightarrow H_0 = \frac{k}{\omega\mu} H_0 = \sqrt{\frac{\varepsilon}{\mu}} E_0 \triangleq \frac{E_0}{\eta}$$

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η : Wave Impedence

3. Energy density:

$$U = \frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B}) \quad (J/m^3)$$

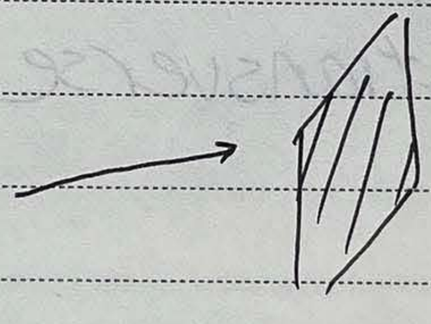
Plane waves:

$$\langle U\rangle = \frac{1}{4}\mathrm{Re}\{\vec{E}\cdot\vec{D}^* + \vec{H}\cdot\vec{B}^*\}$$

$$= \cdots$$

$$= \frac{1}{2}\varepsilon|\vec{E}|^2$$

4. Intensity

Power flux :  $J\cdot S^{-1}\cdot m^{-2} = Watt/m^2$

Poynting vector $\vec{S} = \vec{E}\times\vec{H}$

Intensity $I = \langle S\rangle = \frac{1}{2}\mathrm{Re}\{\vec{E}\times\vec{H}^*\}$

$$= \frac{1}{2}\frac{1}{\eta}|E_0|^2$$

$\because \langle U\rangle = \frac{1}{2}\varepsilon|E_0|^2$

$\therefore \langle U\rangle \cdot v = \langle S\rangle$, $v = \frac{\omega}{k}$

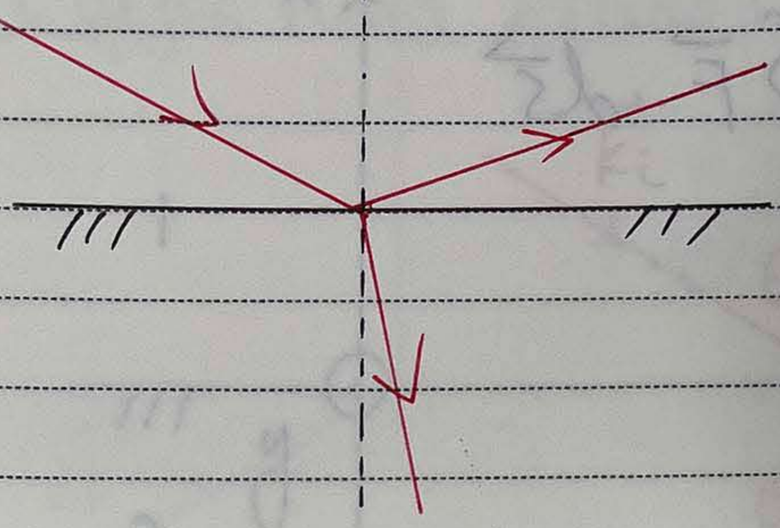
$\Rightarrow v$ = energy propagating speed.

∵ 通常只在意亮暗分佈

∴ 不理常数，常令 $I = |E|^2$.

Lecture 3. Reflection & Refraction

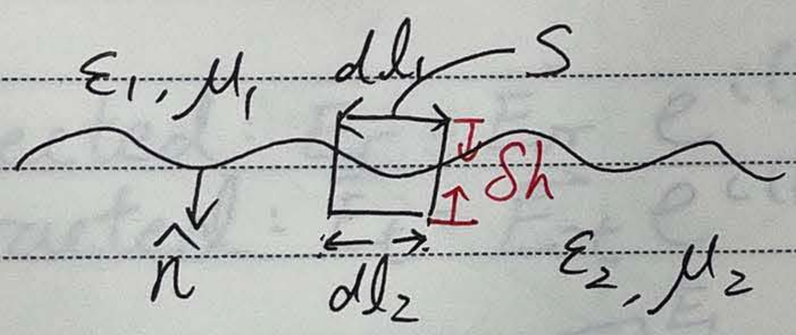
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{ direction
amplitude 之变化?
phase

§1. Boundary Condition (3-3)

1. $\vec{E}$、$\vec{H}$

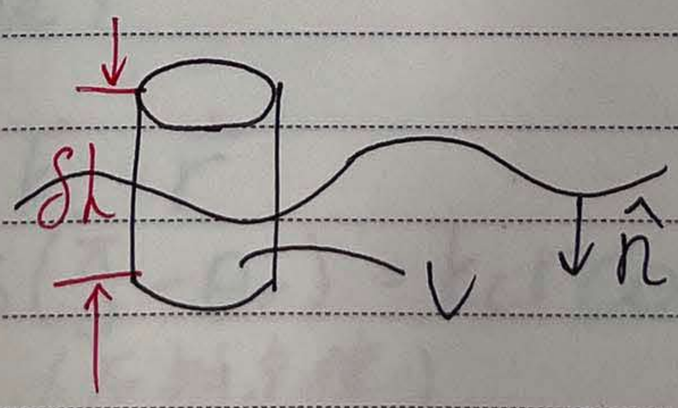


$\because \iint_S \nabla \times \vec{E} \cdot d\vec{S} = \oint_{\partial S} \vec{E} \cdot d\vec{l}$ and $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\therefore \begin{cases} \hat{n} \times (\vec{E_2} - \vec{E_1}) = 0 \\ E_{2t} = E_{1t} \end{cases}$

同理, $H_{2t} = H_{1t}$

2. $\vec{B}$、$\vec{D}$



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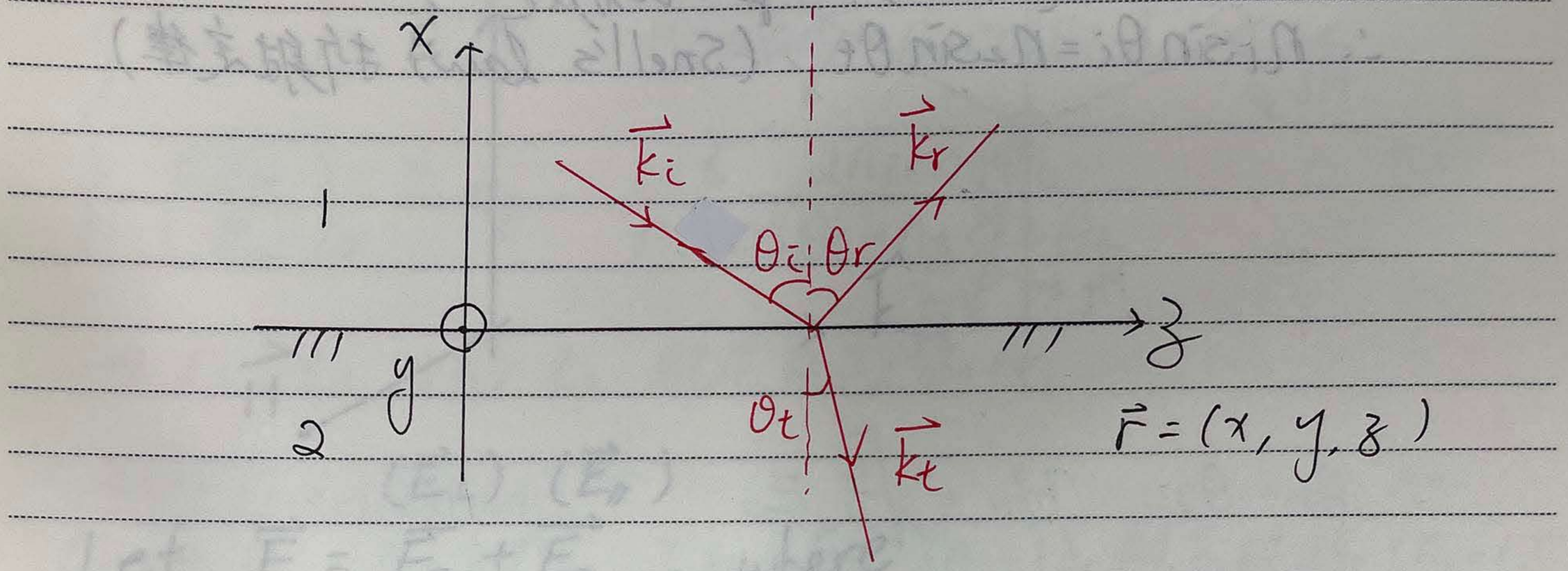
By Divergence Thm,

$$\iiint_V \nabla \cdot \vec{F} \, dV = \oiint_{\partial V} \vec{F} \cdot d\vec{S}$$

$\therefore$ $\begin{cases} \nabla \cdot \vec{D} = 0 \\ \nabla \cdot \vec{B} = 0 \end{cases}$

$\therefore$ $\begin{cases} D_{2n} = D_{1n} \\ B_{2n} = B_{1n} \end{cases}$

§2. The laws of Reflection & Refraction (3-3)



$$\begin{cases} \text{incident}: \vec{E}_i = \vec{E}_{0i}\, e^{i(\vec{k}_i\cdot\vec{r}-\omega_i t)} \\ \text{reflected}: \vec{E}_r = \vec{E}_{0r}\, e^{i(\vec{k}_r\cdot\vec{r}-\omega_r t)} \\ \text{refracted}: \vec{E}_t = \vec{E}_{0t}\, e^{i(\vec{k}_t\cdot\vec{r}-\omega_t t)} \end{cases}$$

$\because$ B.C.: $\hat{n}\times(\underbrace{E_i+E_r}_{E_1}) = \hat{n}\times\underbrace{\vec{E}_t}_{E_2}$ is valid for all t at any $\vec{r}$.

$\therefore \omega_t = \omega_r = \omega_i$

$\Rightarrow \vec{k}_r\cdot\vec{r} = \vec{k}_t\cdot\vec{r} = \vec{k}_i\cdot\vec{r}$

$\Rightarrow (\vec{k}_r - \vec{k}_i)\cdot\vec{r} = 0$

$\Rightarrow \begin{cases} \vec{k}_r - \vec{k}_i \parallel \hat{n} \\ \vec{k}_t - \vec{k}_i \parallel \hat{n} \quad (\text{同理}) \end{cases}$

$\Rightarrow \vec{k}_i, \vec{k}_r, \vec{k}_t, \hat{n}$ coplanar! (on the plane of incidence)

$\because \vec{k}_r\cdot\vec{r} = \vec{k}_i\cdot\vec{r}$

$\therefore k_r r\cos(\frac{\pi}{2}-\theta_r) = k_i r\cos(\frac{\pi}{2}-\theta_i)$

$\Rightarrow \theta_r = \theta_i$ (反射定律)

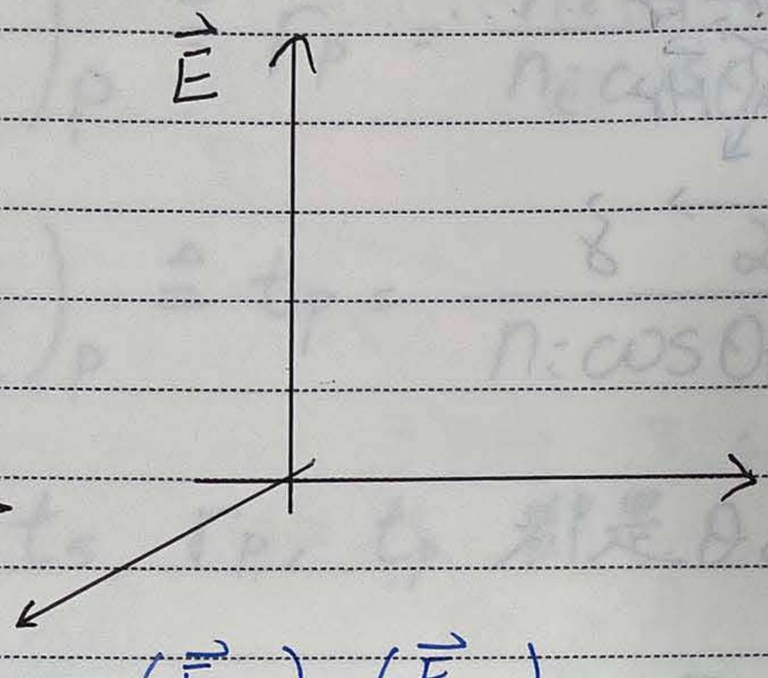
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$\because \vec{k_t} \cdot \vec{r} = \vec{k_i} \cdot \vec{r}$, $k = \frac{\omega}{f} = \omega\sqrt{\mu\varepsilon}$

$\therefore n_1 \sin\theta_i = n_2 \sin\theta_t$ (Snell's law, 折射定律)

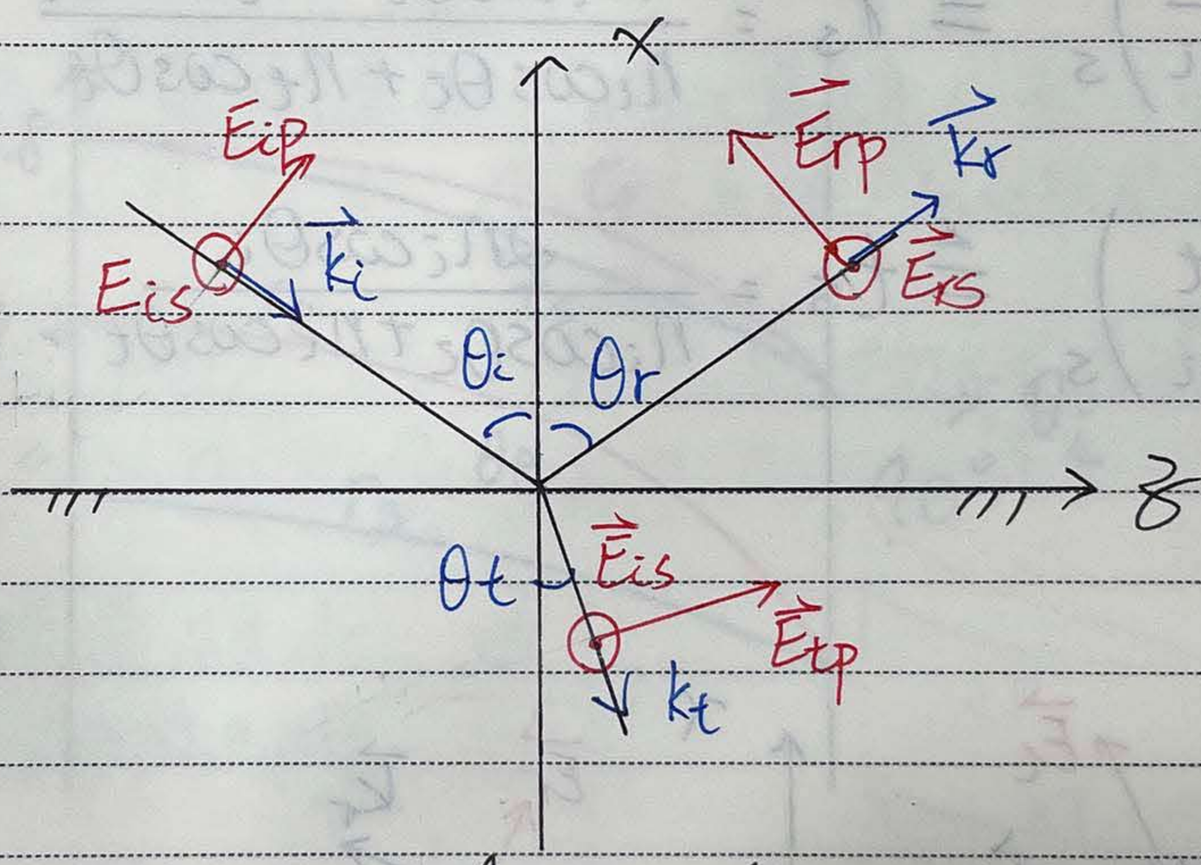
§3. Fresnel Equations

Date. (3-4) Page.



$(\vec{E}_\perp)$ $(\vec{E}_{//})$

Let $\vec{E} = \vec{E}_s + \vec{E}_p$, where



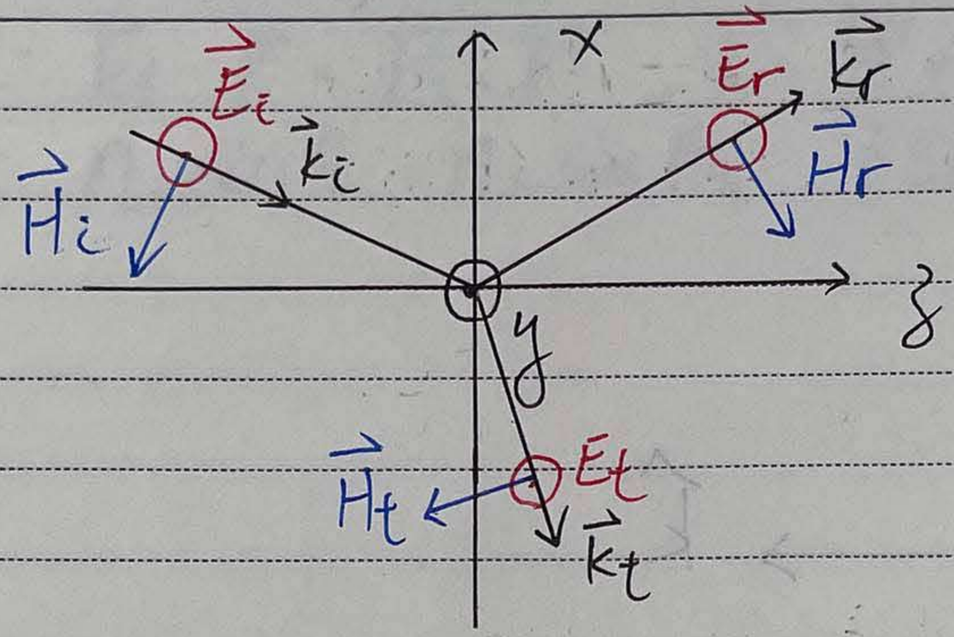
$\begin{cases} \vec{E}_s \perp \text{(plane of incidence, } xz \text{ plane)} \\ \vec{E}_p // xz \text{ plane} \end{cases}$

1. S-wave

$$\begin{cases} \vec{E}_i = \vec{E}_{oi}\, e^{i(\vec{k}_i \cdot \vec{r} - \omega t)} \\ \vec{E}_r = \vec{E}_{or}\, e^{i(\vec{k}_r \cdot \vec{r} - \omega t)} \\ \vec{E}_t = \vec{E}_{ot}\, e^{i(\vec{k}_t \cdot \vec{r} - \omega t)} \end{cases}$$

B.C. : $\begin{cases} E_t \text{ continuous} : E_{oi} + E_{or} = E_{ot} \\ H_t \text{ continuous} : -H_{oi}\cos\theta_i + H_{or}\cos\theta_r = -H_{ot}\cos\theta_t \end{cases}$

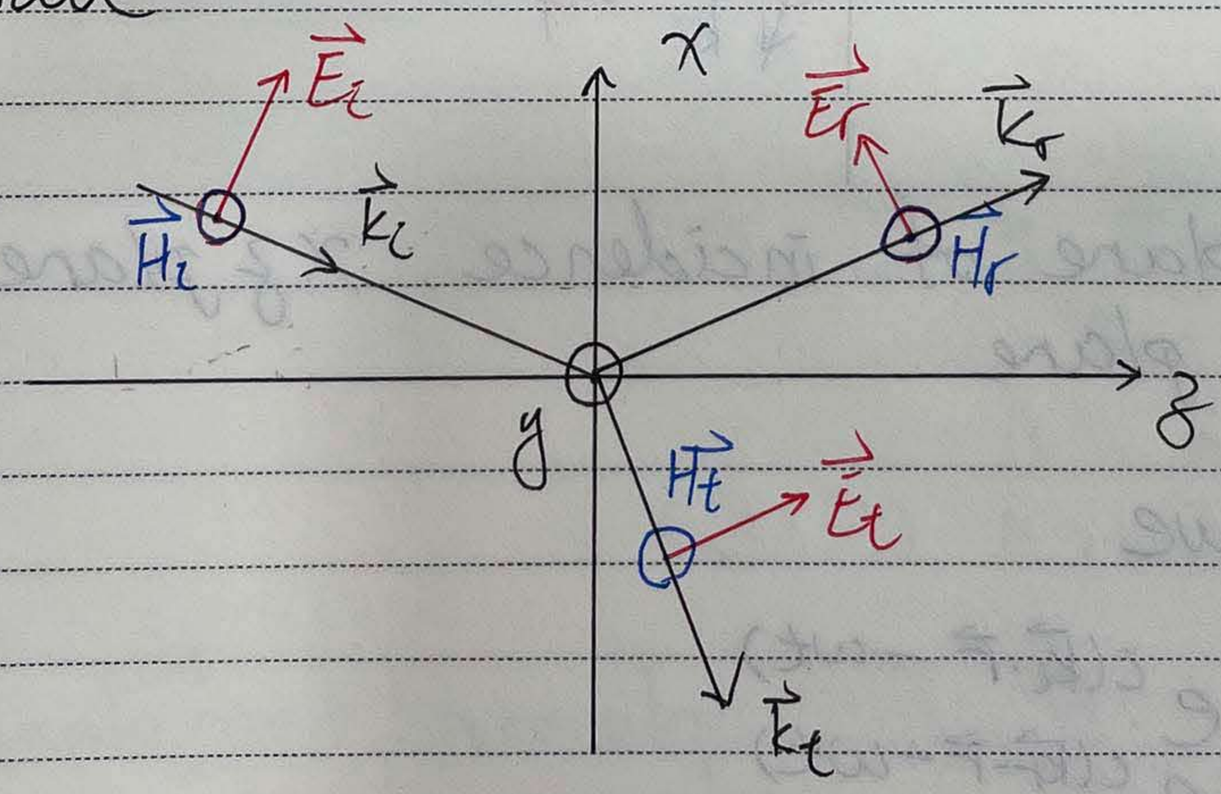
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$\because \theta_r = \theta_i, \quad H_0 = \sqrt{\frac{\varepsilon}{\mu_0}} E_0$

$$\therefore \begin{cases} \left(\frac{E_{0r}}{E_{0i}}\right)_s \triangleq r_s = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \\ \left(\frac{E_{0t}}{E_{0i}}\right)_s \triangleq t_s = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \end{cases}$$

2. p-wave



$\because$ B.C. : $\begin{cases} E_{0i}\cos\theta_i + E_{0r}\cos\theta_i = E_{0t}\cos\theta_t \\ B_{0i} - B_{0r} = B_{0t} \end{cases}$

$\therefore$ 同理,

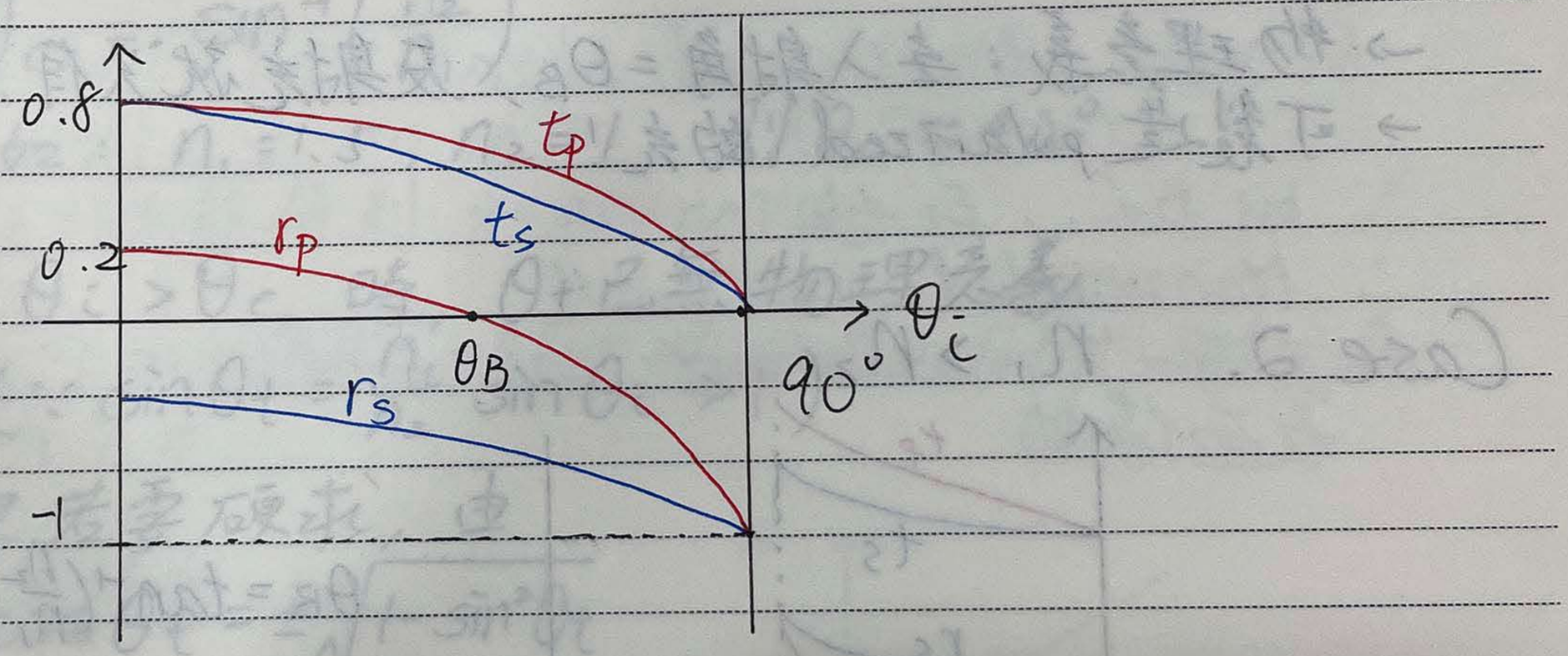
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$$\begin{cases} \left(\frac{E_{or}}{E_{oi}}\right)_p \triangleq r_p = \frac{n_i\cos\theta_t - n_t\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} \\ \left(\frac{E_{ot}}{E_{oi}}\right)_p \triangleq t_p = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} \end{cases}$$

⇒ r_s, t_s, r_p, t_p 都是 θ_i 之函數

3. Discussions

Case 1: $n_1 < n_2$ (如：真空 ($n_1=1$) 射入玻璃 ($n_2=1.5$))



(1) $t_p, t_s > 0$ for $0° < \theta_i < 90°$.

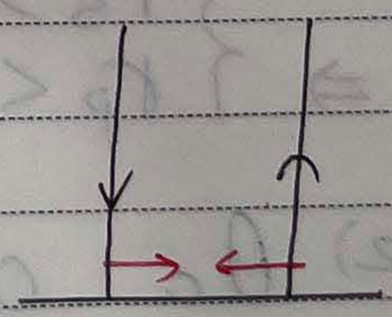
⇒ no phase shift for the transmitted wave

(2) $\theta_i \to 0$.

s-wave, $r_s < 0$

$$r_s = \frac{n_1 - n_2}{n_1 + n_2} \to 0$$

→ p-wave 有 π-phase shift!



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(3). $\theta_i \to 90°$ (glazing incidence, 掠射)
$|r_p|, |r_s| \to 1$

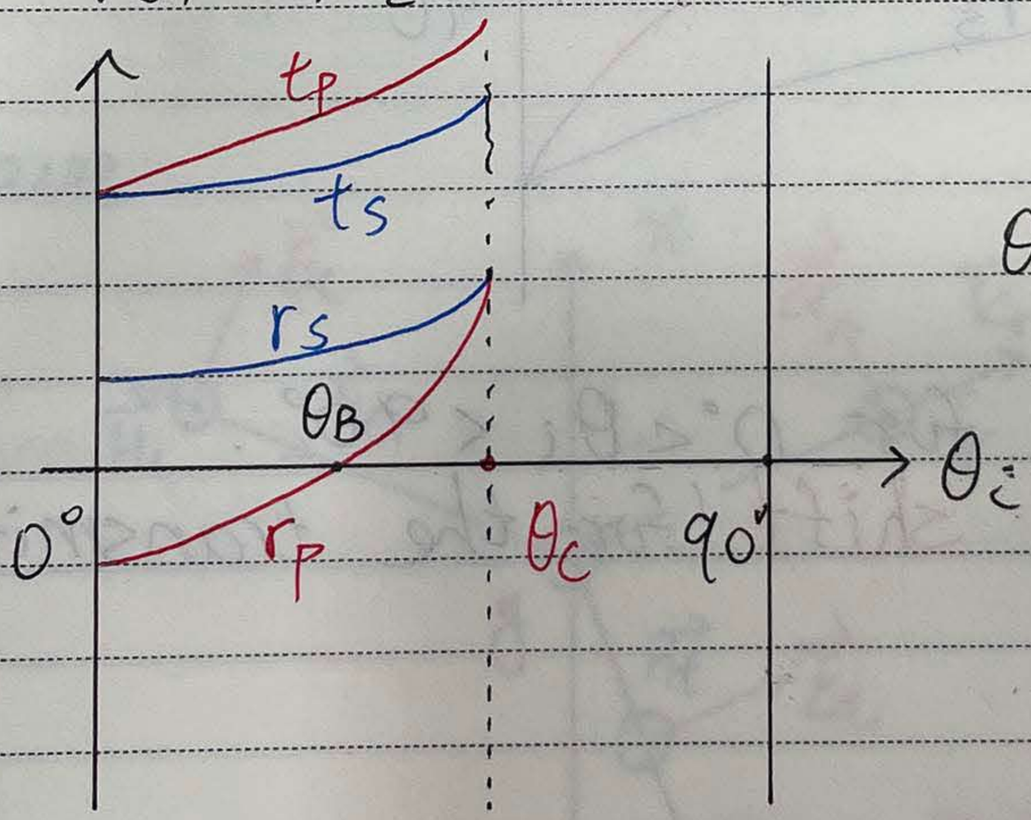
(4). $0 < \theta_i < 90°$

{ s-wave: $r_s < 0 \to \pi$-shift
p-wave: when $\theta_i = \theta_B$, $r_p = 0$

$\to \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$, 称、Brewster angle (or polarizatim angle)

→ 物理意義：当入射角 $= \theta_B$，反射光就只有 s-wave
→ 可製造 polarized 的光！

Case 2. $n_1 > n_2$



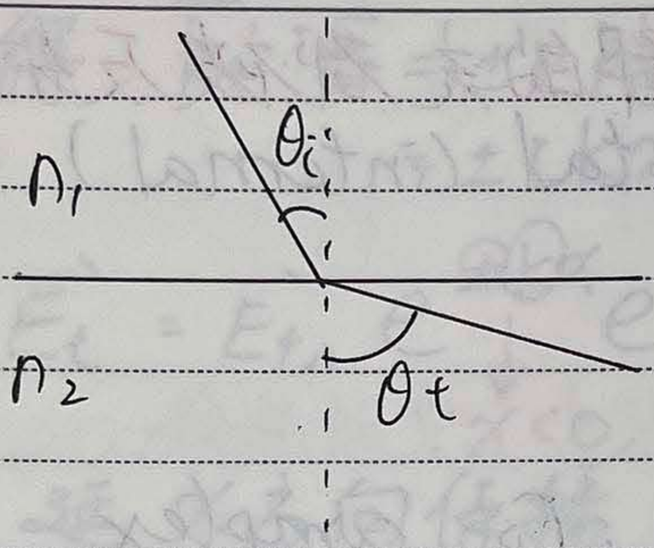
$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

(1) $\theta_i \to 0$

$\Rightarrow$ { $r_s > 0$
$r_p < 0$

(2) θ_c : critical angle : $n_1 \sin\theta_i = n_2 \sin\theta_t$

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$\because n_1 > n_2 \quad \therefore \theta_t > \theta_i$

$\Rightarrow$ 到某一程度，$\theta_i = \theta_c$，$\theta_t = \frac{\pi}{2}$

$\Rightarrow$ 若 $\theta_i > \theta_c$，則 no refraction!，且

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

比如：$n_1 = 1.5$，$n_2 = 1$，則 $\theta_c \approx 42°$

(3). $\theta_i > \theta_c$ 時，θ_t 已無物理意義

$(\because \sin\theta_t = \frac{n_1}{n_2}\sin\theta_i > 1)$

但若要硬求，由

$$\cos\theta_t = \pm\sqrt{1-\sin^2\theta_t}$$

$$= \pm i\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1}$$

代回 r_s, r_p，得

$$\begin{cases} r_s = \cdots = \dfrac{a+ib}{a-ib} = |r_s|e^{i\delta_s} \\ r_p = \cdots = \dfrac{c+id}{c-id} = |r_p|e^{i\delta_p} \end{cases}$$

for some a, b, c, d.

Date. Page.

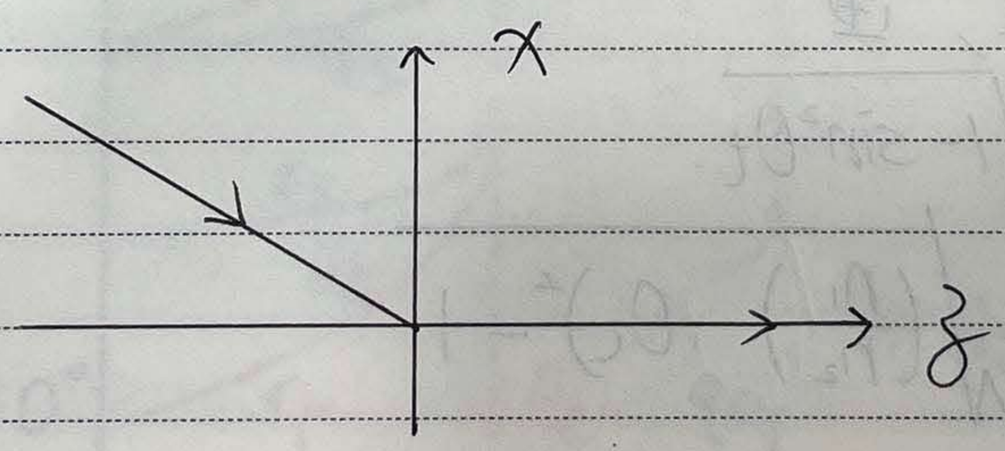
故 $|r_s| = |r_p| = 1$，表全部的光都被反射了！
→ 此現象稱為全反射 (total (internal) reflection, TIR)

(4). When $\theta_i > \theta_c$, ∵ r_s, r_p complex
∴ 有 - phase - shift δ between s.p - waves

If, $\delta = \frac{\pi}{2}$ → the reflected wave is circular polarization

$\theta_i > \theta_c$, transmitted wave = ?

∵ by B.C., $E_t \neq 0$
∴ transmitted wave $\neq 0$!!
→ 原有一个 evanescent wave (倏逝波), 在介面表面傳播



令此波為 $\vec{E_t} = \vec{E_{to}} e^{i(\vec{k_t}\cdot\vec{r} - \omega t)}$, 則

$\vec{k_t}\cdot\vec{r} = k_t x \cos\theta_t + k_t z \sin\theta_t \quad (x<0)$

代入 $\begin{cases} \sin\theta_t = \frac{n_1}{n_2}\sin\theta_1, \\ \cos\theta_t = \pm i\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1} \end{cases}$

Date. Page.

得 $\vec{k_t}\cdot\vec{r} = \pm i k_i \sqrt{\sin^2\theta_i - \sin^2\theta_c}\, x + k_i \zeta \sin\theta_i$

$= \pm i k x + k_i \zeta \sin\theta_i$

$\Rightarrow \vec{E_t} = \vec{E_{t0}}\, e^{\pm kx}\, e^{i(k_i \zeta \sin\theta_i - \omega t)}$

$\because x<0$, k 取正

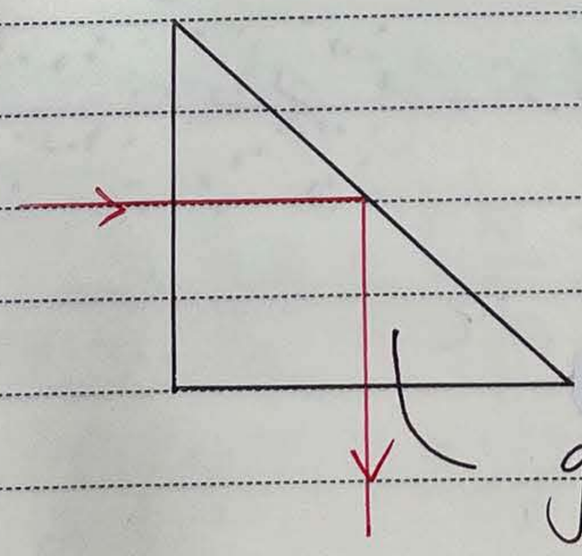
⇒ 延 x 方向传播（介面），amplitude 随着 x 越来越弱.

At $Kx=1$, amplitude $\sim e^{-1}=0.37$, intensity $\sim e^{-2}\approx 0.1$

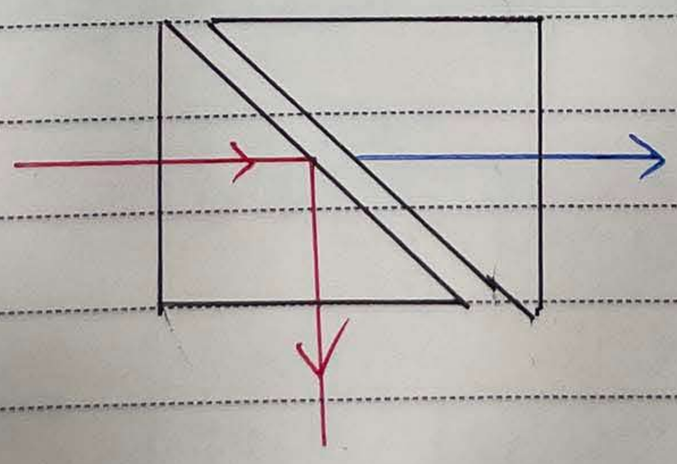
比如：$n_1=1.5$, $n_2=1$, $\theta_i=45°$ 时，当 $|x|=\frac{1}{K}=0.68\lambda_i$, $\lambda_i=550\,nm$, 具大约大能侍到 $x=250\,nm$ 处.

又，$\langle S_t\rangle = \zeta A_0$，即电场延表面分布，不进入另一介质中.

例. 三棱镜.



应用：



Date. Page.

⇒ 取三极镜如此摆放，只要靠够近，
就有来自Ee之光透过去！
⇒ 分光镜！

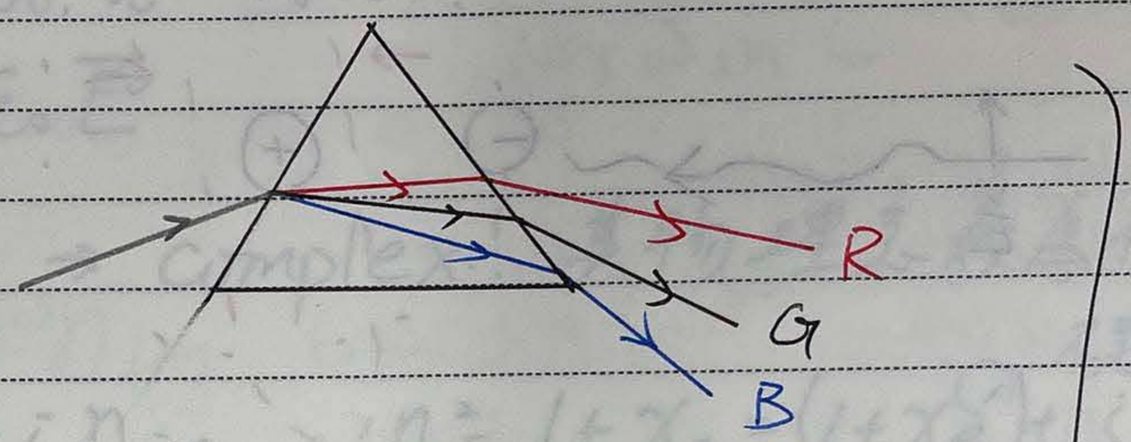
* $|t|^2+|r|^2\neq 1$，但此非能量，故没关你。

Leeture 4. Dispersion, Absorptiun, Scattering

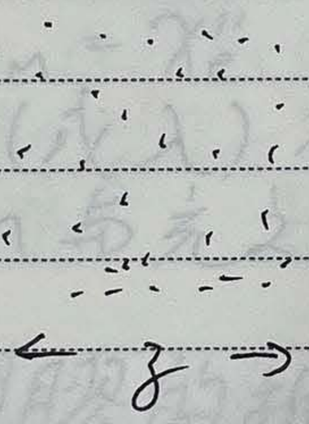
Date. Page.

§1. 名詞解釋 (3-9)

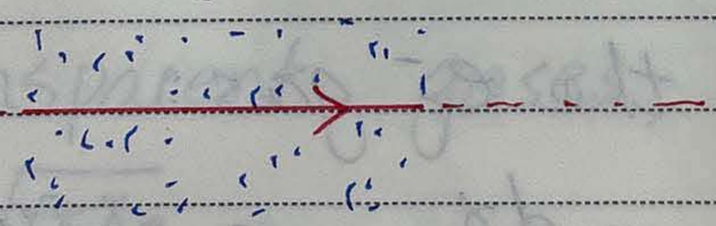
· Dispersion



· Absorptiun



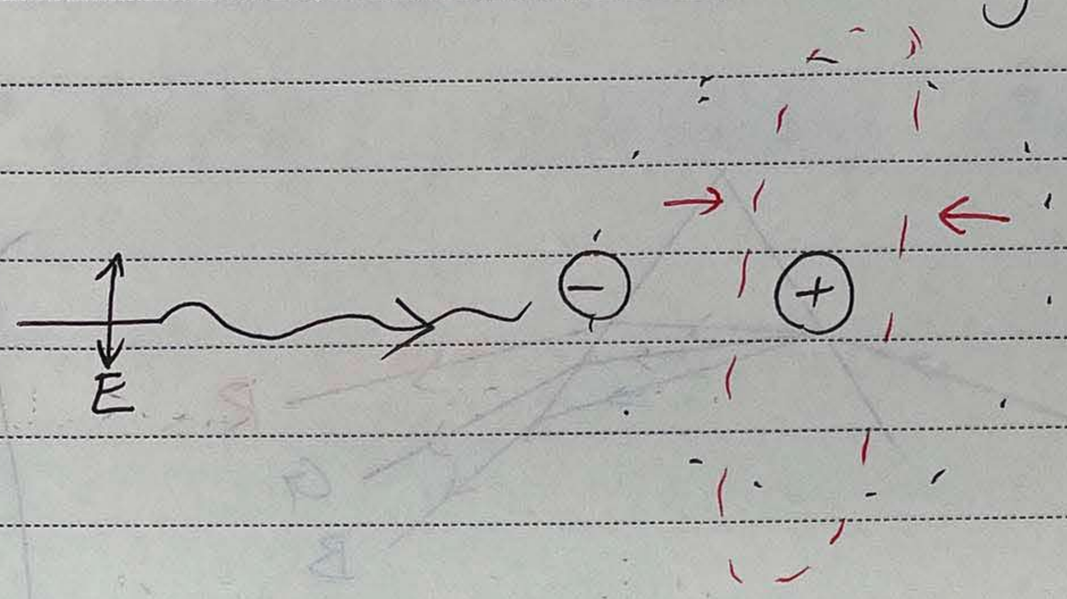
· Scattering



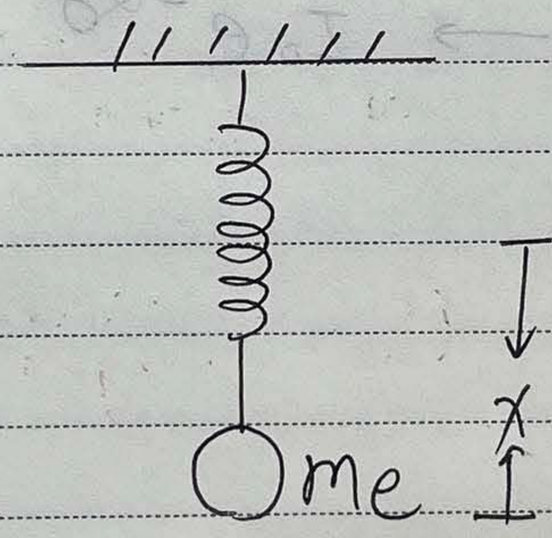
Interaction of light wave, matter, atoms, molecules, $\vec{E}$, $\vec{H}$

Date. §2. Page. Classical Electron Gas Model (3-9)

Matters: Atoms, molecules, ...
~ nucleus + electron gas



⇒ 由於 E 之振盪, e^- 如同被掛在一彈簧上, 使軌道會變形, 振盪.



⇒ Forced motion of the e^- gas:

$$m_e \frac{d^2x}{dt^2} = -k_e x - m_e \Gamma \frac{dx}{dt} + eE_0 e^{-i\omega t}$$

$k_e = m_e \omega_0^2$ (restoring force), $m_e \Gamma$ (damping)

令 $x = x_0 e^{i\omega t}$ 代入可求解. 得

$$x_0 = \frac{\frac{e}{m} E_0}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

Date. Page.

∵ polarization (dipole moment) $\vec{p} = e\vec{x}$

$$\therefore \vec{P} = N\vec{p} = \frac{Ne^2\vec{E}}{m_e(\omega_0^2 - \omega^2 - i\gamma\omega)} \triangleq \chi_e \varepsilon_0 \vec{E}$$

$\Rightarrow n = \sqrt{1+\chi_e}$ → complex! 其物理意義為何？

Let $n = n_r + i n_{im}$, $n^2 = 1 + \chi_e = (1+\chi_e') + i\chi_e''$ 不是微分！

$\Rightarrow (n_r^2 - n_{im}^2) + 2i n_r n_{im} = 1 + \chi_e' + i\chi_e''$

$\Rightarrow \begin{cases} n_r^2 - n_{im}^2 = 1 + \chi_e' \\ 2 n_r n_{im} = \chi_e'' \end{cases}$ 可解 n_r, n_{im}

其中 n 與 ω (或 λ) 有關 → dispersion relation

⇒ dispersion 與光之頻率 (波長) 有關！

* ① 玻璃、塑膠的折射率都是 complex number.

② 透明材料之 $n_{im} = 0$，代表特定波長之光可以通過去

→ n 其實是 $n(\lambda)$

For transparent gases, $|\chi_e| \ll 1$

$$\Rightarrow n_r = \sqrt{1+\chi_e} \simeq 1 + \frac{1}{2}\chi_e = 1 + \frac{1}{2}(\chi_e' + i\chi_e'') = \left(1 + \frac{1}{2}\chi_e'\right) + i\left(\frac{1}{2}\chi_e''\right)$$

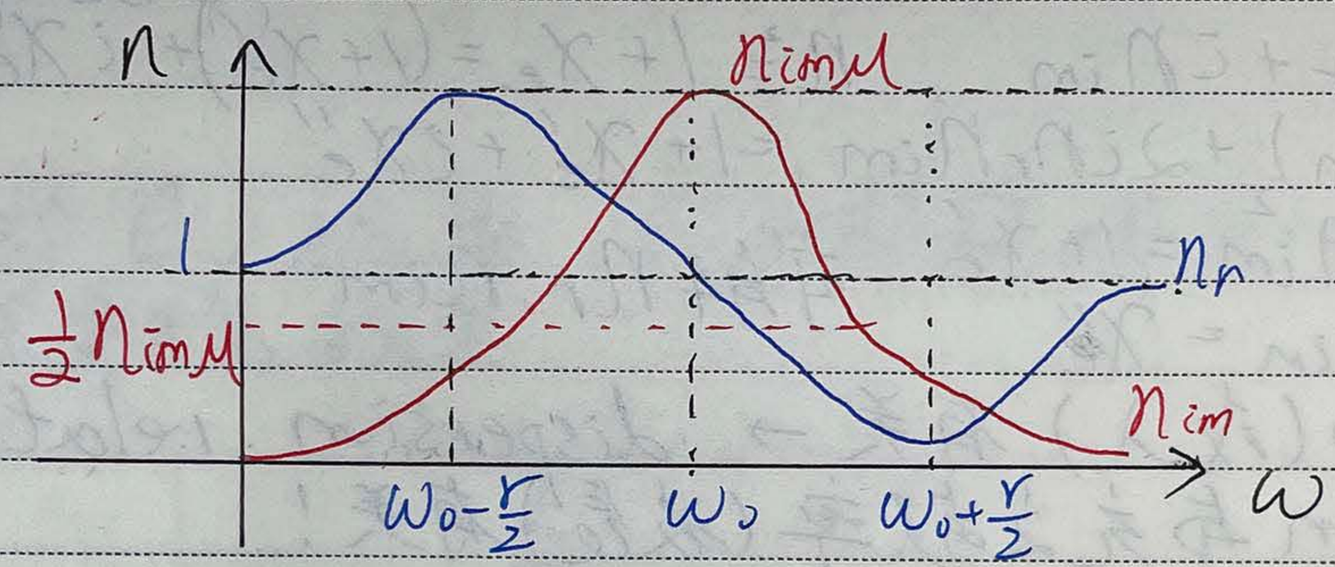
$$\Rightarrow \begin{cases} n_r = 1 + \frac{1}{2}\frac{Ne^2}{m_e\varepsilon_0}\frac{\omega_0^2 - \omega^2}{(\omega_0^2-\omega^2)^2 + (\gamma\omega)^2} \\ n_{im} = \frac{1}{2}\frac{Ne^2}{m_e\varepsilon_0}\frac{\gamma\omega}{(\omega_0^2-\omega^2)+(\gamma\omega)^2} \end{cases}$$

Date. Page.

(1) $\omega = \omega_0$ (characteristic frequency of the matter)

$$\rightarrow \begin{cases} n_r = 1 \\ n_{im} = \frac{1}{2} \frac{Ne^2}{m\varepsilon_0 \gamma \omega_0} \end{cases}$$

(2) $\begin{cases} \omega >> \omega_0 \\ \omega << \omega_0 \end{cases} \Rightarrow \begin{cases} n_r \rightarrow 1 \\ n_{im} \rightarrow 0 \end{cases}$



(3) $\frac{dn_r}{d\omega} = K\left((\omega_0^2 - \omega^2)^2 - (\gamma\omega)^2\right)$

$$\rightarrow \left.\frac{dn_r}{d\omega}\right|_{\omega=\omega_0} < 0$$

(4) When $\frac{dn_r}{d\omega} = 0 \rightarrow \omega \approx \omega_0 \pm \frac{\gamma}{2}$

For n_{im},

(1) $\omega = \omega_0$, $n \in \mathbb{C}$, $\frac{dn_r}{d\omega} < 0$, $\frac{dn_r}{d\lambda} > 0$

→ anomalous dispersion (反常色散)

Date. Page.

⇒ when n complex, $\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$, where

$$\begin{cases} k = \frac{\omega}{c} n = \frac{\omega}{c}(n_r + i n_{im}) \\ \vec{E} = \vec{E}_0 e^{-\frac{\omega}{c} n_{im} z} e^{i(kz-\omega t)} \\ I = I_0 e^{-\frac{2\omega}{c} n_{im} z} \equiv I_0 e^{-\alpha z} \quad \text{(Lambert law)} \end{cases},$$

where $\alpha = \frac{2\omega}{c} n_{im}$ (absorption coefficient)

* Optical materials → transparent to visible light

ex. Glass: $\omega_0 \sim \omega$

⇒ visible λ, $n_{im} \to 0$ (transparent),
$n \approx n_r$

(2) For $\omega < \omega_0 - \frac{\Gamma}{2}$, $\frac{dn_r}{d\omega} > 0$, $\frac{dn}{d\lambda} < 0$

→ Normal dispersion (正常色散)

→ $n_r \approx 1 + \frac{1}{2} \frac{Ne^2}{m_e \varepsilon_0} \frac{1}{\omega_0^2 - \omega^2}$ $(\Gamma = 0)$

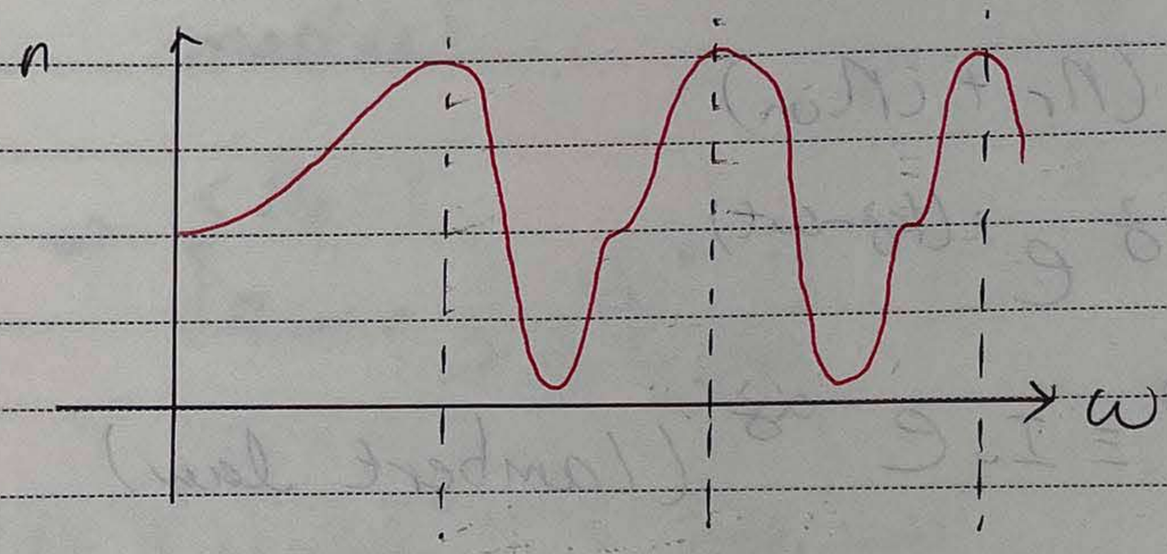
$= (1+A) + \frac{B}{\lambda^2}$

Cauchy dispersion formula: $n = C_1 + \frac{C_2}{\lambda^2} + \frac{C_3}{\lambda^4} + \cdots$

(实验得出的经验公式)

Date. Page.

* 一个材料也可以有很多共振频率



* Solid liquids

$$\frac{n^2-1}{n^2+2} = \frac{1}{3}\frac{N\alpha}{\varepsilon_0}$$

§3 Optical Property of Metals

Date. Page (3-9)

$$\because \begin{cases} \vec{J} = \sigma \vec{E} \\ \nabla \times \vec{E} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} \end{cases}$$

$$\therefore \nabla^2 \vec{E} = \mu_0 \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} + \mu_0 \sigma \frac{\partial \vec{E}}{\partial t}$$

Let $\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$ 代入可求解

$\Rightarrow I(z) = I_0 e^{-\alpha z}$, where $\alpha = \frac{2\omega}{c} n_{im}$

① α 称为 penetration depth (skin depth)

② $z_\alpha \triangleq \frac{1}{\alpha} = \frac{c}{2\omega n_{im}} = \frac{\lambda}{2\pi n_{im}}$

ex. $\begin{cases} Al: n_{im} = 6.8 \\ Ag: n_{im} = 3.45 \end{cases}$, $\lambda = 560\,nm$ → 金属增亮性差!

Good conductors ($\sigma \to \infty$) : free electrons dominate the interaction between light and matter

$$\Rightarrow n^2 = 1 - \frac{N e^2 f_e}{m_e \varepsilon_0 \omega^2} = 1 - \left(\frac{\omega_p}{\omega}\right)^2, \quad \omega >> \omega_p,\ n \simeq 1$$

⇒ X光才穿得过去

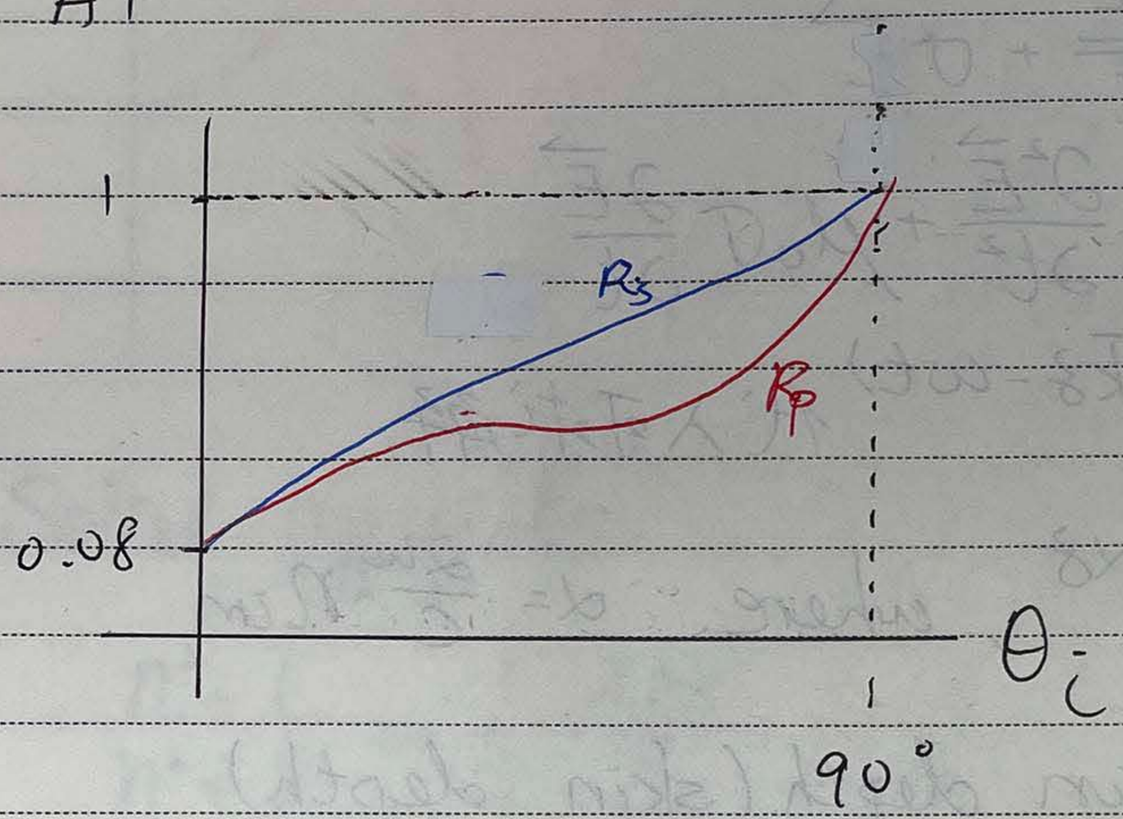
$\omega < \omega_p \to n \simeq i n_{im}$

$$\Rightarrow \begin{cases} r_s = \dfrac{a - ib}{a + ib} \\ r_p = \dfrac{a_1 - ib_1}{a_1 + ib_1} \end{cases}$$ for some a, b, a_1, b_1.

Date. Page.

$\Rightarrow |r_s| = |r_p| = 1$

ex. Al



(定義 $R = \left|\frac{n-1}{n+1}\right|^2$)

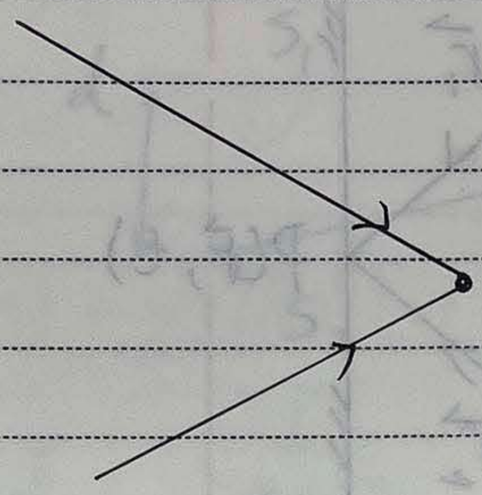
∵ 所有入都反射

∴ 理論上金屬沒有顏色!

Lecture 5 Interference 干涉

Date. Page.

§1 Introduction (7-3)



Consider two monochromatic plane waves

$$\begin{cases} \vec{E}_1(r,t) = \vec{E}_{01} e^{i(\vec{k}_1 \cdot \vec{r}_1 - \omega_1 t + \phi_1)} \\ \vec{E}_2(r,t) = \vec{E}_{02} e^{i(\vec{k}_2 \cdot \vec{r}_2 - \omega_2 t + \phi_2)} \end{cases}$$

At P $\vec{E} = \vec{E}_1 + \vec{E}_2$.

Optical detection: $I = \langle |\vec{S}| \rangle_T = \frac{1}{2} c\varepsilon_0 \langle |\vec{E}|^2 \rangle_T$

$$\Rightarrow I = \langle \vec{E} \cdot \vec{E}^* \rangle_T$$
$$= E_{01}^2 + E_{02}^2 + 2\vec{E}_{01} \cdot \vec{E}_{02} \langle \cos(\delta_2 - \delta_1) \rangle_T$$

To observe interference:

$$\begin{cases} \vec{E}_{01} \cdot \vec{E}_{02} \neq 0 \\ \langle \cos(\delta_2 - \delta_1) \rangle_T = \frac{1}{T} \int_0^T \cos(\delta_2 - \delta_1)\, dt \neq 0 \end{cases}$$

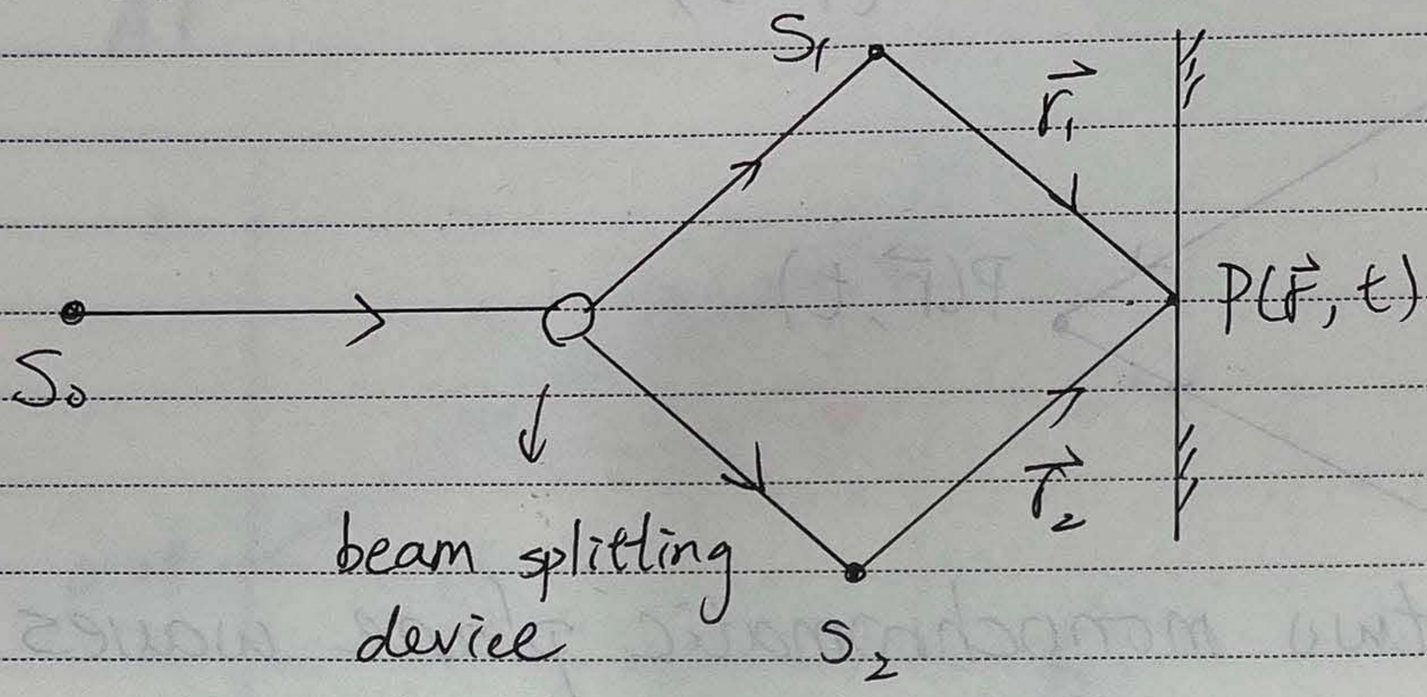
$$\Rightarrow \delta_2 - \delta_1 = \vec{k}_2 \cdot \vec{r}_2 - \vec{k}_1 \cdot \vec{r}_1 - (\omega_2 - \omega_1)t + (\phi_2 - \phi_1)$$

$$\Rightarrow \omega_2 - \omega_1 = 0$$

当 initial phase = const 時，称 coherent light source (同調，相干)

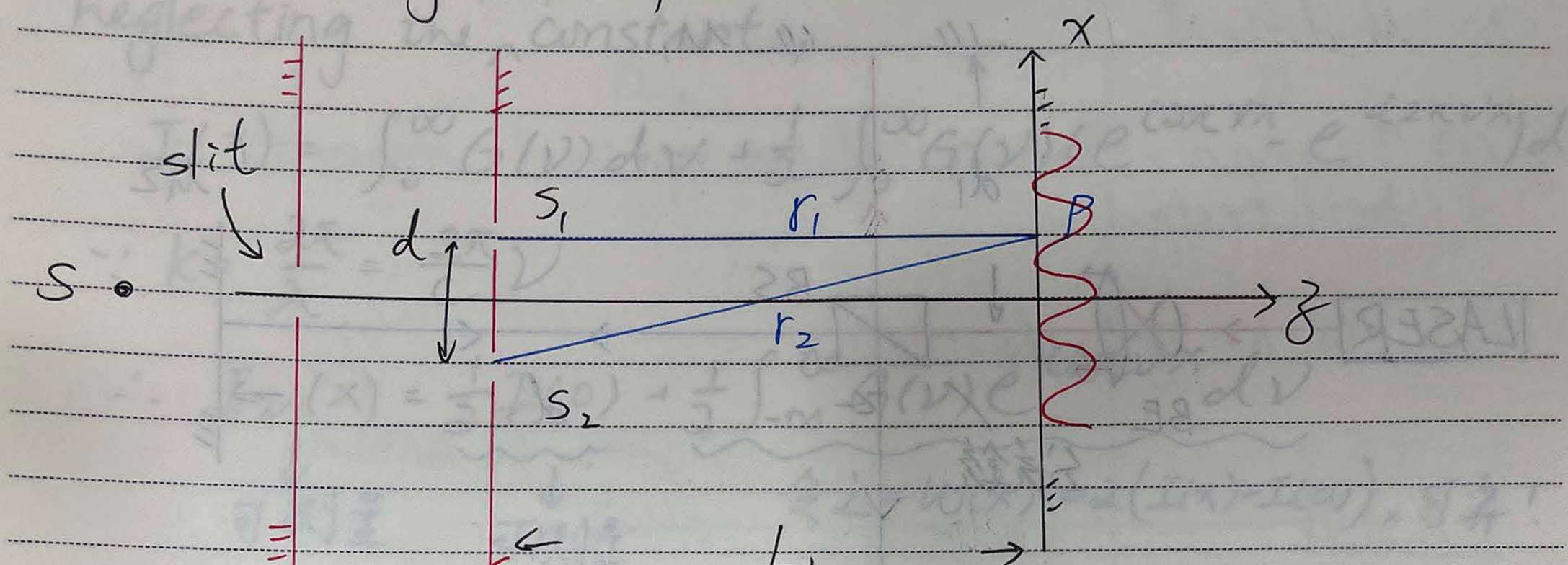
Date. Page.

* 現在沒辦法得到 coherent light，只能用分光的方式的技術



§2. Younge's Experiment

Date (7-3) Page.



光程差 $\delta = k(Y_2 - Y_1)$, slit width $\ll d \ll L$

$\Rightarrow r_2 - r_1 \approx d\frac{x}{L}$

$\Rightarrow I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{2\pi}{\lambda}\frac{dx}{L}\right)$

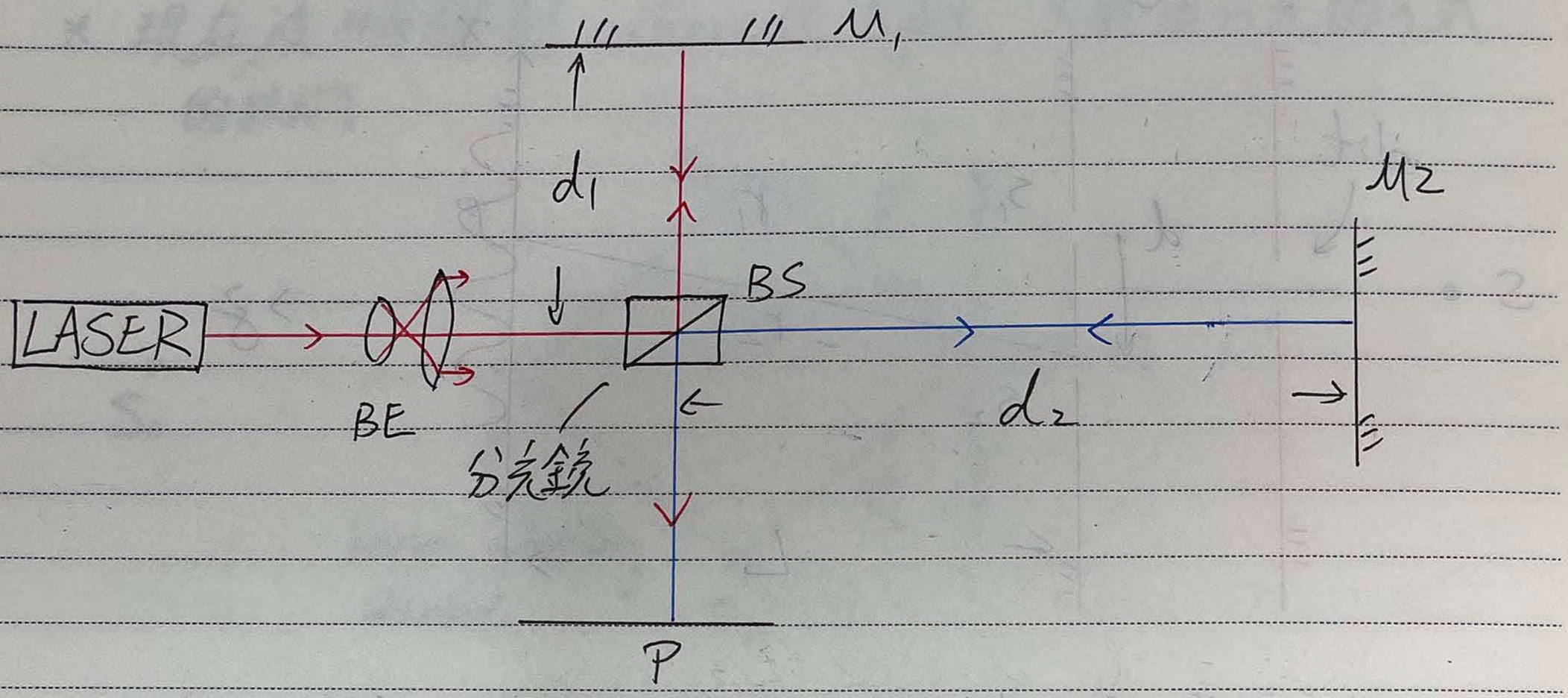
① When $\frac{2\pi}{\lambda}\frac{dx}{L} = m \cdot 2\pi$, $m \in \mathbb{Z}$

$\Rightarrow$ bright fringe

" $(2m+1)\pi$, "

$\Rightarrow$ dark "

Date. § 3 Page. Michelson interferometer (7-12)



$\delta = k(d_2 - d_1) = 2kx$

1. If total phase change = $N \cdot 2\pi = 2kx$ for displacement x (N: fringes)

$\Rightarrow x = N \cdot \frac{\lambda}{2}$

2. Fourier transform spectroscopy $G(\nu)$

Intensity at P: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

Assume $I_1 = I_2 = I_0$, then $I \triangleq I_\nu = 2I_0(1+\cos(2kx))$

3. 用干涉儀可以求光源頻譜

If the power speetrum of the light source is $G(\nu)$

$\Rightarrow$ Total intensity at P = $I = \int_0^\infty I_\nu(x) G(\nu)\, d\nu$

Date. Page.

proportional
neglecting the constant,

$$I(x) = \int_0^\infty G(\nu)\,d\nu + \frac{1}{2}\int_0^\infty G(\nu)\left(e^{i2\pi\nu x} - e^{-i2\pi\nu x}\right)dx$$

$\because k = \frac{2\pi}{\lambda} = \frac{2\pi}{c}\nu$

$$\therefore I_2(x) = \frac{1}{2}I(0) + \frac{1}{2}\int_{-\infty}^{\infty} G(\nu)\,e^{i2\pi\nu x}\,d\nu$$

$I_2(x)$: 可以测量　$\frac{1}{2}I(0)$: 可以测量　$\int_{-\infty}^{\infty} G(\nu)e^{i2\pi\nu x}d\nu$: 令之 $= W(x) = 2\left(I(x) - I(0)\right)$, 可算!

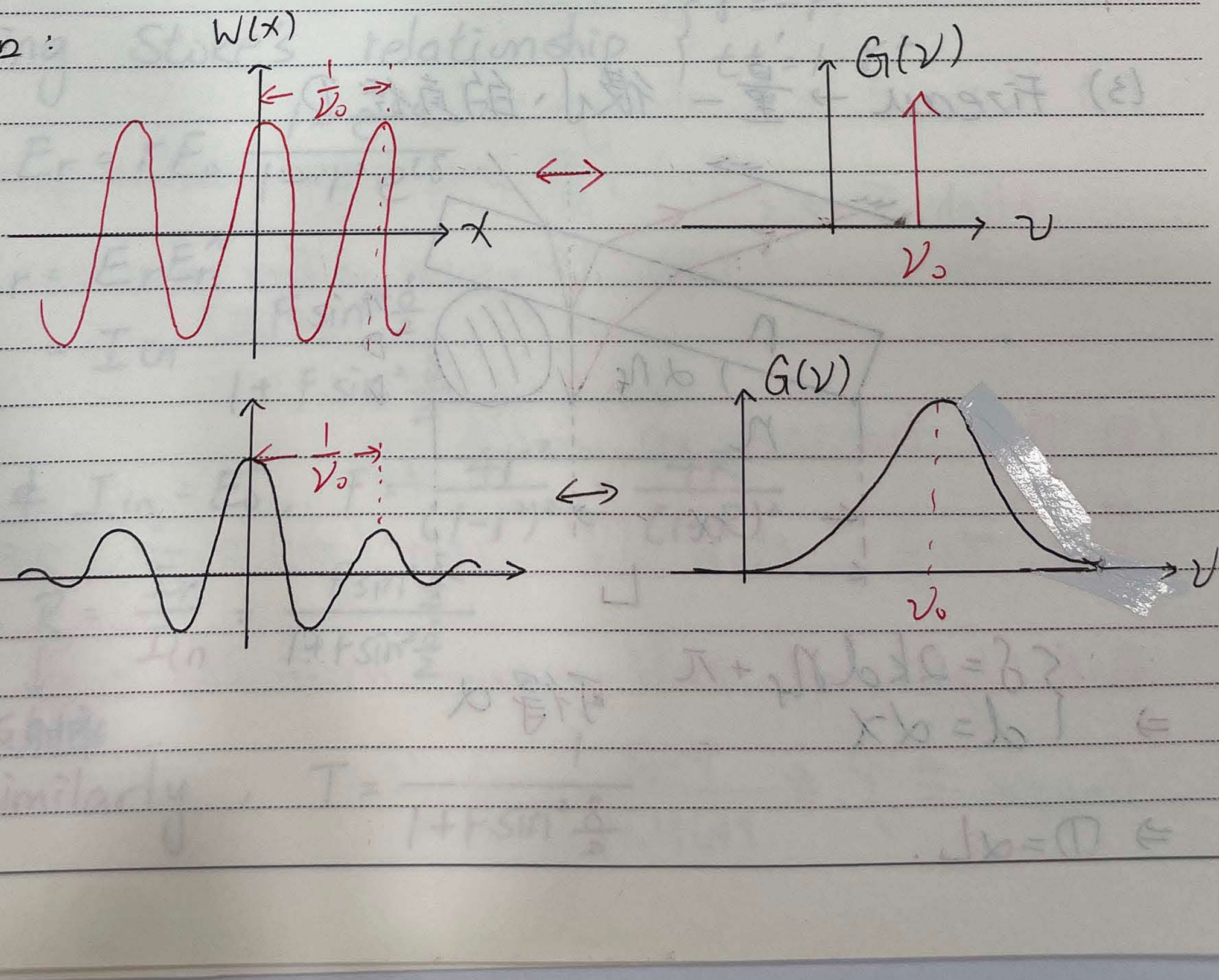
$\Rightarrow$ 只需量 d_1, d_2 处之强度!, 用

$$G(\nu) = \mathcal{F}^{-1}\{W(x)\}$$

物理系的 FT 改电机系不同

$$= \int_{-\infty}^{\infty} W(x)\,e^{-i2\pi\nu x}\,dx$$

比如:

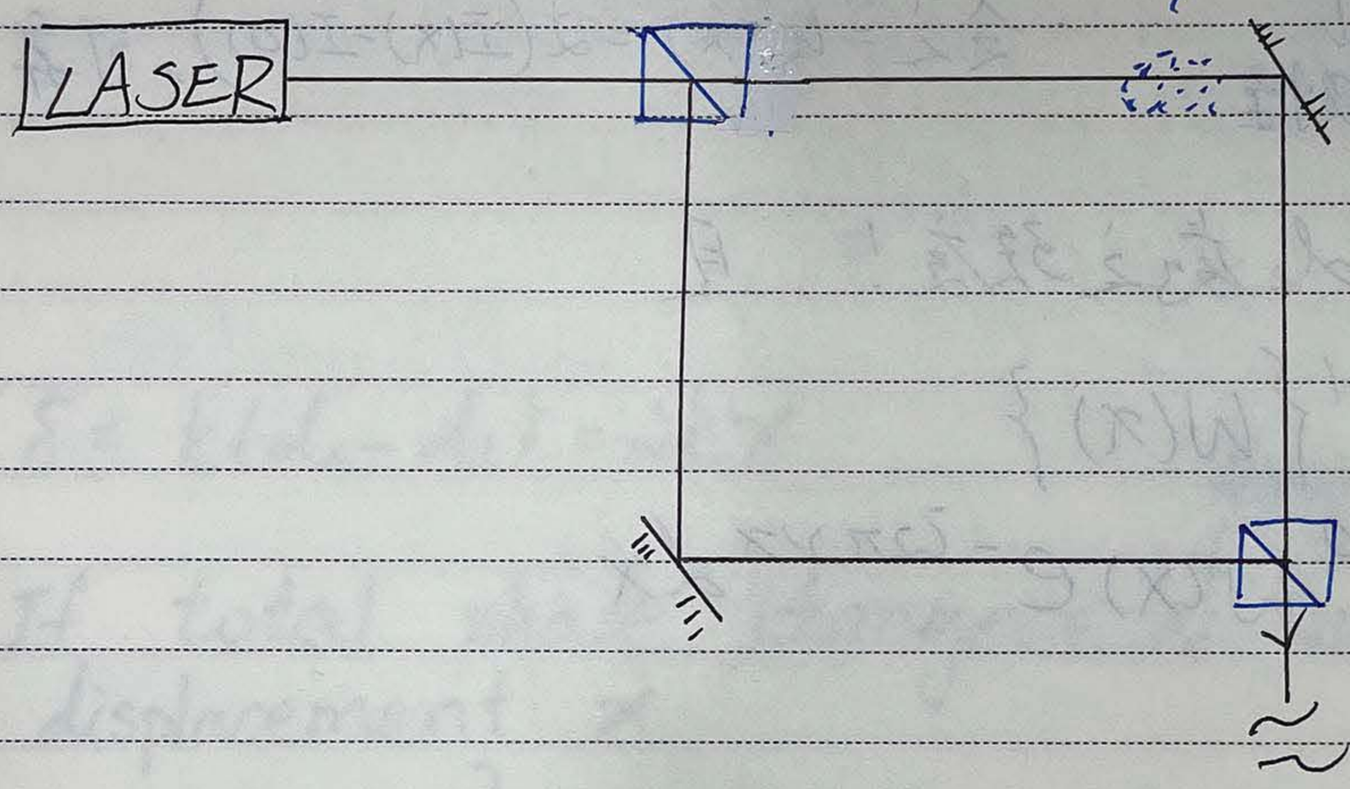


Date. Page.

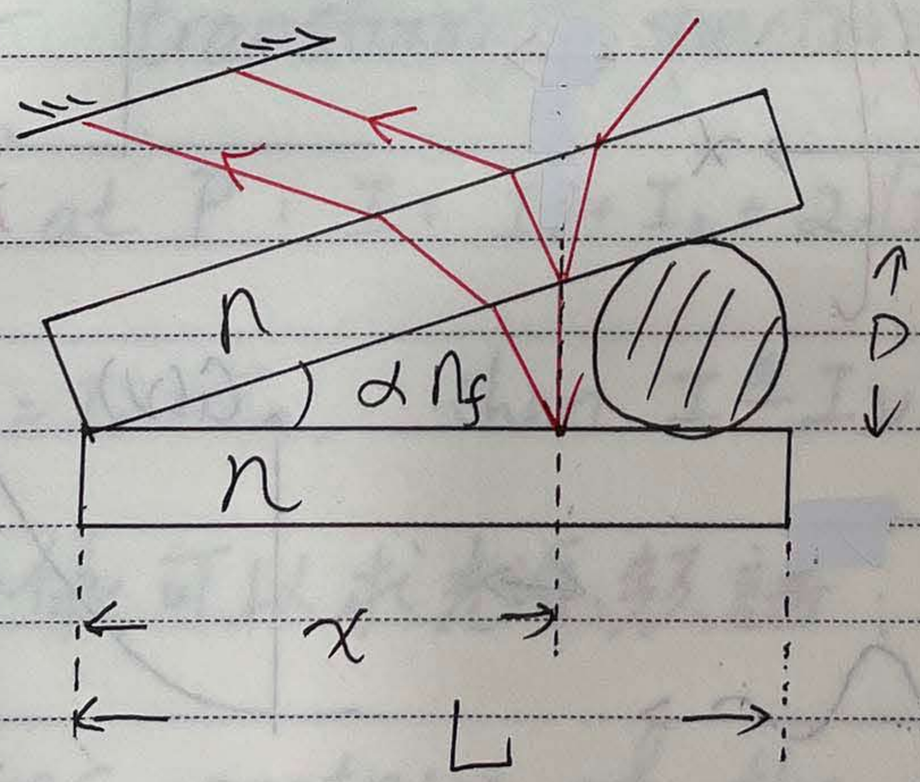
3. Modified

(1) Twyman-Green: M_2 → optical elements to be tested

(2) Mach-Zehnder



(3) Fizeau → 量一很小的直徑 D

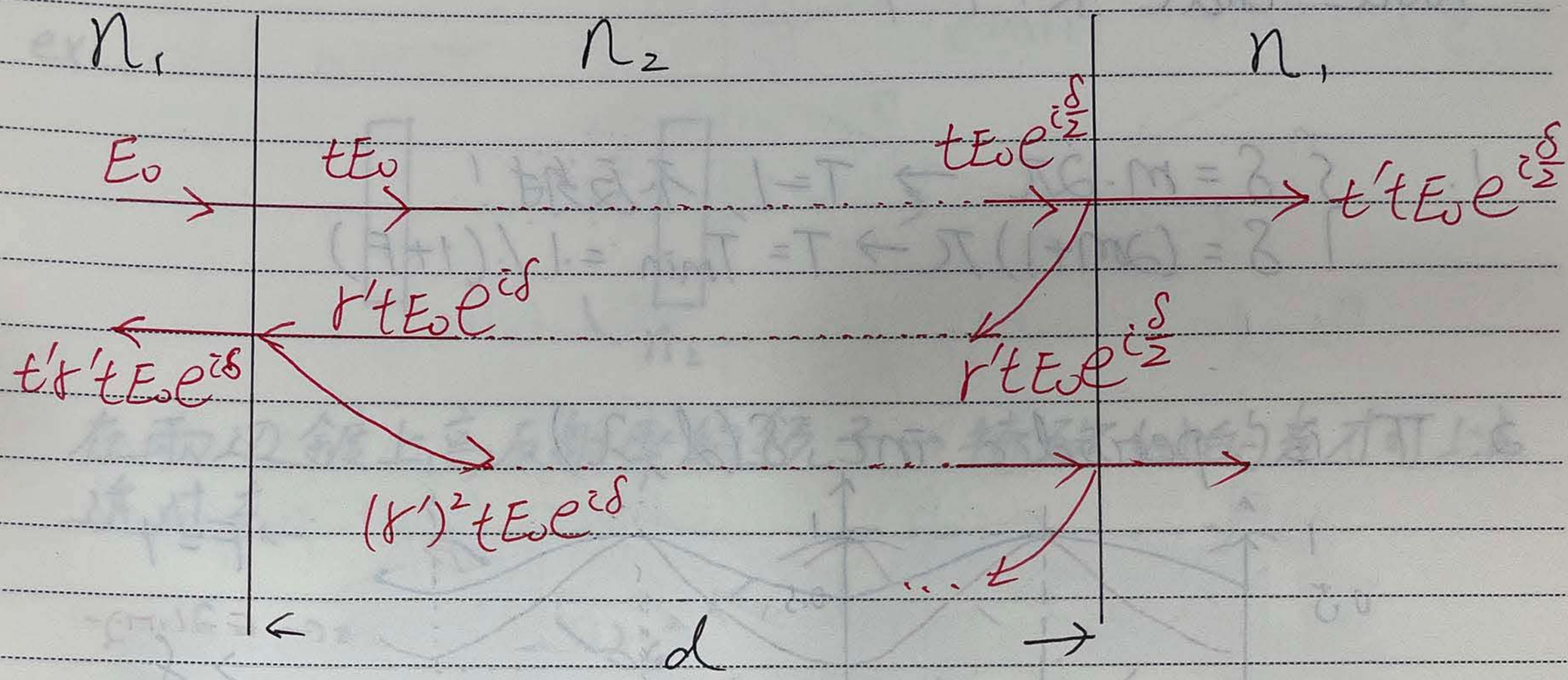


$$\Rightarrow \begin{cases} \delta = 2kd n_f + \pi \\ d = \alpha x \end{cases}$$ 可得 α

$\Rightarrow D = \alpha L$.

§4. Multiple-beam interferometer

Date. (7-10) Page.



Total reflected wave

$$E_r = rE_0 + tt'r'E_0e^{i\delta}\left(1+(r')^2e^{i\delta}+\left((r')^2e^{i\delta}\right)^2+\cdots\right)$$

using Stoke's relationship $\begin{cases} r'=-r \\ tt'=1-r^2 \end{cases}$

$$\Rightarrow E_r = rE_0\frac{1-e^{i\delta}}{1-r^2e^{i\delta}}$$

$$I_r = E_rE_r^* = I_{in}\frac{F\sin^2\frac{\delta}{2}}{1+F\sin^2\frac{\delta}{2}}$$

其中 $I_{in}=E_0^2$, $F=\frac{4r^2}{(1-r^2)^2}=\frac{4R}{(1-R)^2}$

$$\Rightarrow R=\frac{I_r}{I_{in}}=\frac{F\sin^2\frac{\delta}{2}}{1+F\sin^2\frac{\delta}{2}}$$

↓

反射率

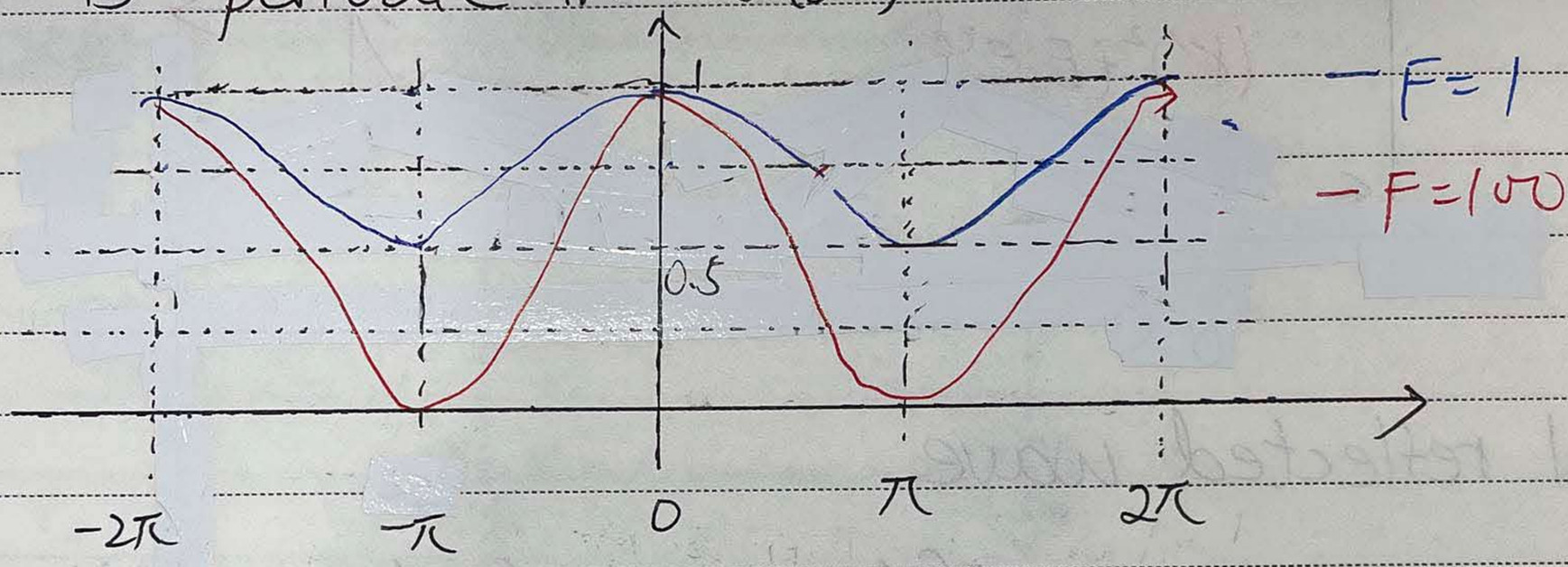
Similarly, $T=\frac{1}{1+F\sin^2\frac{\delta}{2}}$

Date. Page.

Note that $R+T=1$

1. $\begin{cases} \delta = m\cdot 2\pi \rightarrow T=1, \text{不反射!} \\ \delta = (2m+1)\pi \rightarrow T=T_{min}=1/(1+F) \end{cases}$

2. T is periodic in $\delta(d,\lambda)$



(1) $T=1$ if $\delta = m\cdot 2\pi$

(2) Low $F \rightarrow$ T always $\rightarrow 1$

High $F \rightarrow \begin{cases} T=1, \delta = 2m\pi \rightarrow \text{transmission spectra} \\ T\rightarrow 0, \text{otherwise} \rightarrow \text{Free spectra range (FSR)} \end{cases}$

(3) The transmission width.

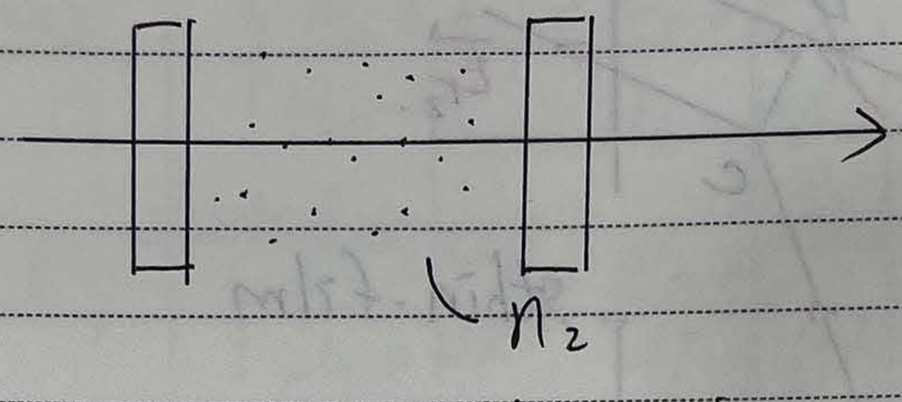
Let $T=\frac{1}{2}$, then $\delta_{\frac{1}{2}} = 2\sin^{-1}\left(\frac{1}{\sqrt{F}}\right) \approx \frac{2}{\sqrt{F}}$ $(\because F >> 1)$

Full Width at half-maximum $\triangleq 2\delta_{\frac{1}{2}} \approx \frac{4}{\sqrt{F}}$

finesse $\triangleq \mathcal{F} \triangleq \frac{FSR}{FWHM} = \frac{\pi}{2}\sqrt{F}$

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ex.



在两边镀上高反射率的镜子 → 特殊 W 的光才可以透过去.

$\because \delta = 2\pi m = k \cdot 2d$

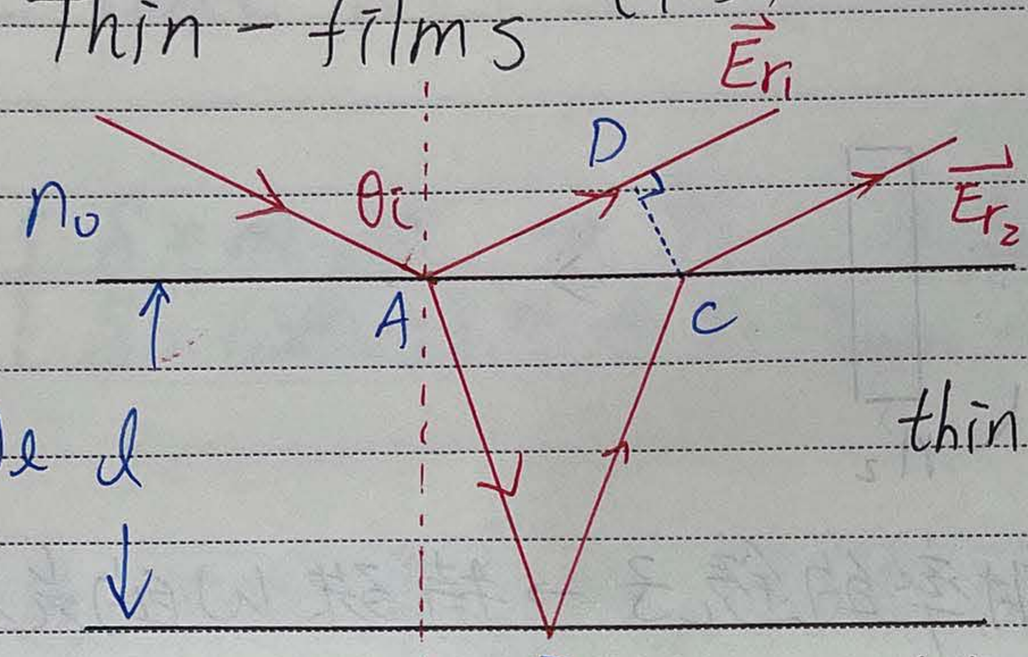
$\therefore \lambda_m = \frac{2n_2 d}{m}$ = 透出来的光的波长

⇒ 干涉可以镀高反射率的物质透出特定频率的光!

Lecture 6. Thin-Film Interference

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§1 Thin-films (7-3)

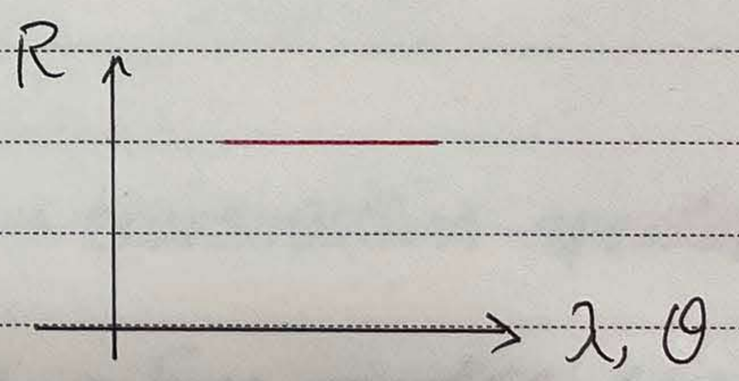


$$R = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2, \quad F = \frac{4R}{(1-R)^2}$$

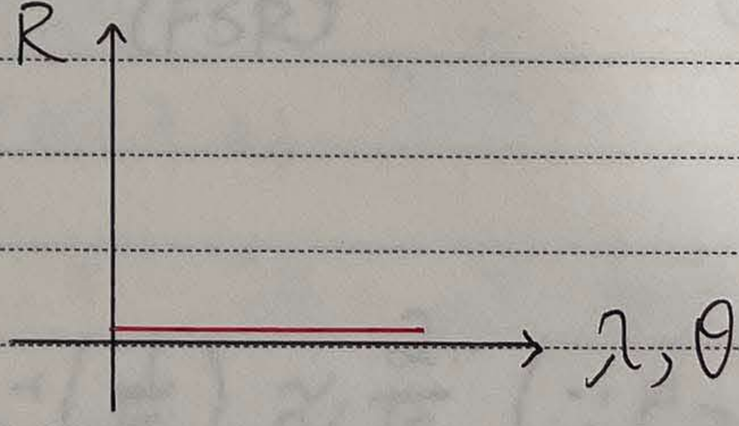
理論上來說会有多次反射 ($\vec{E}_{r_1}$, $\vec{E}_{r_2}$, $\vec{E}_{r_3}$, ...)
但取 E_{r_1}、E_{r_2} 已足夠逼近.

由 substrate 上鍍一層 thin-film 之方法可做出類似 filter 的東西

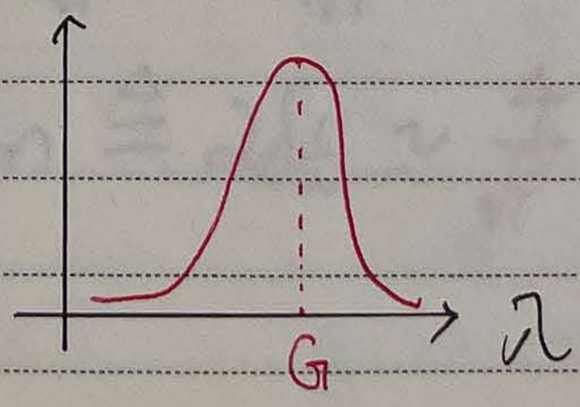
hi-R



Low-R

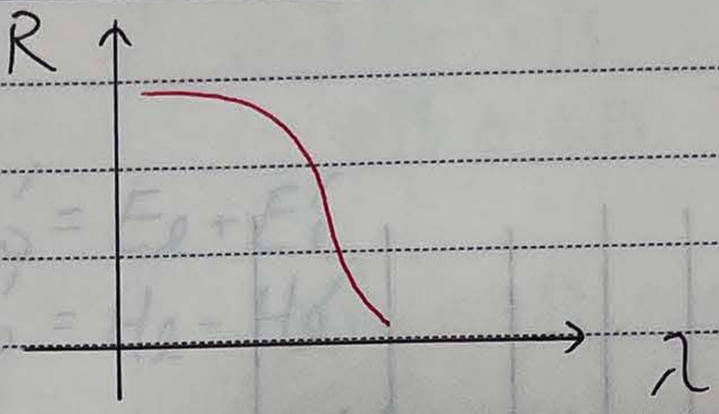


color filter

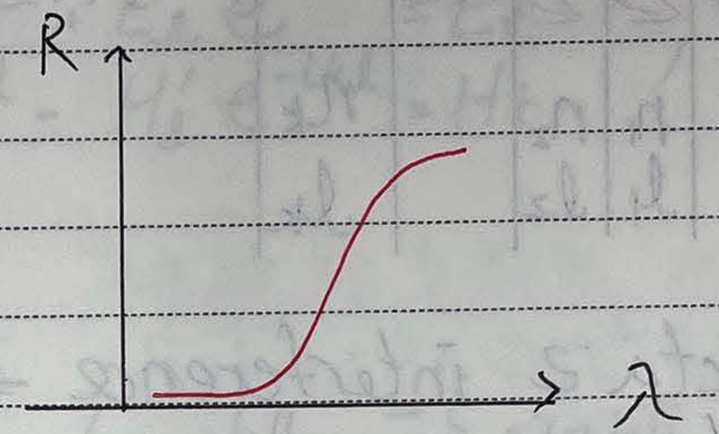


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cold-mirror



hot-mirror



Interference between the reflactions of the boundaries

$I_r = |E_{r_1} + E_{r_2}|^2 = I_{r_1} + I_{r_2} + 2\sqrt{I_{r_1} I_{r_2}} \cos\delta$, where

$\delta = k \cdot (\overline{AB} + \overline{BC}) n_e - k\overline{AD} + \delta$ reflections

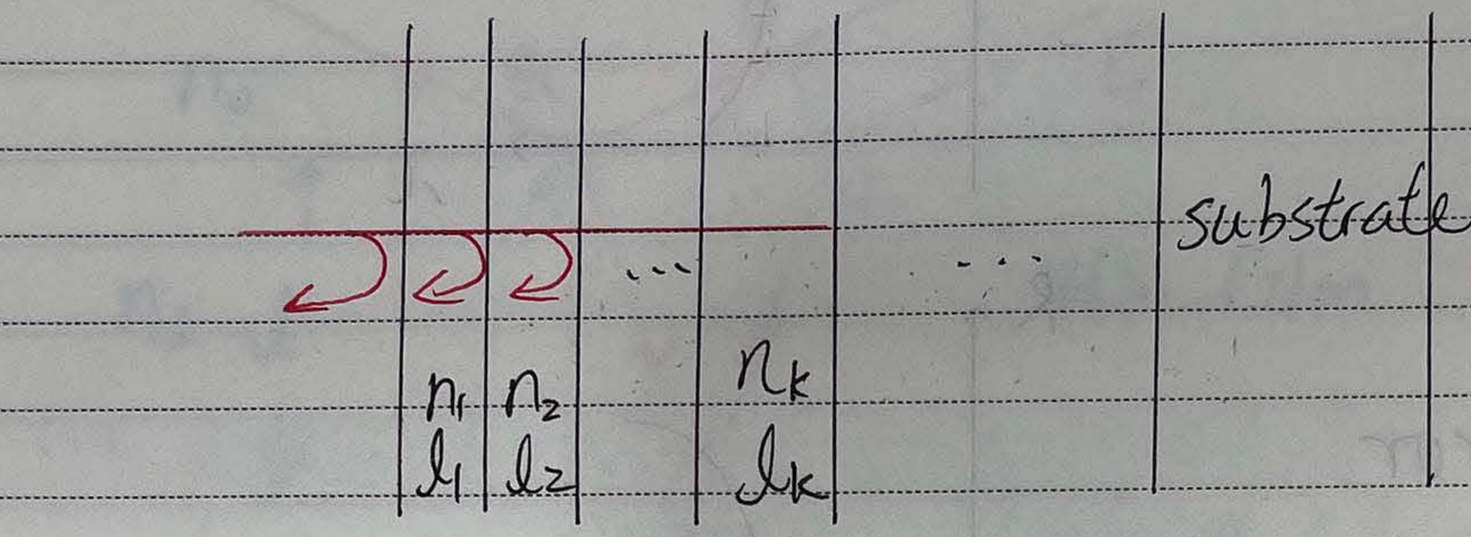
$$\Rightarrow \begin{cases} \delta = \pi \rightarrow I_r = I_{min} \\ \delta = 2\pi \rightarrow I_r = I_{max} \end{cases}$$

$\Rightarrow$ Given n_e, n_s, n_o, l, 則 I_{min}, I_{max} 可知

$\Rightarrow$ 可算 R

Date. Page. § 2 Multilayer films ('17-11)

1. Idea:

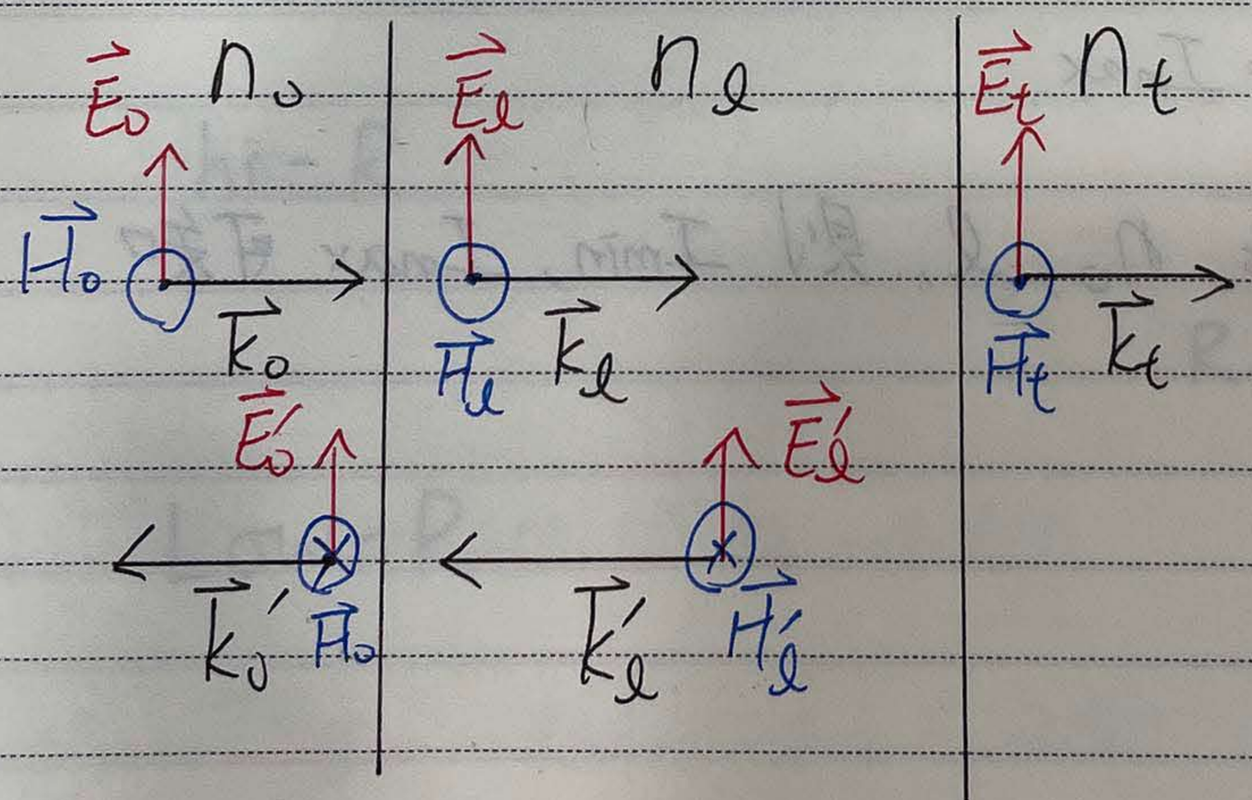


< constructive interference → hi-R
destructive " → low-R

2. Analysis.

(1) Summation of many reflected waves
$\delta = k\Delta + \phi_{reflections}$

(2) boundary conditions: consider 2 layers first



Given $\vec{E}_0$, n_1, l → find $\vec{E}_0$, $\vec{E}_t$.

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∵ B.C.s:

When $z=0$, $\begin{cases} E_0 + E_0' = E_1 + E_1' \\ H_0 - H_0' = H_1 - H_1' \end{cases}$

" $z=l$, $\begin{cases} E_1 e^{ikl} + E_1' e^{-ikl} = E_t \\ H_1 e^{ikl} - H_1' e^{-ikl} = H_t \end{cases}$,

where $k = \frac{2\pi}{\lambda} n_1$

$$\therefore \begin{pmatrix} 1 \\ n_0 \end{pmatrix} + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix} \frac{E_0'}{E_0} = \begin{pmatrix} \cos kl & \frac{-i}{n_1}\sin kl \\ -in_1 \sin kl & \cos kl \end{pmatrix} \begin{pmatrix} 1 \\ n_t \end{pmatrix} \frac{E_t}{E_0} \quad \text{—— (1)}$$

($\frac{E_0'}{E_0} = r$; the matrix $= M$; $\frac{E_t}{E_0} = t'$)

⇒ 推廣到 N layers:

$$\begin{pmatrix} 1 \\ n_0 \end{pmatrix} + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix} r = M_1 \cdots M_N \begin{pmatrix} 1 \\ n_t \end{pmatrix} t \quad \text{—— (2)}$$

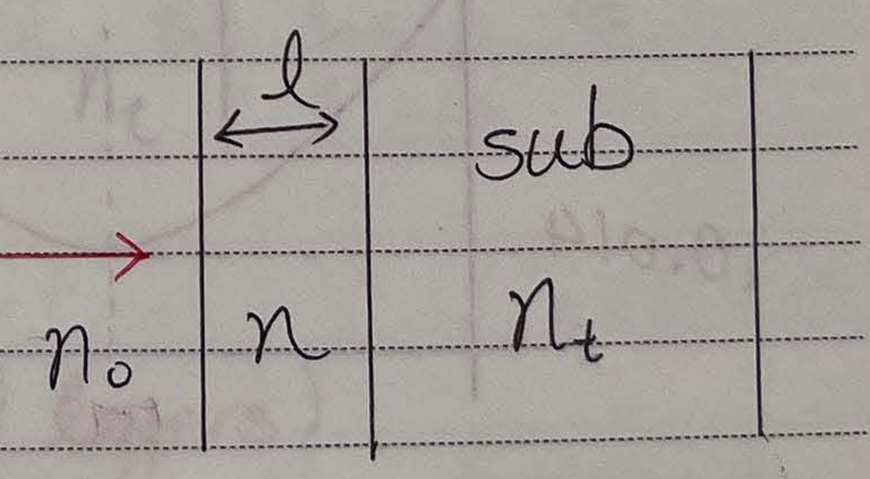
→ 解 r, t, 得垂直入射之 $R = |r|^2$, $T = |t|^2$.

Case I: Single layer

$\therefore kl = \left(\frac{2\pi}{\lambda} n_1\right) \cdot l$

$\therefore$ when $kl = \frac{\pi}{2}$, then (→ 破坏型干涉 (不反光))

$$M = \begin{pmatrix} 0 & \frac{-i}{n_1} \\ -in_1 & 0 \end{pmatrix}$$



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$$\Rightarrow r = \frac{n_t^2 - n_l^2}{n_t + n_l}$$

When $r=0$, $n_l = \sqrt{n_t}$.

比如：取 $n_0=1$, $n_t=1.5$ (glass)，则欲使 $r=0$（不反射），应取 $n_l=\sqrt{1.5}\approx 1.22$ 之材料

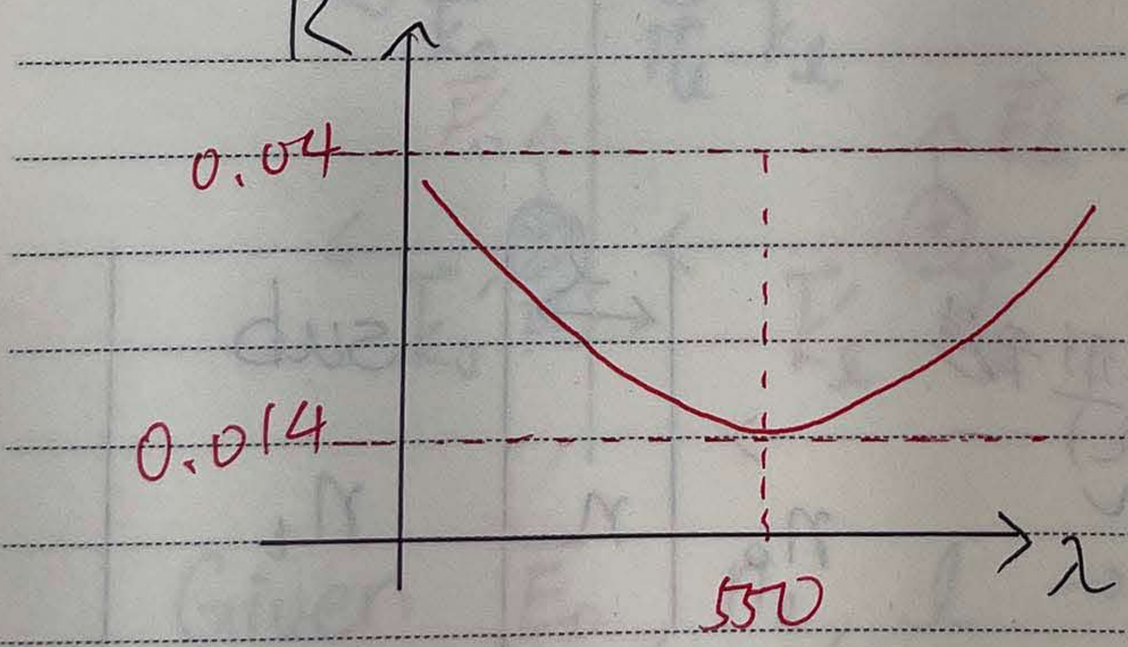
* 1. 除 n_l 外，应考虑该材料
① 是否能黏在玻璃上？
② " 加工？
③ " 会吸光？

2. 若一没有 $n_l=1.22$ 之材料，只能找比较接近的来加工，比如 MgF ($n_l=1.38$)
→ 所做出之 $|R|=|r|^2=0.014$，虽不为 0，但能从玻璃的 0.04 降到 0.014 已经很好了.

thin-film 要多厚

→ $\because kl = \frac{2\pi}{\lambda} n_l = \frac{\pi}{2}$

$\therefore l = \frac{\lambda}{4n_l}$



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Case II. 2-layer of $\frac{\lambda}{4}$ stacks

n_0	n_1	n_2	n_t

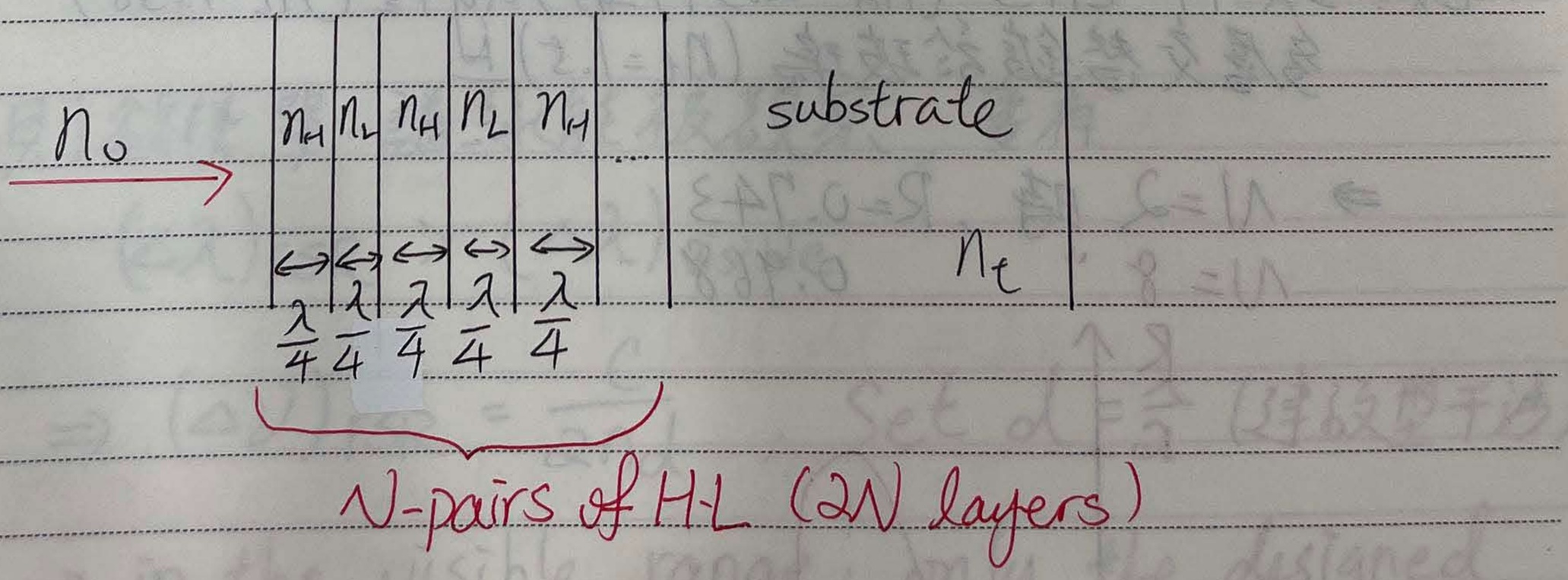
$\Rightarrow M = M_1 M_2 = \begin{pmatrix} m_{11} & 0 \\ 0 & m_{22} \end{pmatrix}$, m_{11}, m_{22} are functions of n_2, kd. solve for r by (2) gives

$$r = \frac{n_2^2 - n_t n_1^2}{n_2^2 + n_t n_1^2}$$

$\Rightarrow r = 0$ 時, $n_2 = \sqrt{n_t} \cdot n_1$

比如前例中，取 n_1 为 MgF_2 (1.38)，则可取 $n_2 = 1.69$.
最接近物质为 MgO ($n_2 = 1.75$)
$\Rightarrow$ 算出 $R = 0.00$[illegible]，更接近 0.

Case III. High-Reflectance Films by $\frac{\lambda}{4}$ stacks



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$$\Rightarrow M = M_H \cdot M_L \cdot M_H \cdot M_L \cdots$$

$$= \begin{pmatrix} 0 & -\frac{i}{n_H} \\ i n_H & 0 \end{pmatrix} \begin{pmatrix} 0 & -\frac{i}{n_L} \\ -i n_L & 0 \end{pmatrix} \cdots\cdots$$

$$= \begin{pmatrix} -\frac{n_L}{n_H} & 0 \\ 0 & -\frac{n_H}{n_L} \end{pmatrix}^N$$

$$= \begin{pmatrix} \left(-\frac{n_L}{n_H}\right)^N & 0 \\ 0 & \left(-\frac{n_H}{n_L}\right)^N \end{pmatrix}$$

$$\Rightarrow \text{可算出 } R = \left| \frac{n_t \left(\frac{n_H}{n_L}\right)^{2N} - 1}{n_t \left(\frac{n_H}{n_L}\right)^{2N} + 1} \right|$$

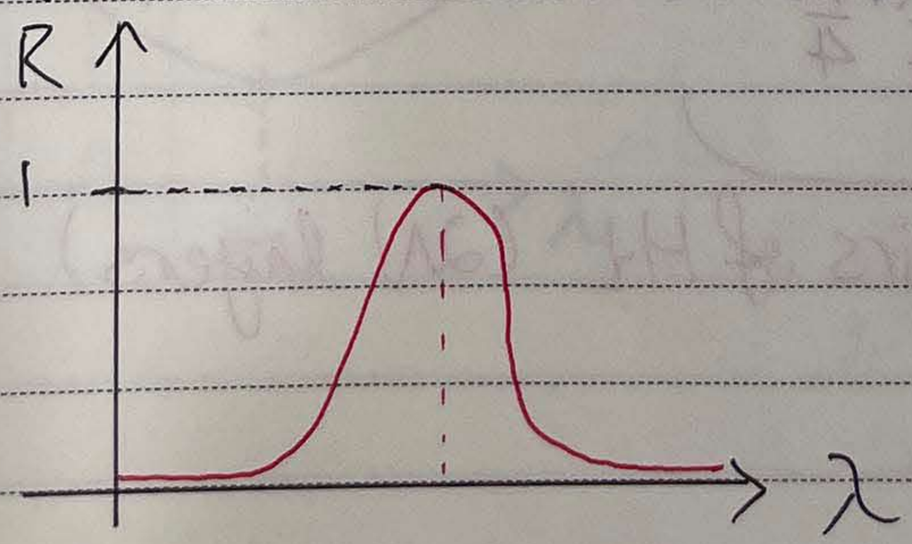
$\because \frac{n_H}{n_L} > 1, \left(\frac{n_H}{n_L}\right)^{2N} >> 1$

$\therefore$ When $N \to \infty$, $R \to 1$.

ex. 以 ZnS ($n_H = 2.39$) 与 MgF_2 ($n_L = 1.38$) 多层交替镀於玻璃 ($n_t = 1.5$) 上

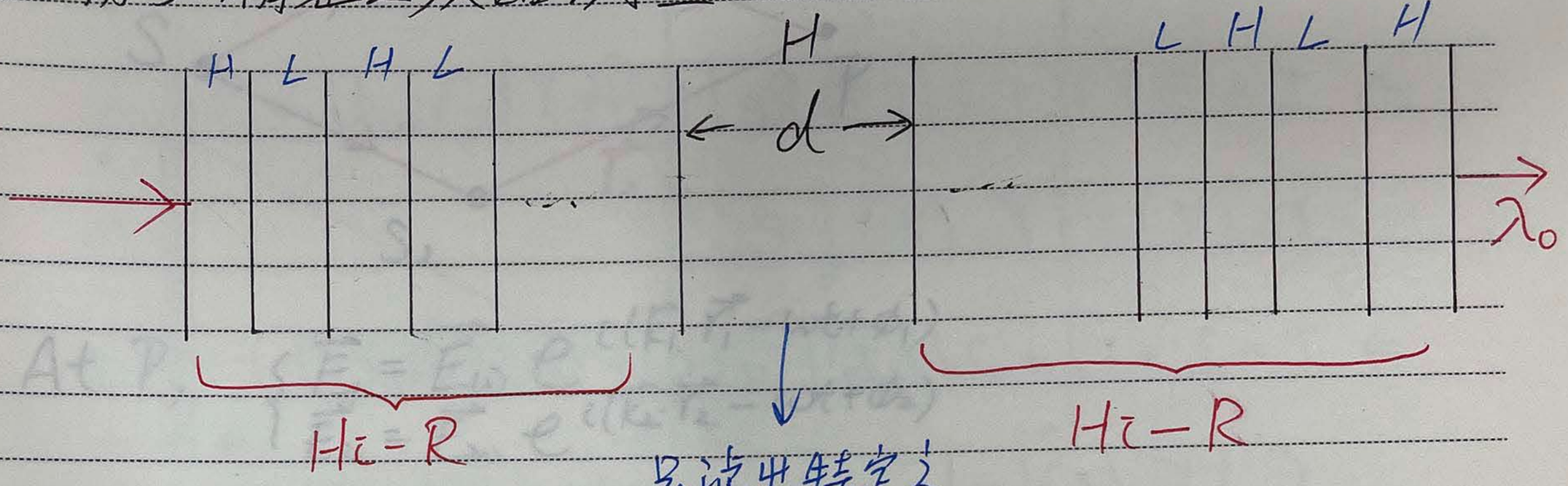
$\Rightarrow N=2$ 時, $R = 0.743$

$N = 8$ " 0.968



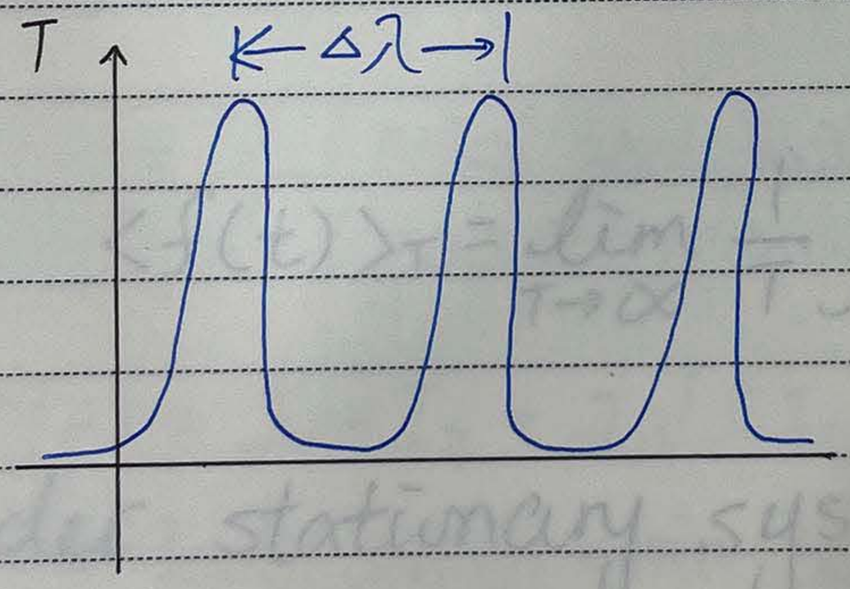
Case IV. Fabry-Perot interference filter

→用於精密之頻譜測量



只濾出特定入
一條光譜線.

假設光譜上兩譜線間距為 $\Delta\lambda$ (称 Free Spectral Range)



則欲使單一譜線能被觀察，要有

$$(\Delta\lambda)_{FSR} > (\Delta\lambda)_{visible}$$

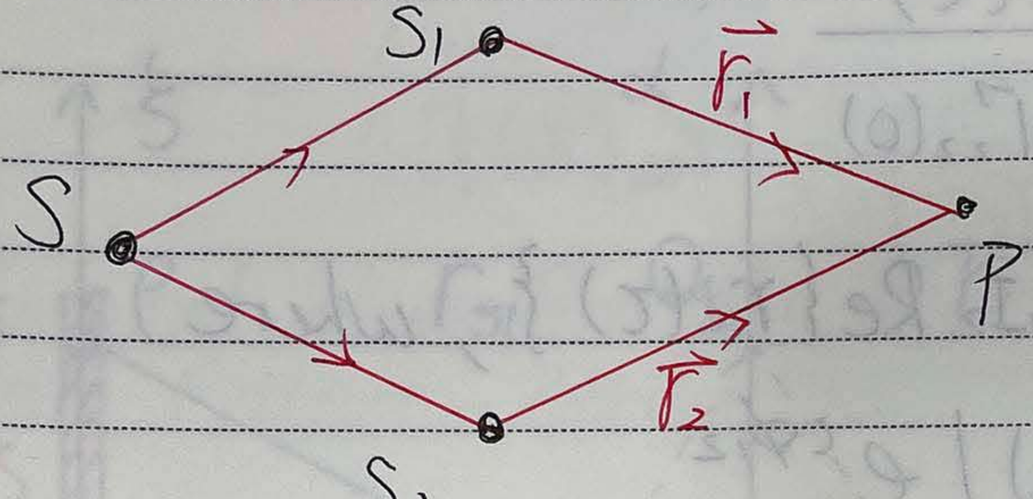
$$\Rightarrow (\Delta\nu)_{FSR} = \frac{c}{2nd}, \text{ Set } d = \frac{\lambda}{2} \text{ (建設型干涉)}$$

→ in the visible range, only the designed wavelength λ can be selected

Lecture 7 Coherence

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§1. Introduction (7-12)



At P, $\begin{cases} \vec{E}_1 = \vec{E}_{1\omega}\, e^{i(\vec{k}_1\cdot\vec{r}_1 - \omega t + \phi_1)} \\ \vec{E}_2 = \vec{E}_{2\omega}\, e^{i(\vec{k}_2\cdot\vec{r}_2 - \omega t + \phi_2)} \end{cases}$

$$I = \langle |\vec{E}_1 + \vec{E}_2|^2 \rangle_T$$
$$= \underbrace{\langle |\vec{E}_1|^2 \rangle_T}_{I_1} + \underbrace{\langle |\vec{E}_2|^2 \rangle_T}_{I_2} + \underbrace{2\,\mathrm{Re}\{\langle \vec{E}_1 \cdot \vec{E}_2^{*} \rangle_T\}}_{\triangleq \Gamma_{12}\ \text{interference / mutual coherence}}, \text{ where}$$

$$\langle f(t) \rangle_T = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)\, dt$$

Consider stationary systems,

$$\begin{cases} \Gamma_{12}(\tau) = \lim\limits_{T\to\infty} \dfrac{1}{T} \displaystyle\int_0^T \vec{E}_1(t)\, \vec{E}_2^{*}(t+\tau)\, dt \\ \Gamma_{11}(0) = \lim\limits_{T\to\infty} \dfrac{1}{T} \displaystyle\int_0^T |\vec{E}_1(t)|^2\, dt = I_1 \\ \Gamma_{22}(0) = \lim\limits_{T\to\infty} \dfrac{1}{T} \displaystyle\int_0^T |\vec{E}_2(t)|^2\, dt = I_2 \end{cases}$$

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Define the degree of coherence,

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}}$$

$$\Rightarrow I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, \mathrm{Re}\{\gamma_{12}(\tau)\}, \text{ where}$$

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)|\, e^{i\phi_{12}}$$

$$\Rightarrow I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma_{12}(\tau)|\cos(\phi_{12}(\tau))$$

$$\Rightarrow \begin{cases} |\gamma_{12}(\tau)| = 1 & : \text{complete coherence} \\ |\gamma_{12}(\tau)| = 0 & : \text{“} \quad \text{incoherence} \\ 0 < |\gamma_{12}(\tau)| < 1 & : \text{partial coherence (ex. 日光灯)} \end{cases}$$

Define the visibity (清晰度) to be

$$v \triangleq \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

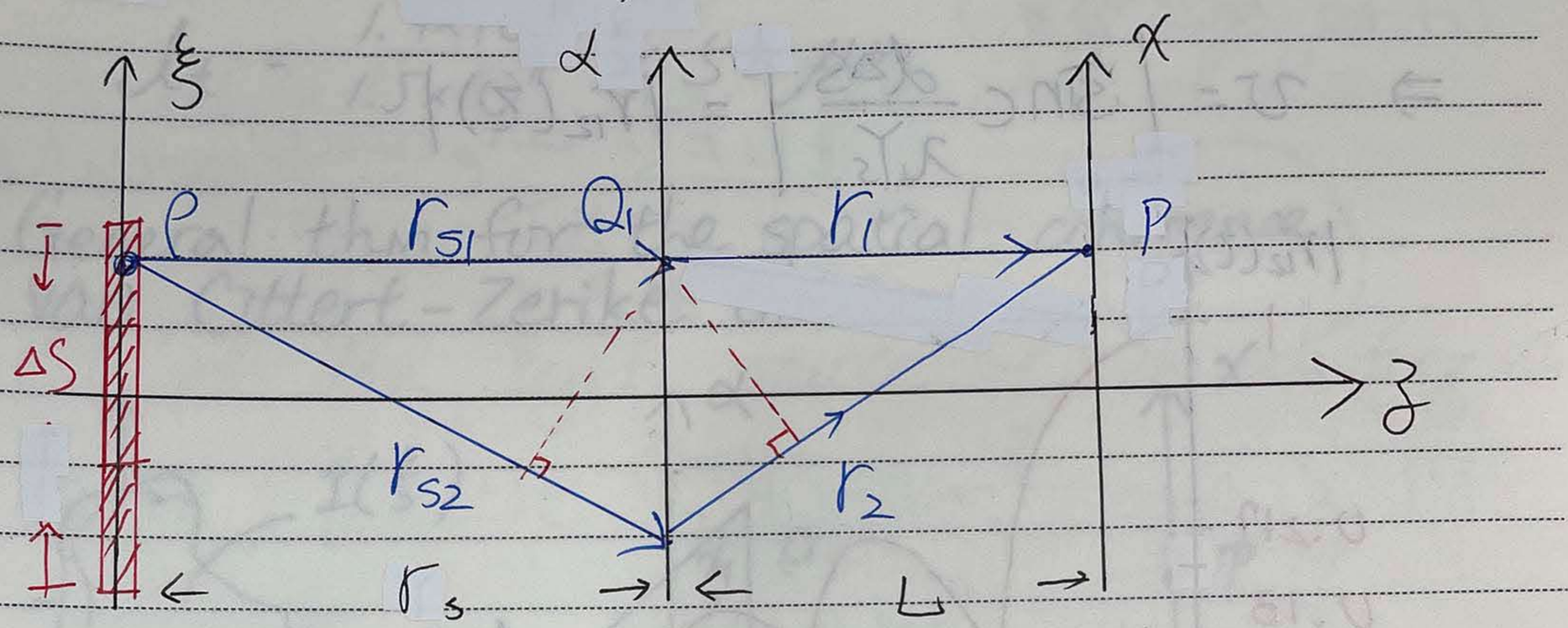
$$= \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}\,|\gamma_{12}(\tau)|$$

① Measure $v \rightarrow |\gamma_{12}(\tau)|$ found

② $I_1 = I_2 = I_0 \rightarrow v = v_{max} = |\gamma_{12}(\tau)|$

§2. Spatial Coherence ('7-12)

1. 考慮一長 ΔS 之面光源.



測量 U 可得 $|\gamma_{12}(\tau)|$

Assume ΔS: extended source (擴展光源, 即很多点光源構成)

$\underline{I} = \sum_i$ (point sources of equal intensity)$_i$ · (quasi-monochromatic).

$\Rightarrow dI(\rho) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$, where

$\delta = kd(\sin\theta_1 + \sin\theta_2)$

$\approx kd\left(\frac{x}{L} + \frac{\xi}{r_s}\right)$ if $L, Y_s \gg d \gg \lambda$

$\Rightarrow$ Total intensity

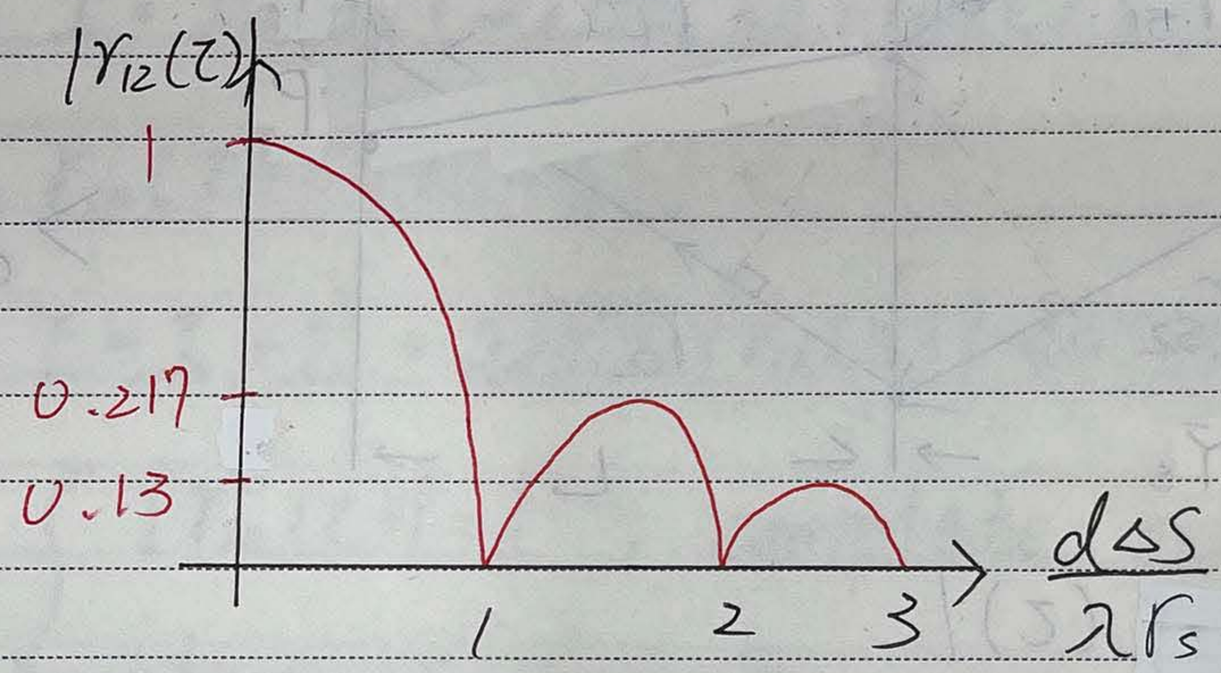
$$\underline{I} = \int_{-\frac{\Delta S}{2}}^{\frac{\Delta S}{2}} dI(\rho)\cdot d\rho$$

$= \cdots$

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$$= \Delta S\left(I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{sinc}\frac{d\Delta s}{\lambda r_s}\cos\frac{kdx}{L}\right)$$

$$\Rightarrow v = \left|\mathrm{sinc}\frac{d\Delta s}{\lambda r_s}\right| = |\gamma_{12}(\tau)|$$



(i) $\Delta S \to 0$, $|\gamma_{12}(\tau)| \to 1$

(ii) $\Delta S \uparrow \to |\gamma_{12}(\tau)| \downarrow$

When $\frac{d\Delta s}{\lambda r_s} = 1 \to |\gamma_{12}(\tau)| = 0$, d為兩光源之距離(或狹縫間距)

$\Rightarrow d = \frac{\lambda}{\Delta S} r_s$ (比此值小才會有干涉)

Define $\theta_s = \frac{\Delta S}{r_s}$ (光源張角), then 称此 d 為 transverse coherence distance

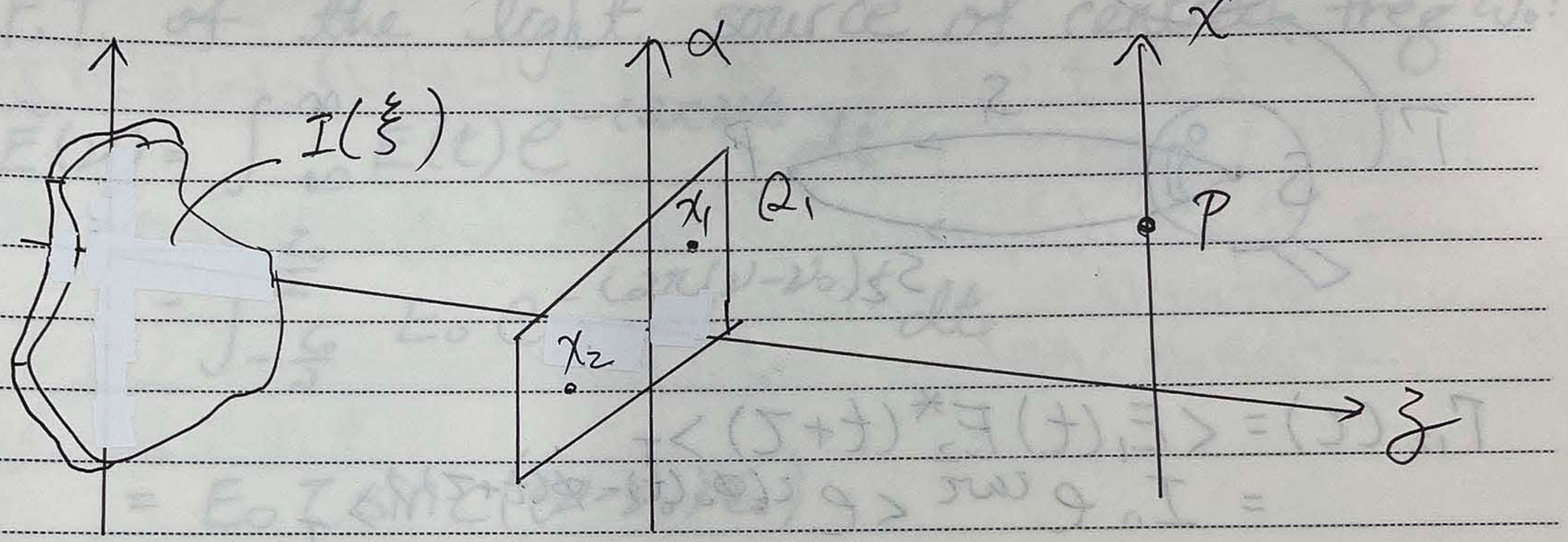
$\triangleq l_t = d = \frac{\lambda}{\theta_s}$

↑
空間多久平均以同調之範圍

比如：太陽直徑 $\Delta S = 1.4\times10^{9}\,m$，距地球 $r_s = 1.5\times10^{11}\,m$，
則想用陽光做干涉實驗，狹縫間距要小於

$$l_t = \frac{1.4\times10^{9}}{1.5\times10^{11}} \approx 50\,\mu m$$

2. General thm for the spatial coherence:
van Cittert-Zerike thm



$$\gamma_{12} = \frac{\int_{\Delta S} I(\xi)\, e^{-i\frac{2\pi}{\lambda r_s}(\vec{x}_2-\vec{x}_1)\xi}\, d\xi}{\int_{\Delta S} I(\xi)\, d\xi}$$

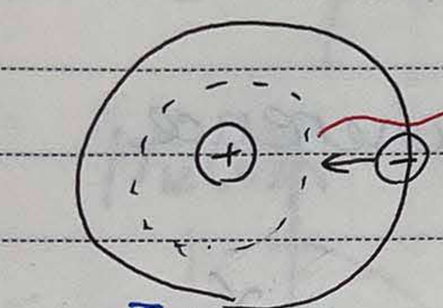
If ΔS : circular $= \mathrm{circ}\left(\frac{\xi}{w/2}\right)$, then

$$\gamma_{12} = \left(\frac{w}{2}\right)^2 \frac{J_1(\pi w \rho)}{w\rho/2}, \quad \text{where } \rho = \frac{\gamma_0}{\lambda r_s}$$

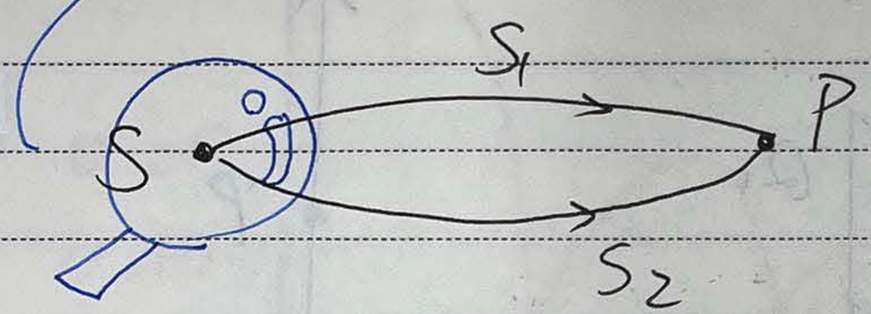
$$\Rightarrow l_t \doteq 1.22\,\frac{\lambda}{\theta_s}.$$

Date. Page. § 3. Temporal Coherence (7-12)

1. Assume quasi-monochromatic light source
(比如：原子發光)



$\vec{E}(t) = \vec{E}_0 e^{-i\omega t} e^{i\phi_0(t)}$, ϕ_0: initial phase
$\in [-\pi, \pi]$ 為 r.v.
一次發光時是定。

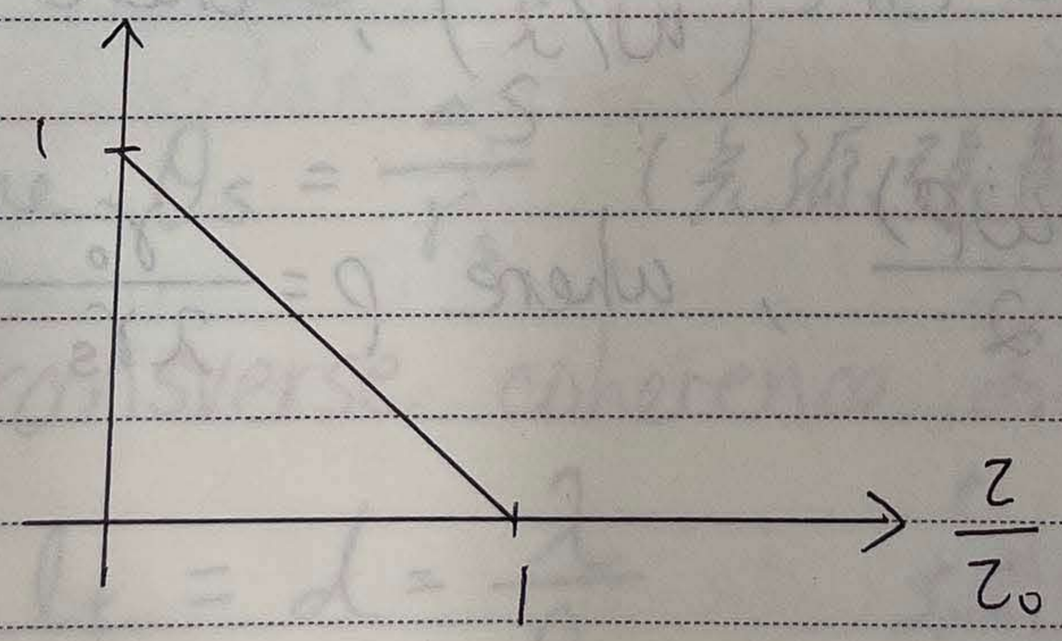


$$\Gamma_{12}(\tau) = \langle E_1(t) E_2^*(t+\tau) \rangle_T$$
$$= \underline{I_0} e^{i\omega\tau} \langle e^{i(\phi_0(t) - \phi(t+\tau))} \rangle_T$$

$$\gamma_{12}(\tau) = \frac{\langle E(t) E^*(t+\tau) \rangle}{\langle |E|^2 \rangle}$$

$$= e^{i\omega\tau} \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{i(\phi_0(t) - \phi_0(t+\tau))} dt$$

If $\tau < \tau_0$, $\gamma_{12}(\tau) = e^{i\omega\tau}\left(1 - \frac{\tau}{\tau_0}\right)$



⇒ 光程不可超过 τ_0，光程差不能超过 $l_c = c\tau_0$，
時間差

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τ_0 称为 coherence time

l_c " length.

τ_0 is related to the linewidth of the light source

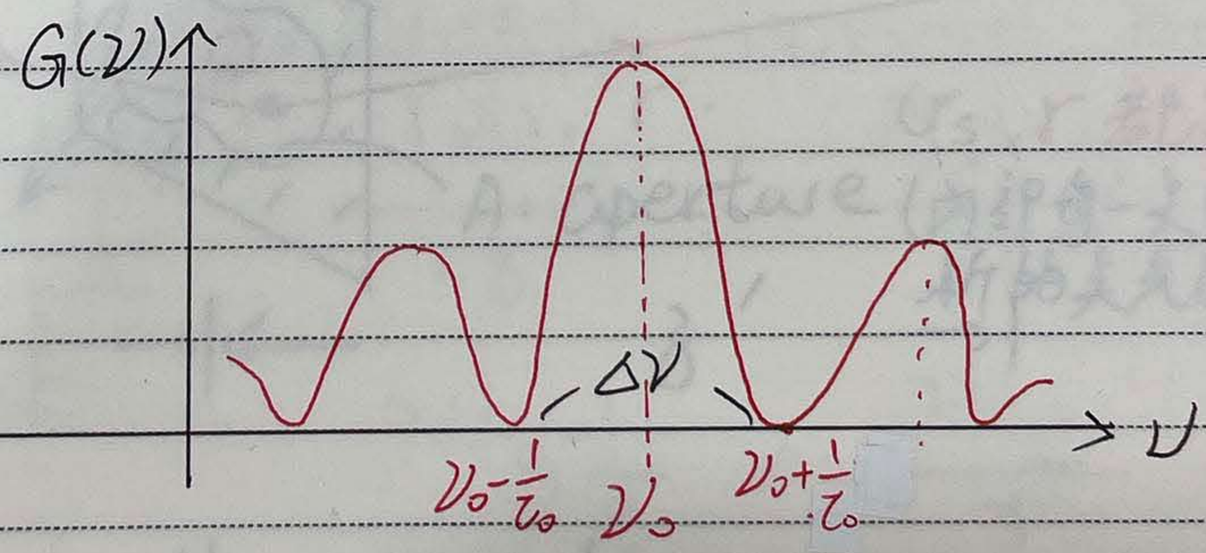
2. F.T. of the light source of central freq ω_0:

$$\tilde{E}(\nu) = \int_{-\infty}^{\infty} E(t) e^{-i2\pi\nu t} dt$$

$$= \int_{-\frac{\tau_0}{2}}^{\frac{\tau_0}{2}} E_0 e^{-i2\pi(\nu-\nu_0)t} dt$$

$$= E_0 \tau \operatorname{sinc}((\nu-\nu_0)\tau_0)$$

Power spectrum $G(\nu) = |\tilde{E}(\nu)|^2 = E_0^2 \tau_0^2 \operatorname{sinc}^2((\nu-\nu_0)\tau_0)$



$\Rightarrow$ linewidth $\Delta\nu = \frac{1}{\tau_0}$, 来自于原子單次時長 τ_0 之發光

$$\Rightarrow l_c = c\tau_0$$

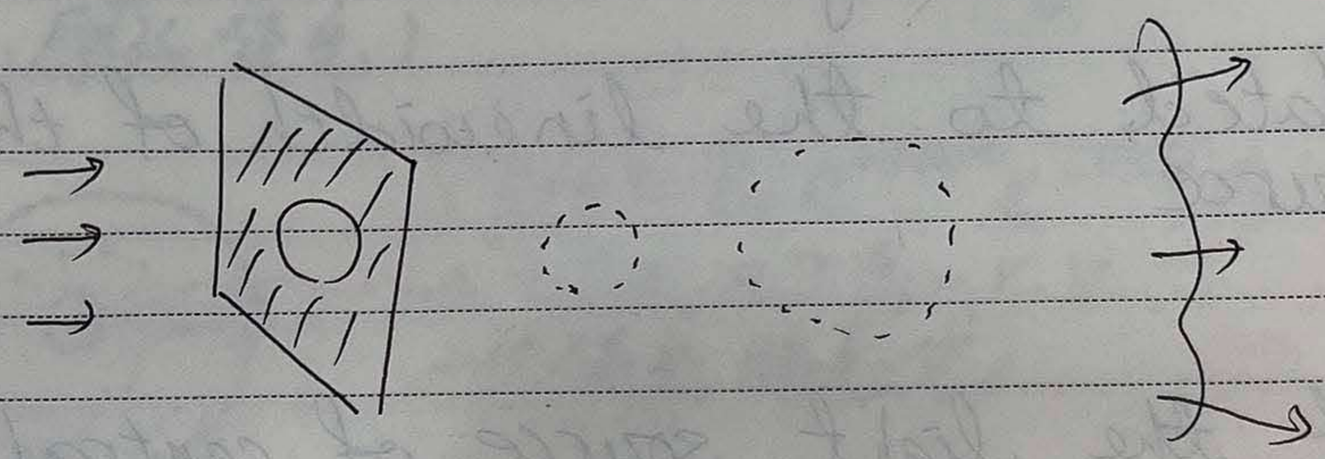
$$= \frac{c}{\Delta\nu}$$

$$= \frac{\lambda^2}{\Delta\lambda}$$

Date. Page.

Lecture 8. Diffraction.

§1. Introduction (8-1)



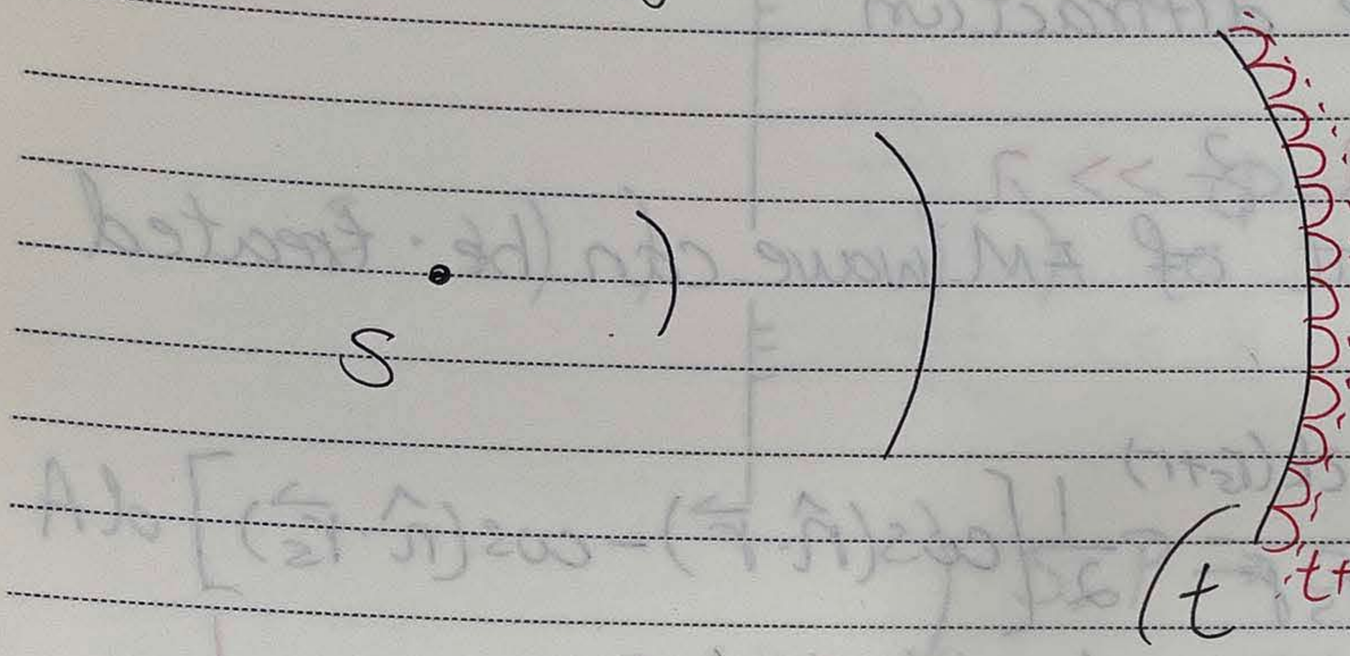
光通过小孔後，每个位置上的像如同被散开3-标

⇒ light propagation that deviates from the geometric prediction.

→ wave nature of light.

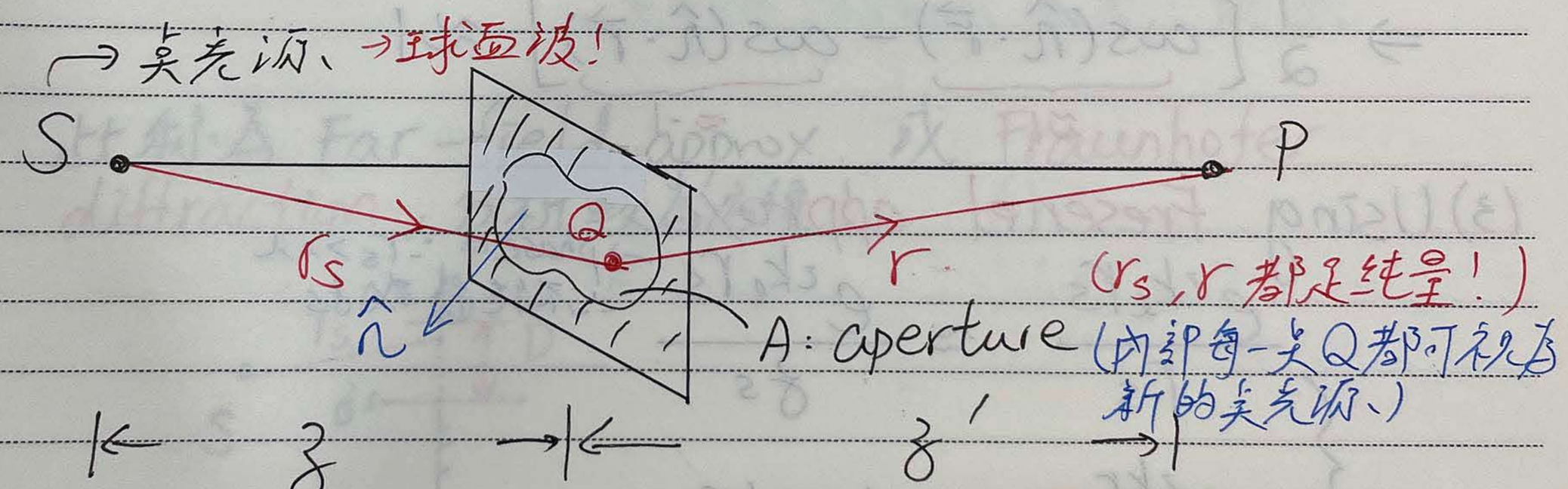
§2. Huygine's Principle

Date. (8-2) Page.



→ wavefront上每一点都可视为新的点光源.

令P点之 wavefront 为 $E(P, t)$, 此时E为纯量。
考虑以下装置:



(r_s, r都是纯量!)

∵ For any Q on the aperture,

$$E_Q = \frac{E_s}{r_s} e^{ikr_s}$$

∴ At P, $E_P = \int_A E_Q \cdot \frac{e^{ikr}}{r} dA$

$$= E_s \iint_A \frac{e^{ik(r_s+r)}}{r_s r} dA$$

Date. Page.

1. Scalar-wave diffraction

(1) Assume $A \gg \lambda$, $z \gg \lambda$

∴ Each component of EM wave can be treated independently

$$\therefore E_p = \textcircled{i}\frac{k}{\lambda}\iint_A \frac{e^{ik(r_s+r)}}{r_s r}\frac{1}{2}\left[\cos(\hat{n}\cdot\vec{r}) - \cos(\hat{n}\cdot\vec{r_s})\right] dA$$

90° 相位差

└ obliquity factor (傾斜因子)

(2) Consider paraxial approximation.

region of interest to the small angle with the optical axis

$$\Rightarrow \frac{1}{2}\left[\underbrace{\cos(\hat{n}\cdot\vec{r})}_{\approx 1} - \underbrace{\cos(\hat{n}\cdot\vec{r})}_{\approx -1}\right] \approx 1$$

(3) Using Fresenel approx

$$\frac{e^{ik_e r_s}}{r_s} \approx \frac{e^{ik_e r_s}}{z_s}$$ → phase中 ∵ $r_s \gg \lambda$ ∴不能取成 z_s

$$\begin{cases} \frac{e^{ikr}}{r} \approx \frac{1}{z}e^{ikr} \end{cases}$$

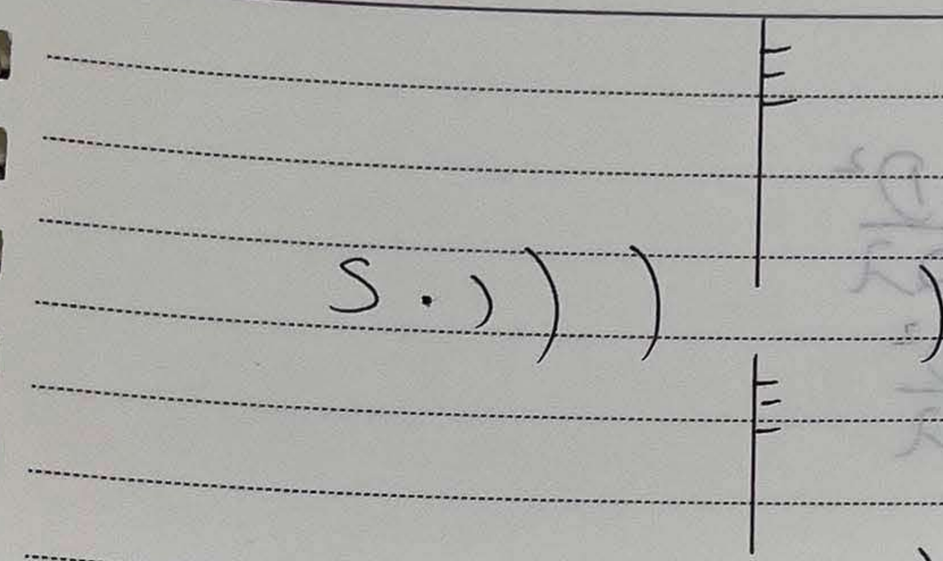
$$\Rightarrow E_p = C\int_A e^{ik(r_s+r)}\,dA$$ → 但一般仍難算!

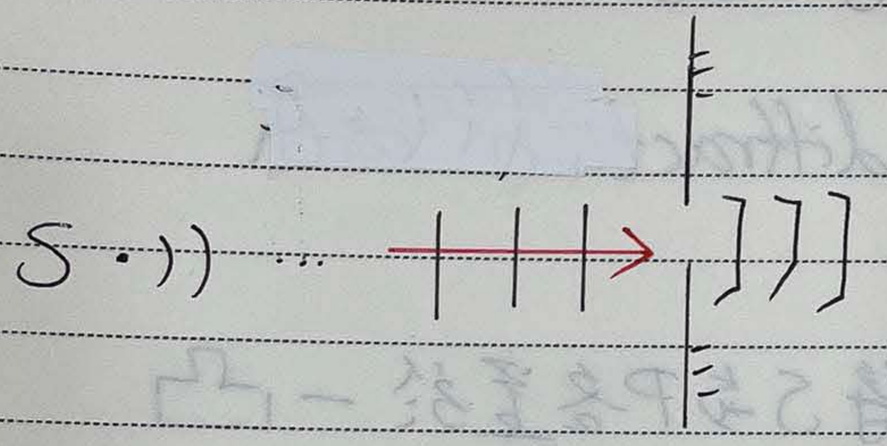
2. 点光源 S 放出之波为球面波

→ 若 S「夠遠」，可以將球面波以平面波近似嗎?

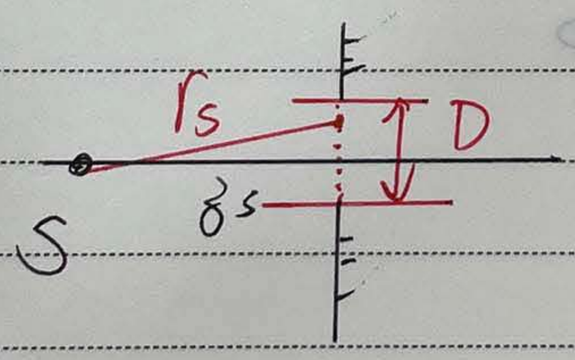
→ 可以! 但多遠才算「夠遠」?

Date. Page.





此称为 Far-field approx. 或 Fraunhofer diffraction，分析較为方便！



When spherical wave ≈ plane wave,

$r_s - \delta_s << \lambda$

$$\Rightarrow \sqrt{E_s^2 + \left(\frac{D}{2}\right)^2} - \delta_s << \lambda$$

$$\Rightarrow \frac{D^2}{2\delta_s} << \lambda$$

Date. Page.

⇒ 要用此 approx.,

{ 光源要距小孔 $z_s >> \frac{D^2}{2\lambda}$

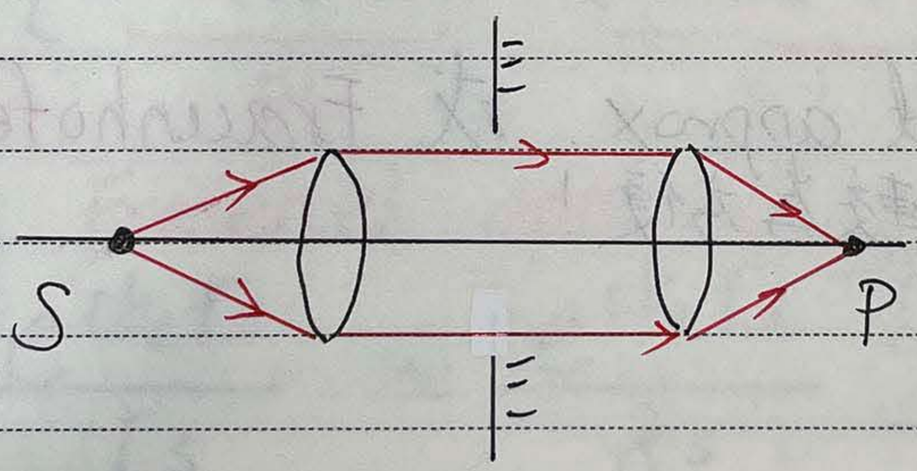
成像位置 $z >> \frac{D^2}{2\lambda}$

才算夠遠!

ex. $D = 5\,mm$, $\lambda = 500\,nm \rightarrow z > 25\,m$

Q. How to make far field diffraction with a compact system?

A: Add focusing lense! 將 S 與 P 各置於一凸透鏡之焦点上形成平行光.



§3 Fraunhofer diffraction

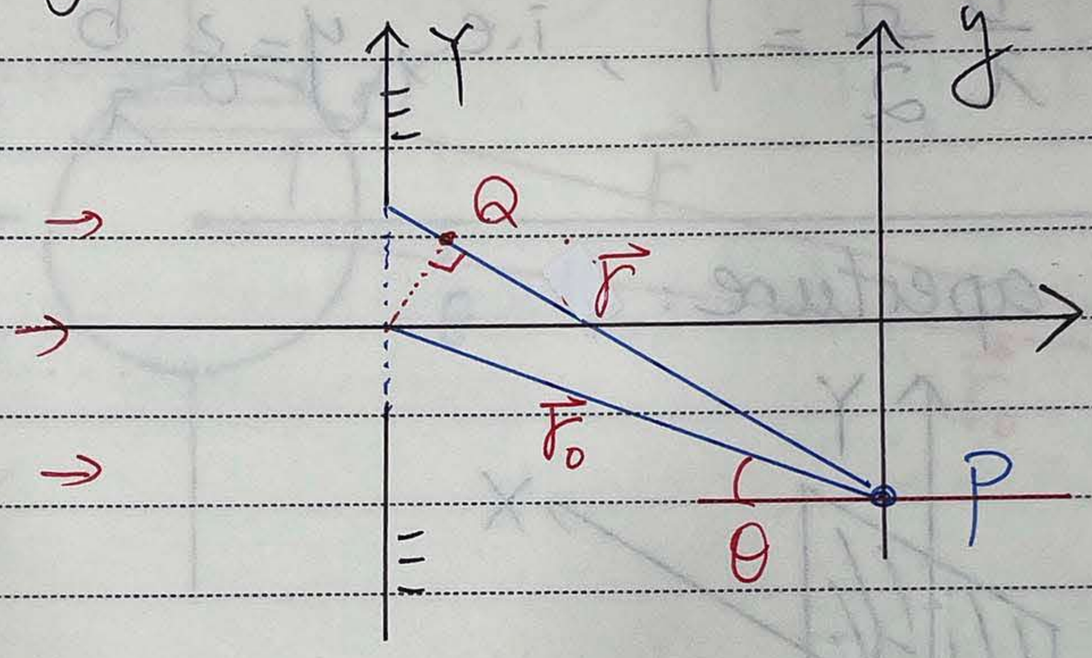
Date. (8.3) Page.

Assume plane wave illumination

$E_p = c\int_A e^{ikr}\,dA$, where

$c = \frac{1}{z_s} e^{ikr_s}$ is a const.

1. Single slit



∵ Far field, $z \to \infty$, $\vec{r} \parallel \vec{r_0}$

∴ $r = r_0 + Y\sin\theta$

$\Rightarrow E_p = C\int_{-\frac{b}{2}}^{\frac{b}{2}} e^{ik(r_0 + Y\sin\theta)}\,dY$

$= C' \operatorname{sinc}\left(\frac{b}{\lambda}\sin\theta\right)$, where

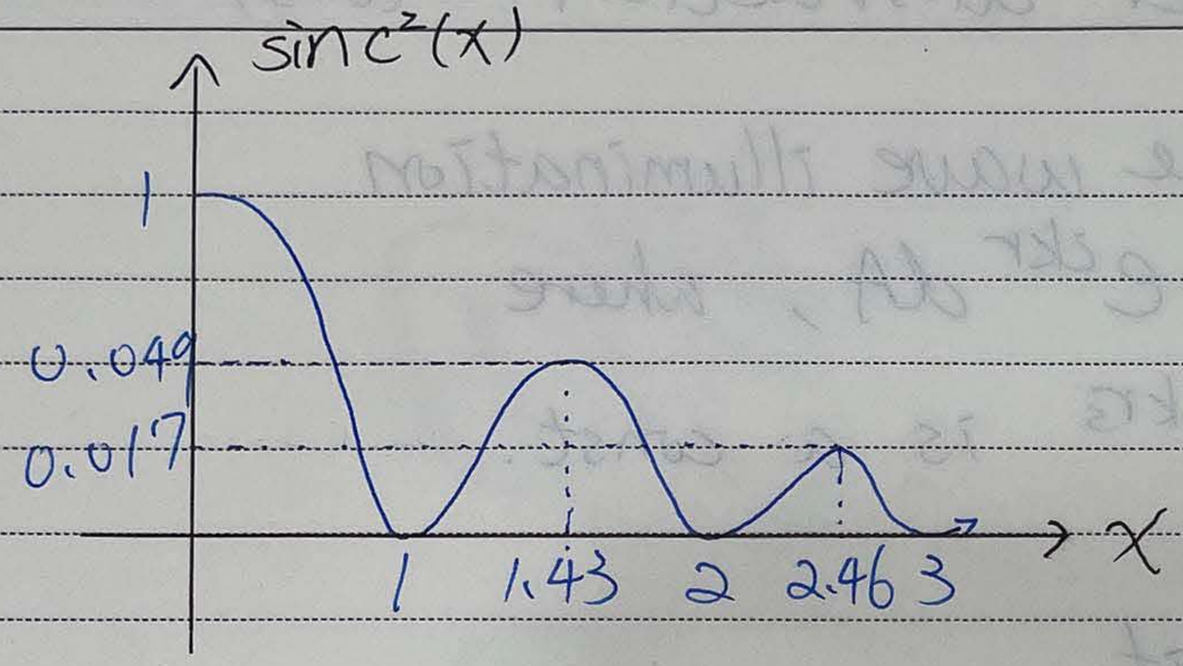
$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

$\Rightarrow I_p = I_0 \operatorname{sinc}^2\left(\frac{b}{\lambda}\sin\theta\right)$

∵ $\sin\theta \approx \tan\theta = \frac{y}{z}$

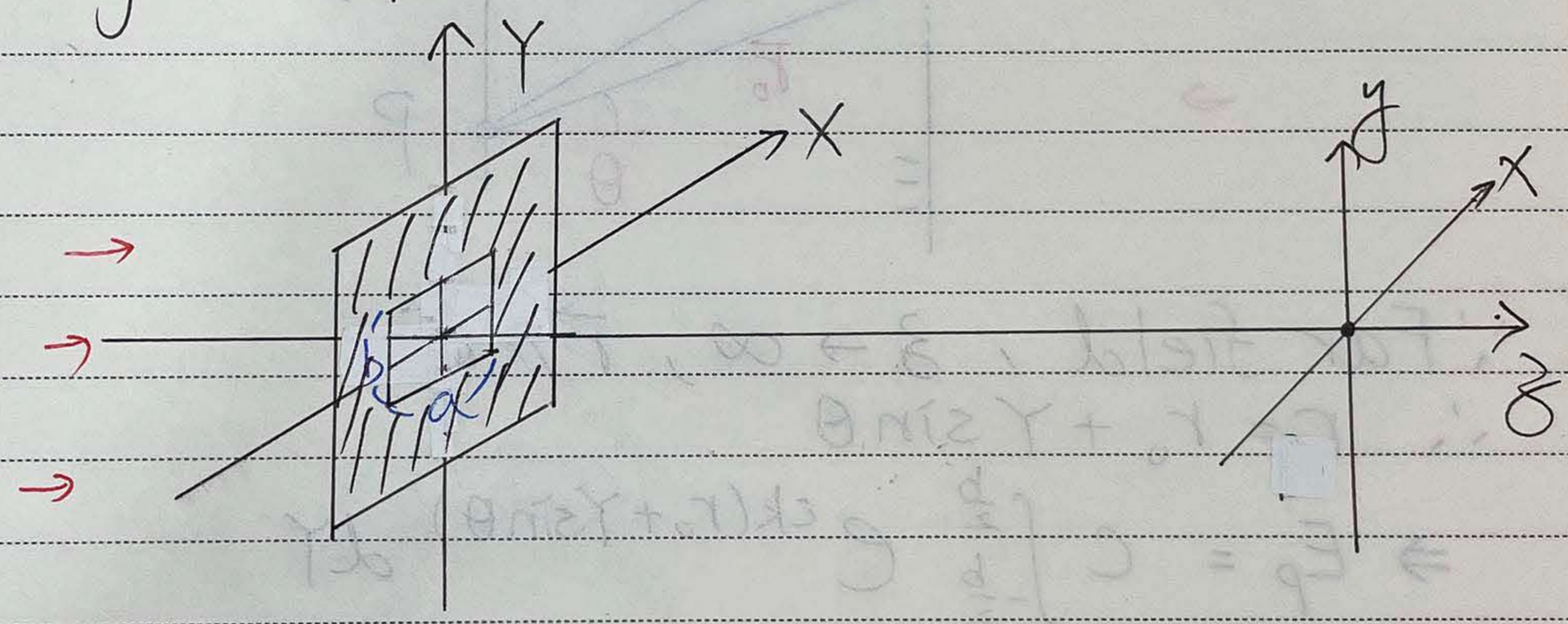
∴ $I_p \approx I_0 \operatorname{sinc}^2\left(\frac{b}{\lambda}\frac{y}{z}\right)$.

Date. Page.



$\Rightarrow 1^{st}$ zero: $\frac{b}{\lambda}\frac{y}{z} = 1$, i.e., $y = z\frac{b}{\lambda}$

2. Rectangular aperture:



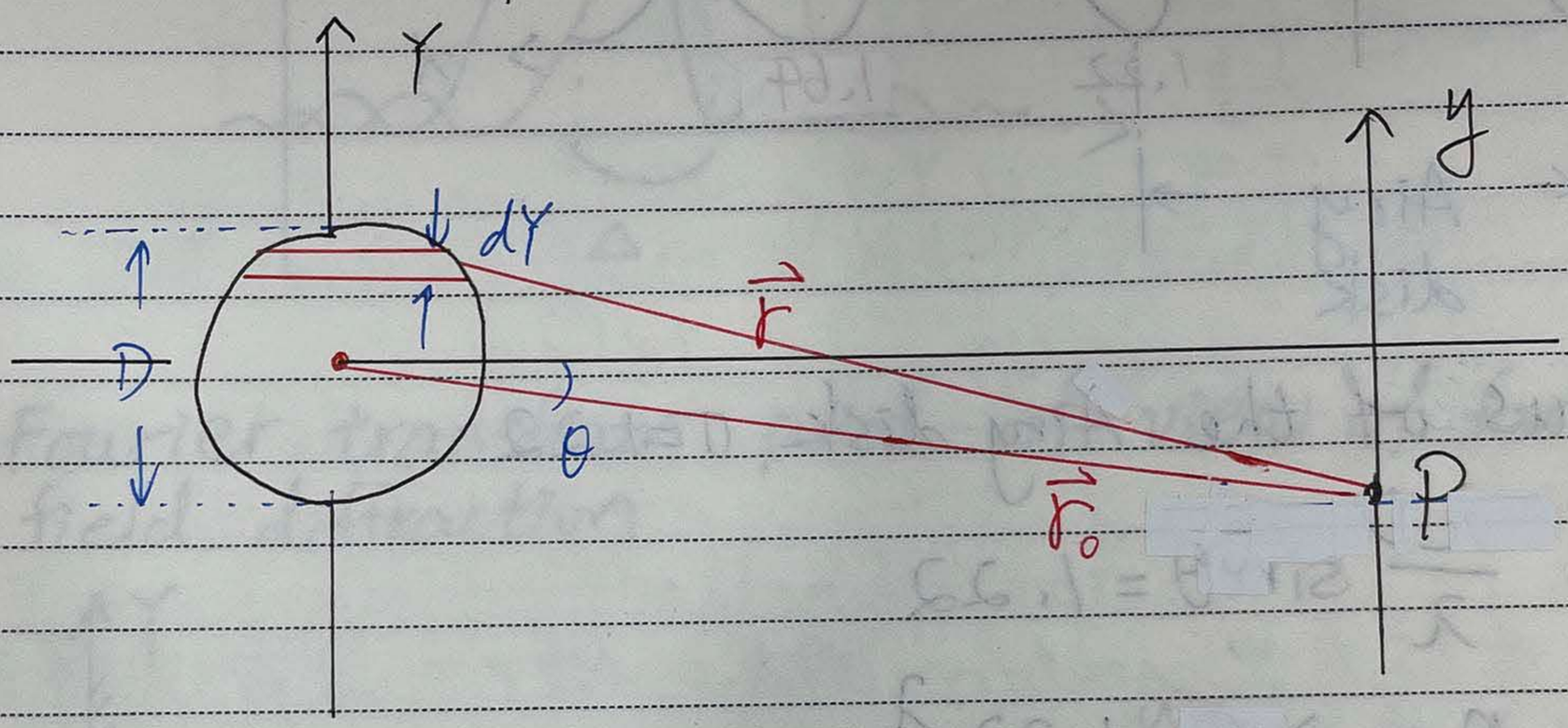
$$\Rightarrow E_p = c\int_A e^{ikr}\,dA$$

$$= c\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{\frac{a}{2}}^{\frac{a}{2}} e^{ik(r_0 + x\sin\alpha + y\sin\theta)}\,dx\,dy$$

$$\Rightarrow \begin{cases} I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2 \\ E_p = E_0\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin\beta}{\beta}\right) \end{cases}, \text{ where}$$

$$\begin{cases} a = \frac{1}{2} k a \sin\phi \\ \beta = \frac{1}{2} k b \sin\theta . \end{cases}$$

3. Ciraular aperture



將圓孔看成很多小狹縫

$$\text{apeture} = \sum_i (\text{single slits})$$

$$\Rightarrow dA = 2\sqrt{\left(\frac{D}{2}\right)^2 - Y^2}\, dY, \quad r = r_0 + Y\sin\theta$$

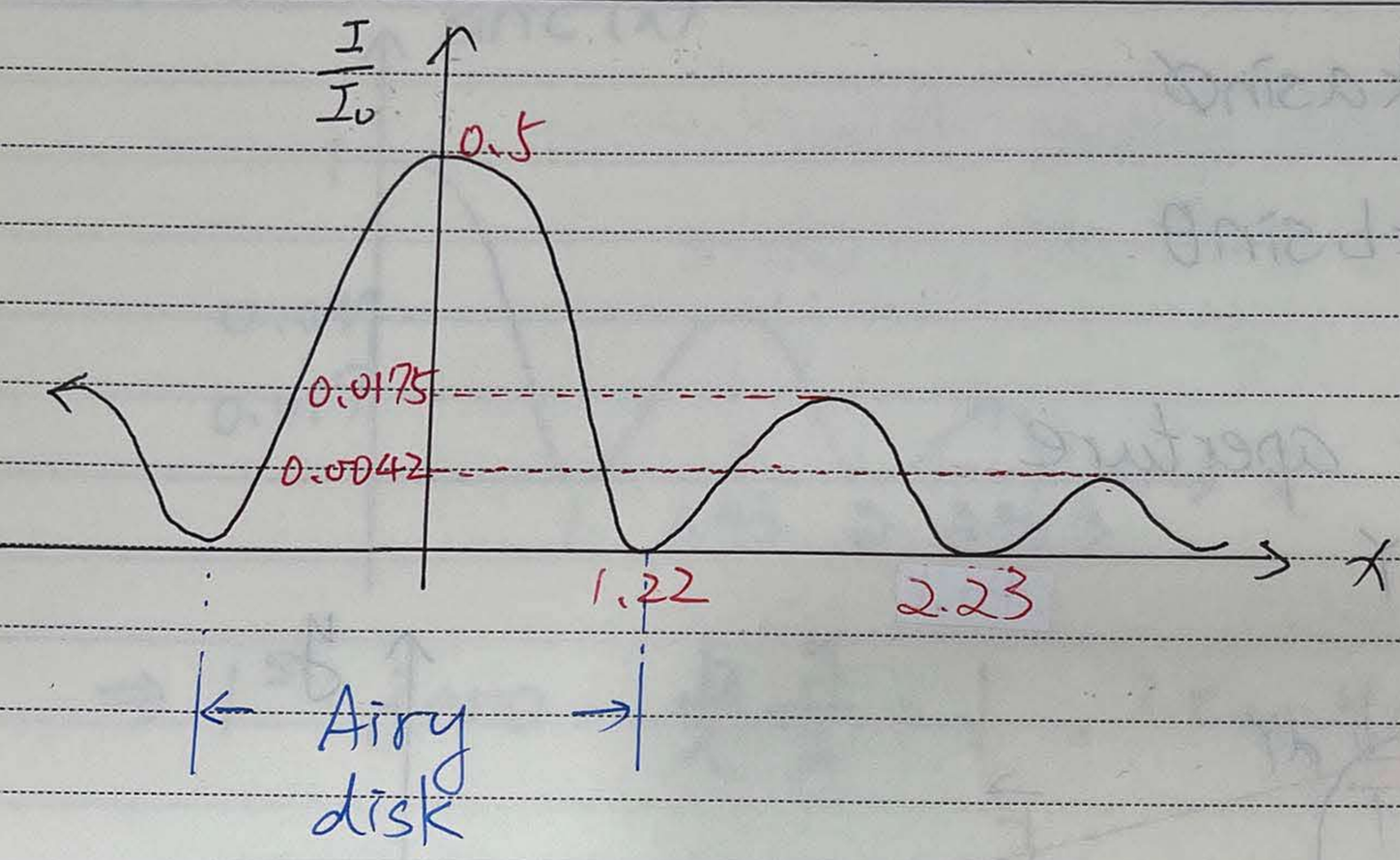
$$\Rightarrow E_p = C\int_A e^{ikr} dA$$

$$= Ce^{ikr_0}\int_{-\frac{D}{2}}^{\frac{D}{2}} e^{iky\sin\theta}\left(\sqrt{\left(\frac{D}{2}\right)^2 - y^2}\right) dy$$

$$= E_0\left(\frac{2J_1(\sigma)}{\sigma}\right), \quad \sigma = \frac{\pi D}{\lambda}\sin\theta, \ E_0 \text{ const}$$

$$\Rightarrow I_p = I_0\left(\frac{2J_1(\sigma)}{\sigma}\right)^2, \quad I_0 \text{ const.}$$

Date. Page.



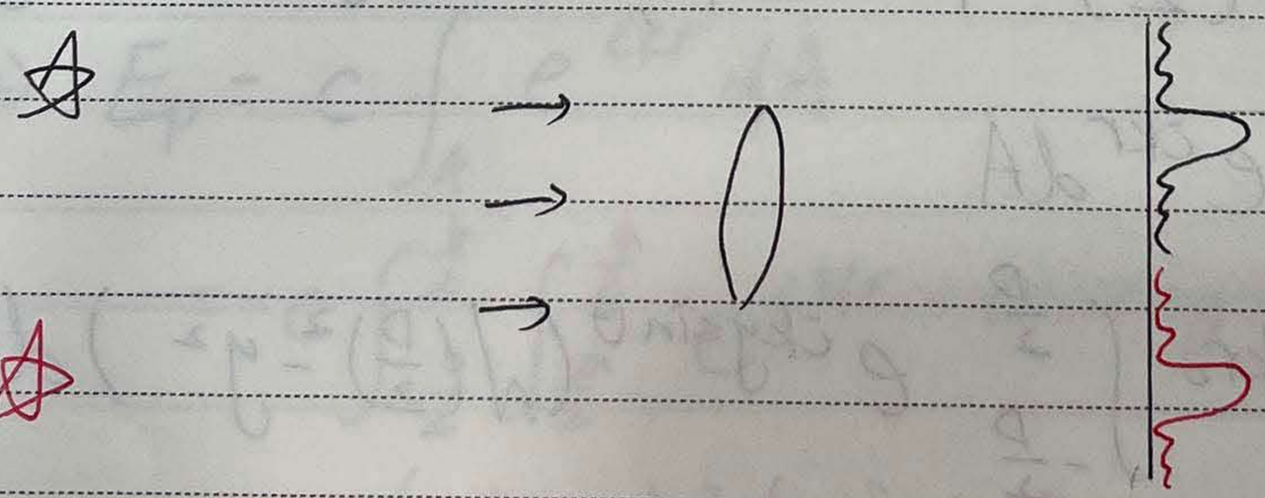
radius of the Airy disk $\sigma = 1.22$

$$\Rightarrow \frac{\pi D}{\lambda} \sin\theta = 1.22$$

$$\Rightarrow \theta \approx \sin\theta = 1.22 \frac{\lambda}{D}$$

diameter $\Delta R = 20 \cdot f \approx 2.44 \frac{\lambda}{D} f$

ex. 用望遠鏡看星星



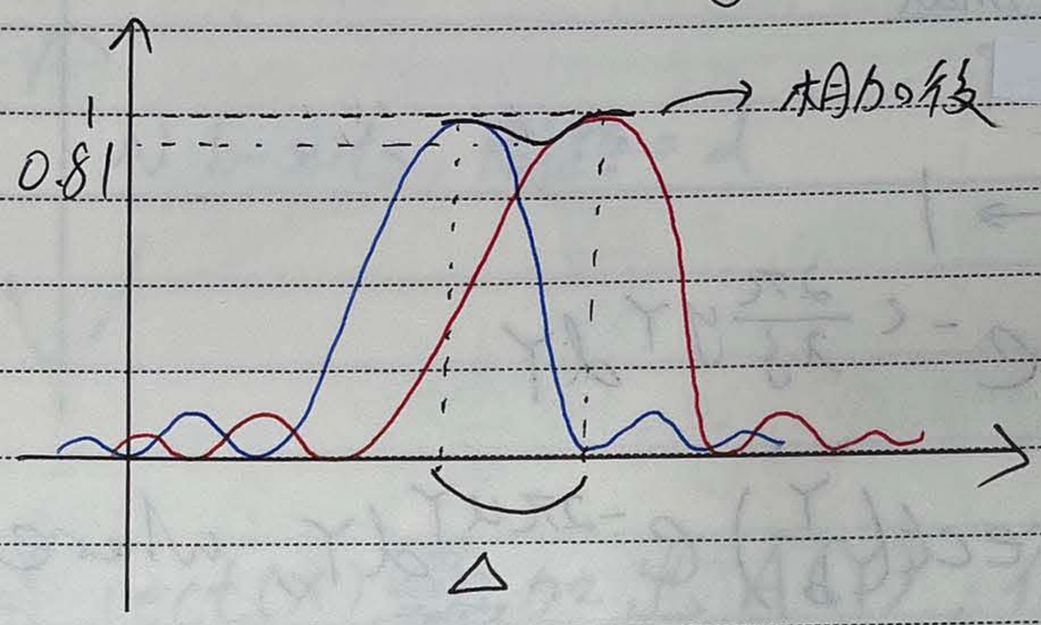
⇒ 2顆太靠近，会分辨不出来！

* ① 任意 Imaging system 都有 diffraction - limit，与做工好坏無关

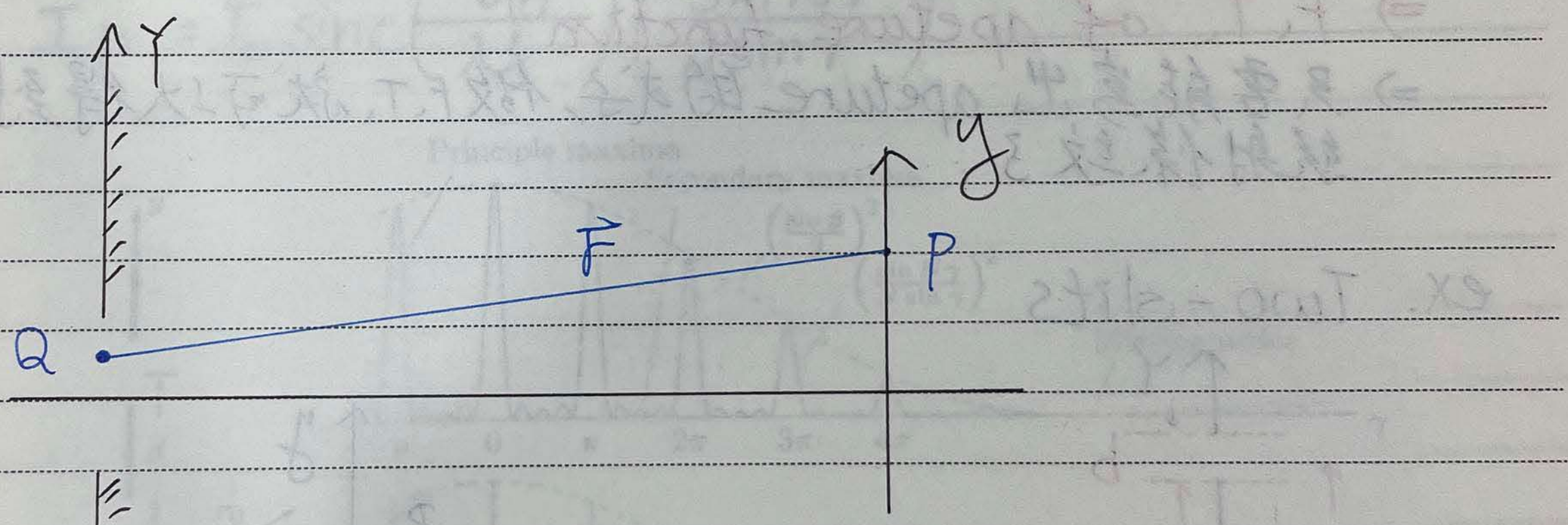
② 〃 对 point object 都有其 diffraction pattern

Date. Page.

③ Rayleigh criterion: resolvable if
$\Delta \geq$ radius of Airy disk (由实验得出)



4. Fourier transform point-of-view of far-field diffraction



$\because r = \sqrt{(y-Y)^2 + z^2}$

$\approx z + \frac{1}{2z}(y-Y)^2$

$= z + \frac{y^2}{2z} + \frac{Y^2}{2z} - \frac{yY}{z}$

$\therefore E_P = C_0 e^{ikz} e^{ik\frac{y^2}{2z}} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{i\frac{\pi Y^2}{\lambda z}} e^{-i\frac{2\pi}{\lambda z} yY} dY$

$\because$ Far field

Date. Page.

$\therefore \frac{\pi Y^2}{\lambda z} << \pi$

$\Rightarrow z >> \frac{(Y^2)_{max}}{\lambda}$

$\Rightarrow e^{i\frac{\pi Y^2}{\lambda z}} \to 1$

$\Rightarrow E_p = C' \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-i\frac{2\pi}{\lambda z} yY} dY$

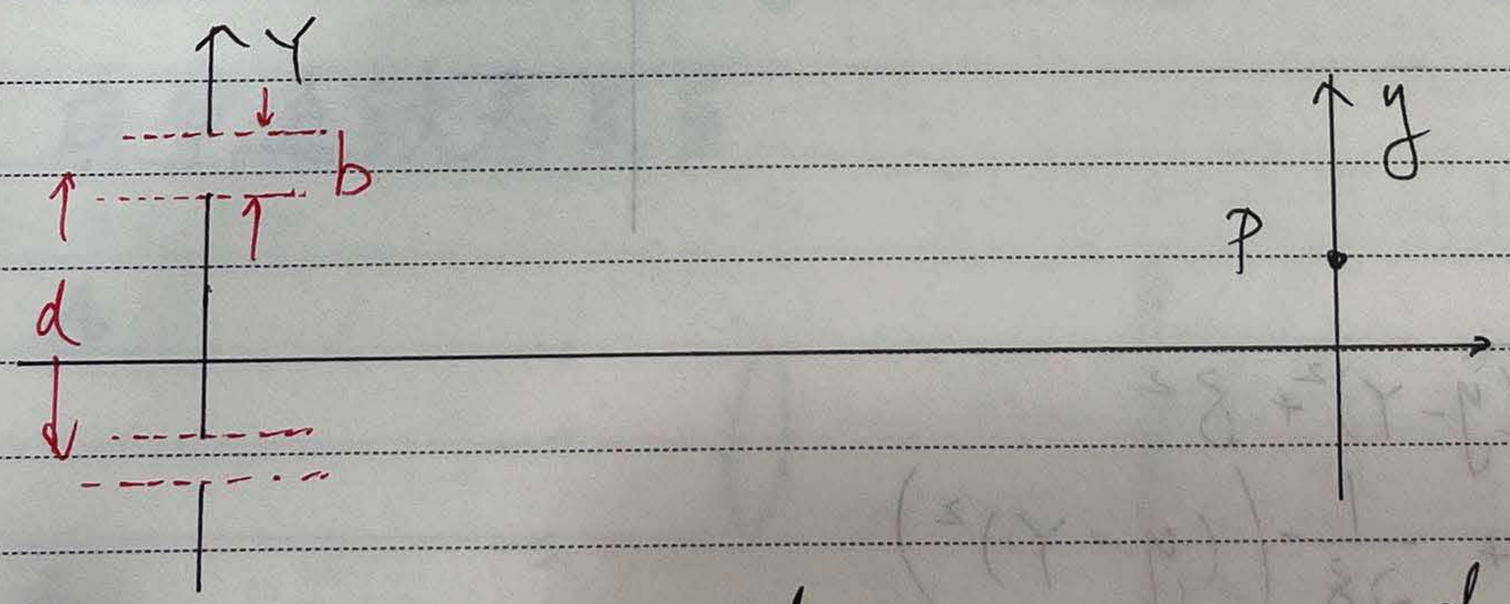
$= C' \int_{-\infty}^{\infty} rect\left(\frac{Y}{b}\right) e^{-2\pi u Y} dY$, where

$u = \frac{y}{\lambda z}$, $rect(x) = \begin{cases} 1, & |x| \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$.

$\Rightarrow$ F.T. of apeture function!

$\Rightarrow$ 只要能写出 apeture 的式子，做 F.T. 就可以得到繞射條紋了.

ex. Two-slits



$\Rightarrow A(Y) = rect\left(\frac{Y - \frac{d}{2}}{b}\right) + rect\left(\frac{Y + \frac{d}{2}}{b}\right)$

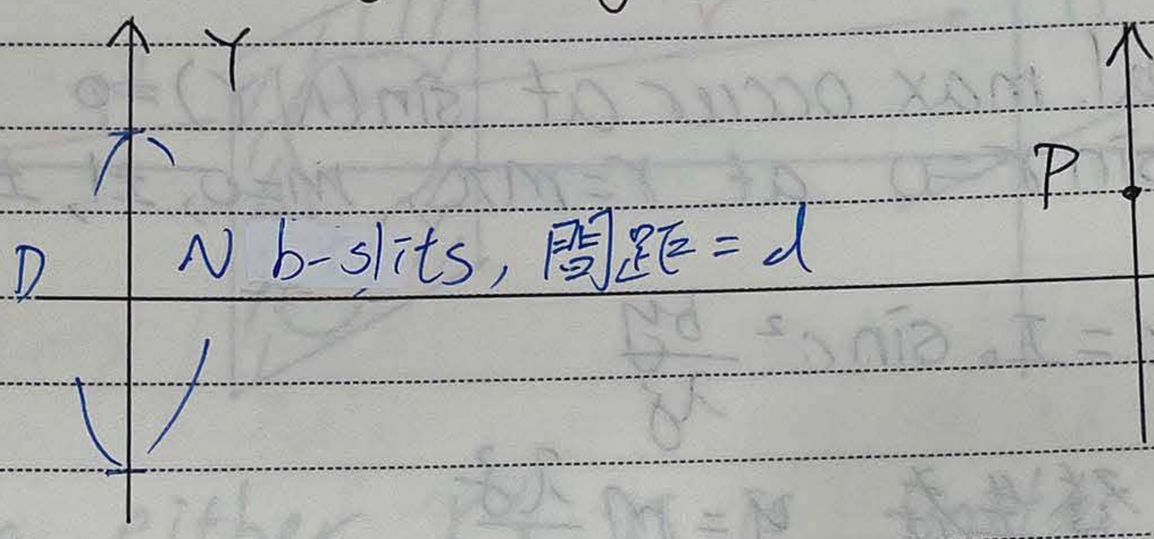
$\Rightarrow E_p(y) = \mathcal{F}\{A(Y)\}$

$= \cdots$

$= C' sinc\left(\frac{1}{2} kb \sin\theta\right) \cos\left(\frac{1}{2} kd \sin\theta\right)$

Date. Page.

ex. N-slits grating



⇒ 可用 rect(X), 當加出 A(Y), 做 F.T. 得

$$\begin{cases} E_p = C' \operatorname{sinc}(bu)\dfrac{\sin(N\gamma)}{\sin\gamma} \\ I_p = I_0 \operatorname{sinc}\left(\dfrac{by}{\lambda z}\right)\left(\dfrac{\sin N\gamma}{N\sin\gamma}\right)^2 \end{cases}, \quad \gamma = \frac{dy}{\lambda z}$$

單狹縫 ← → N 狹縫互相干涉

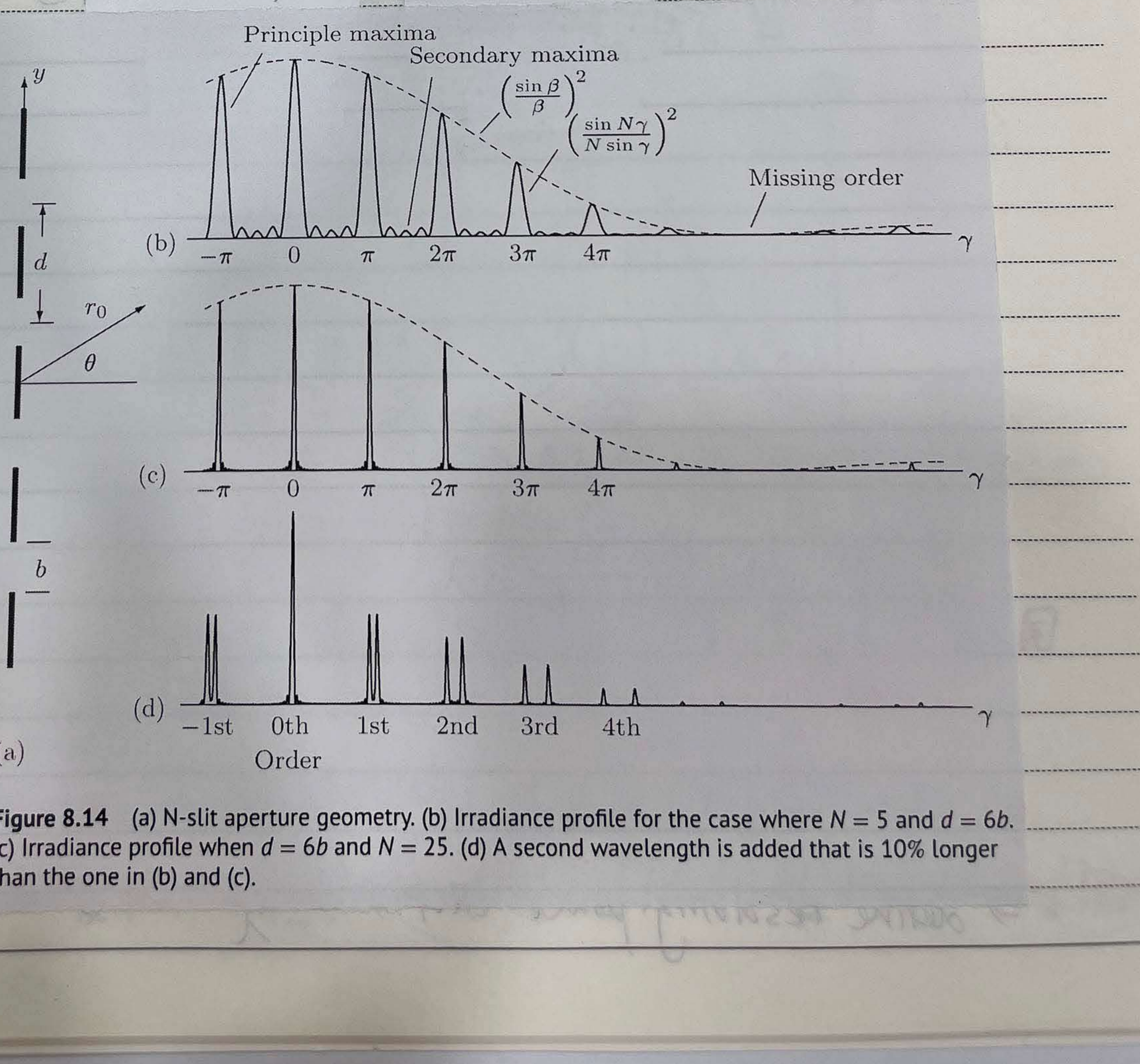


Figure 8.14 (a) N-slit aperture geometry. (b) Irradiance profile for the case where $N = 5$ and $d = 6b$. (c) Irradiance profile when $d = 6b$ and $N = 25$. (d) A second wavelength is added that is 10% longer than the one in (b) and (c).

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① $y=0$, $r=0$, $I_p(0)=I_0$

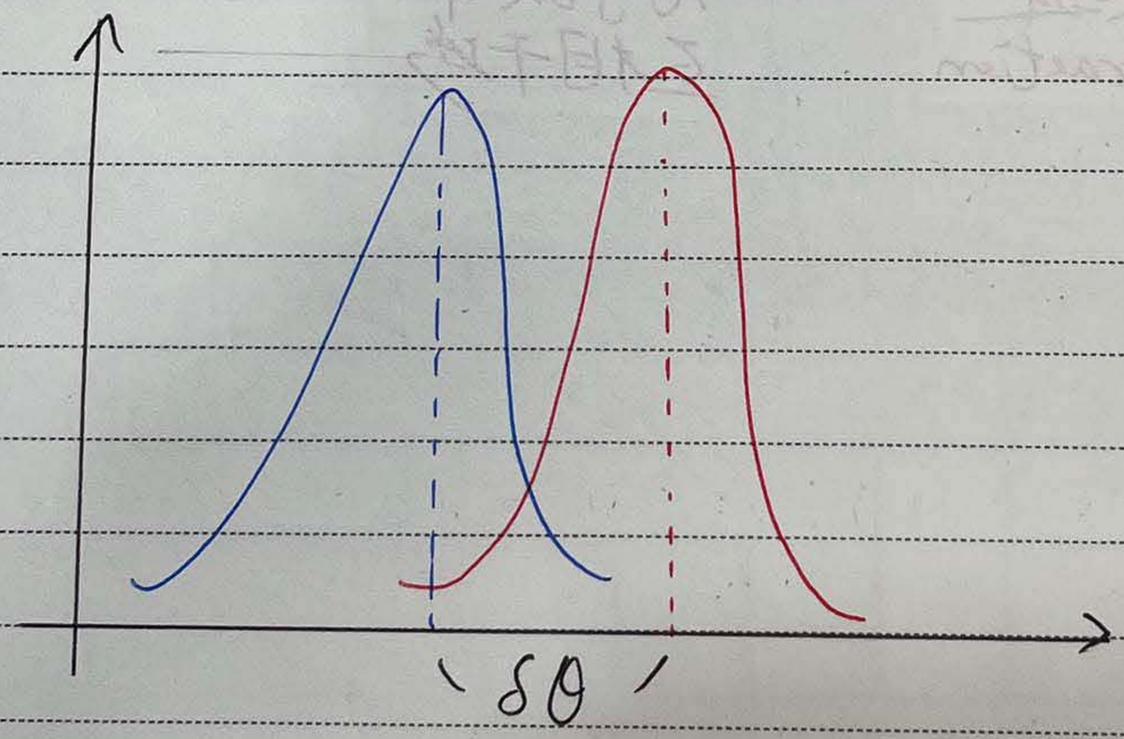
⇒ principal max occur at $\sin(N\gamma)=0$
and $\sin\gamma=0$ at $\gamma=m\pi$, $m=0,\pm1,\pm2,\dots$,

$$I_{p,max}=I_0\,\mathrm{sinc}^2\frac{by}{\lambda s}$$

⇒ 各极位發生在 $y=m\frac{\lambda s}{d}$.

但由於 envelope 也有自身之零点，有些 principle maximum 会被压掉！

② Rayleigh criterion:
∵ 波長不同
∴ diffracted lines are resolvable
if $\delta\theta\geq\Delta\theta$



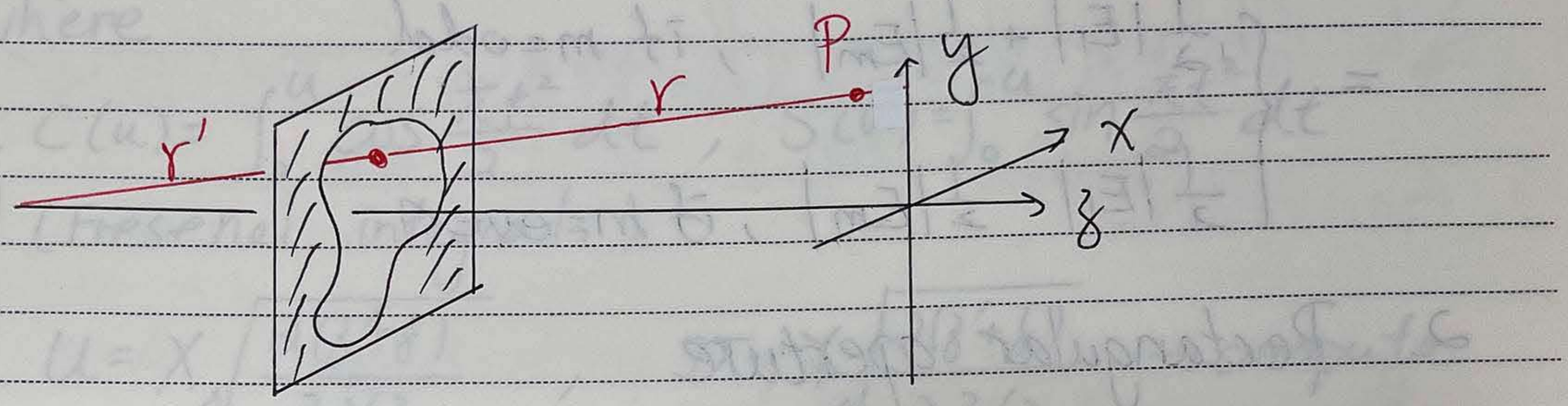
$$\begin{cases}\delta\theta=\text{(?)}\\ \Delta\theta=\dfrac{\lambda}{Nd\cos\theta}\end{cases},\quad \lambda=\frac{d}{m}\sin\theta$$

⇒ 最小可分辨之波長差 $\Delta\lambda=\frac{d}{m}\cos\theta\Delta\theta$,

⇒ define resolving power (R.P.) $\triangleq\frac{\lambda}{\Delta\lambda}$

§4. Fresnel diffraction

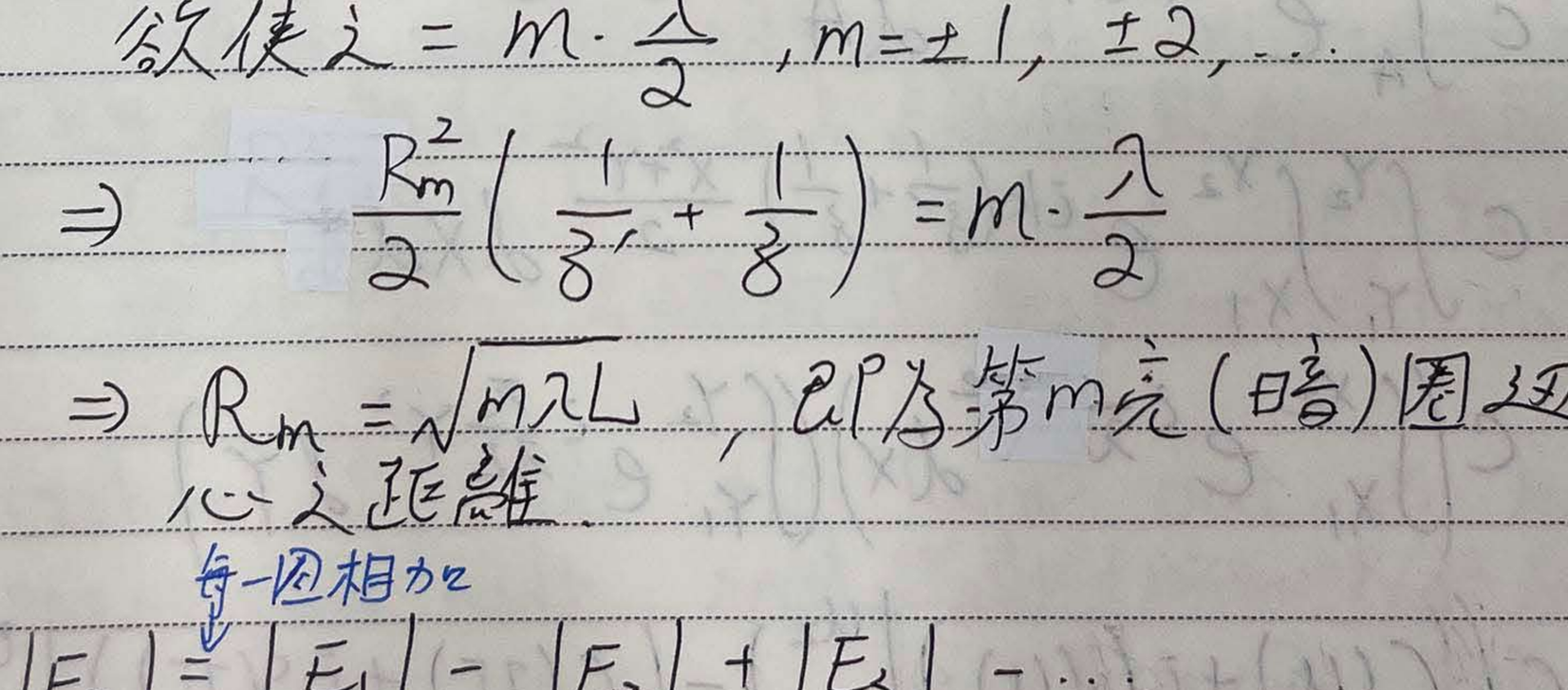
Date. (8.4) Page.



When either r', r is finite
→ 非≡ far-field!
⇒ $E_p = C\int_A e^{ik(r'+r)}\,dA$, 一般只能是 numerical calculation

1. Circular apeture: Fresnel zone

(1)



↳ 看成同心圓

⇒ 每一圈上任一点之光程差 $=(r'+r)-(z'+z)$,
欲使之 $= m\cdot\frac{\lambda}{2}$, $m=\pm1, \pm2, \ldots$

⇒ $\frac{R_m^2}{2}\left(\frac{1}{z'}+\frac{1}{z}\right)=m\cdot\frac{\lambda}{2}$

⇒ $R_m=\sqrt{m\lambda L}$, 即为第 m 亮(暗)圈边界距圆心之距離.

每一圈相加

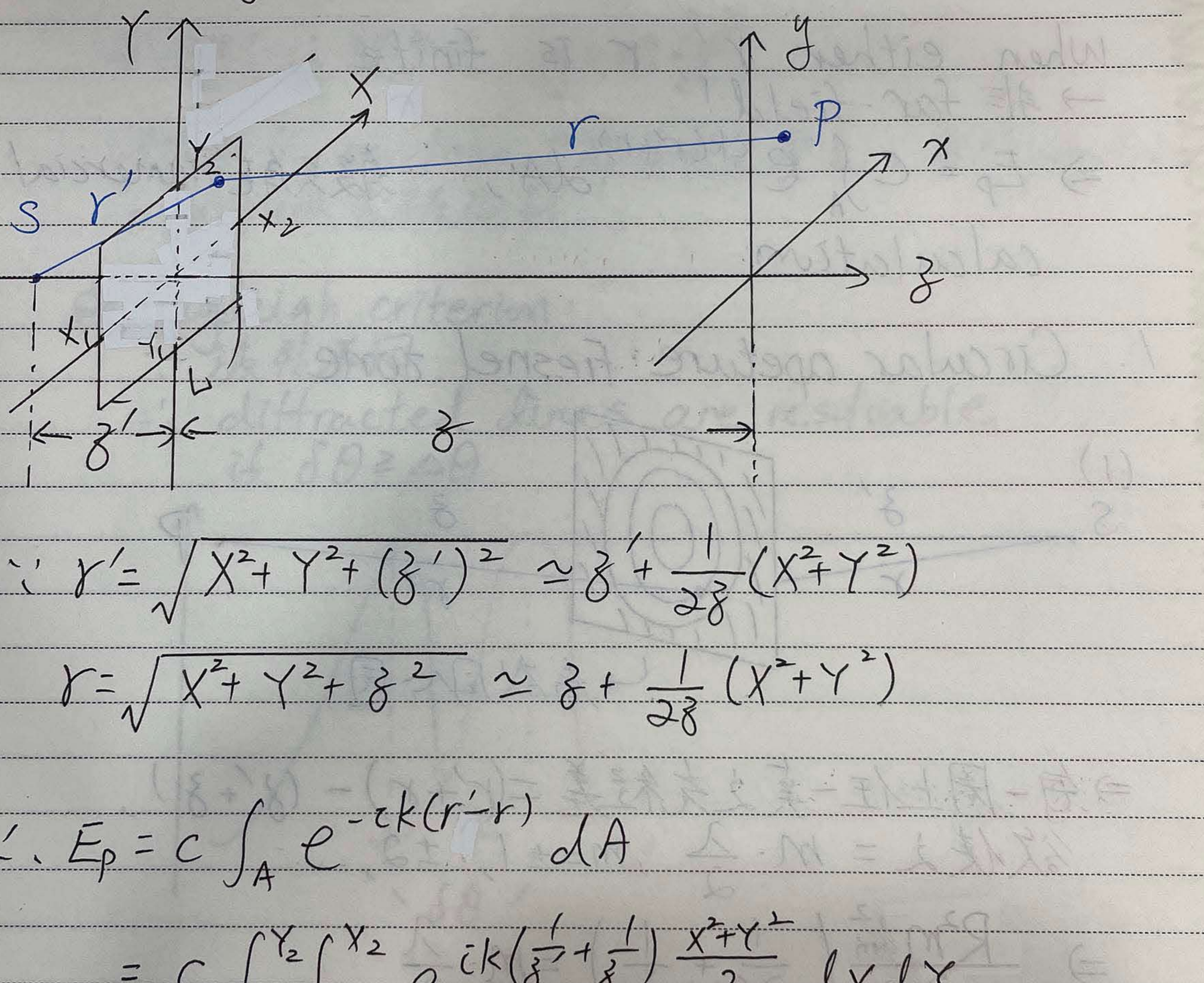
(2) $|E_p| = |E_1| - |E_2| + |E_3| - \cdots$

$= \frac{1}{2}|E_1| + \left(\frac{1}{2}|E_1| - |E_2| + \frac{1}{2}|E_3|\right) + \left(\frac{1}{2}|E_3| - |E_4| + \frac{1}{2}|E_5|\right)$

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$$= \begin{cases} \frac{1}{2}|E_1| + \frac{1}{2}|E_m| & , \text{if } m = \text{odd} \\ \frac{1}{2}|E_1| - \frac{1}{2}|E_m| & , \text{if } m = \text{even} \end{cases}$$

2. Rectangular aperture



$$\therefore r' = \sqrt{X^2 + Y^2 + (z')^2} \simeq z' + \frac{1}{2z}(X^2 + Y^2)$$

$$r = \sqrt{X^2 + Y^2 + z^2} \simeq z + \frac{1}{2z}(X^2 + Y^2)$$

$$\therefore E_p = C \int_A e^{-ik(r'-r)} dA$$

$$= C \int_{Y_1}^{Y_2} \int_{X_1}^{X_2} e^{ik\left(\frac{1}{z'} + \frac{1}{z}\right)\frac{X^2+Y^2}{2}} dX dY$$

$$= C \left(\int_{X_1}^{X_2} e^{i\frac{\pi}{\lambda L}X^2} dX \right) \left(\int_{Y_1}^{Y_2} e^{i\frac{\pi}{\lambda L}Y^2} dY \right)$$

$$= C \left[\left(C(u) + iS(u) \right)\Big|_{u_1}^{u_2} - \left(C(v) + iS(v) \right)\Big|_{v_1}^{v_2} \right]$$

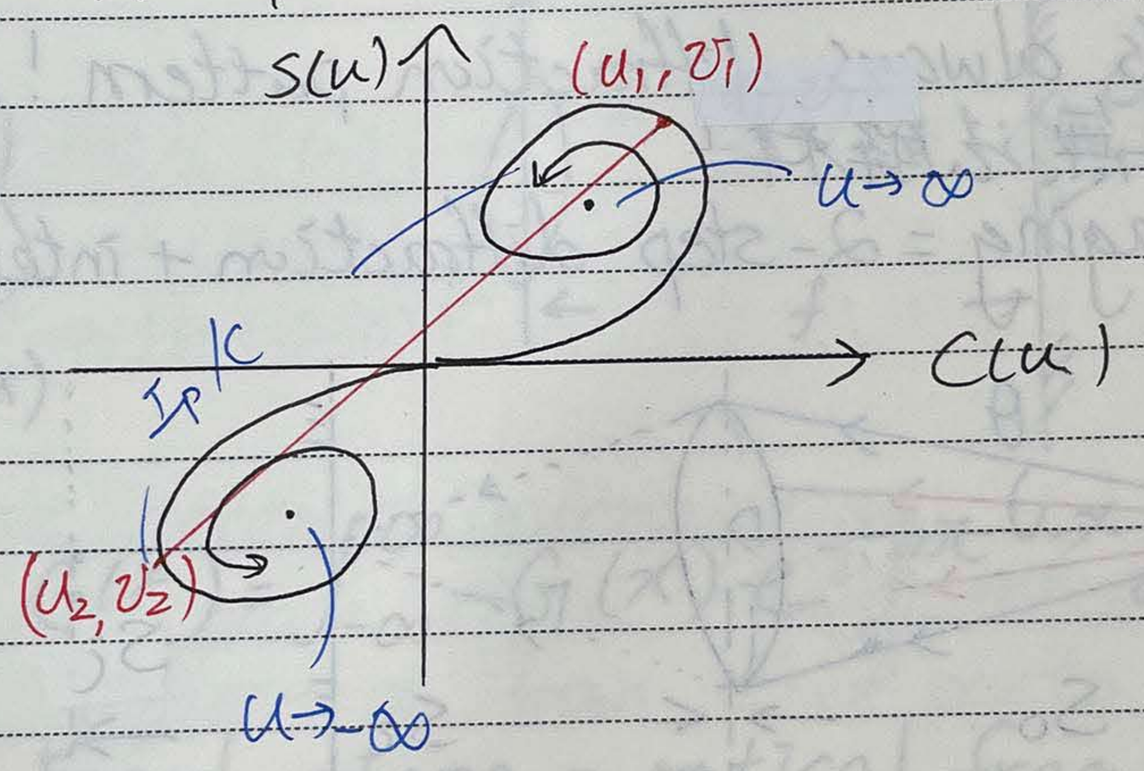
Date. Page.

where

$$\begin{cases} C(u)=\int_0^u \cos\frac{\pi t^2}{2}\,dt\ ,\ S(u)=\int_0^u \sin\frac{\pi t^2}{2}\,dt \\ \text{(Fresenel integrals)} \\ u=X\sqrt{\frac{2(\beta'+\beta)}{\lambda\beta'\beta}}\ ,\quad v=Y\sqrt{\frac{2(\beta'+\beta)}{\lambda\beta'\beta}} \rightarrow \text{compute} \end{cases}$$

$u_1,\ u_2,\ v_1,\ v_2$

$$\Rightarrow I_p=|E_p|^2\propto\left|C(u_2)-C(u_1)\right|^2+\left|S(v_2)-S(v_1)\right|^2$$

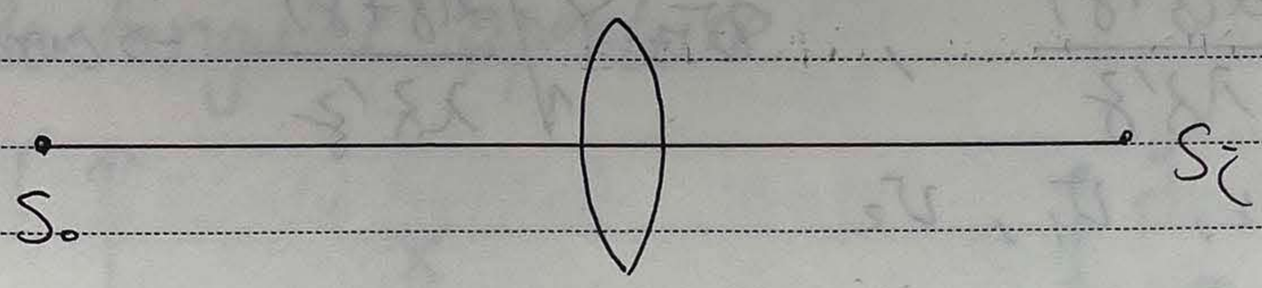


⇒ I_p 之物理意義：積分上下限在 Cornu spiral 上對應之兩點間長度

Date. Page. Leeture 9. Optical Imaging

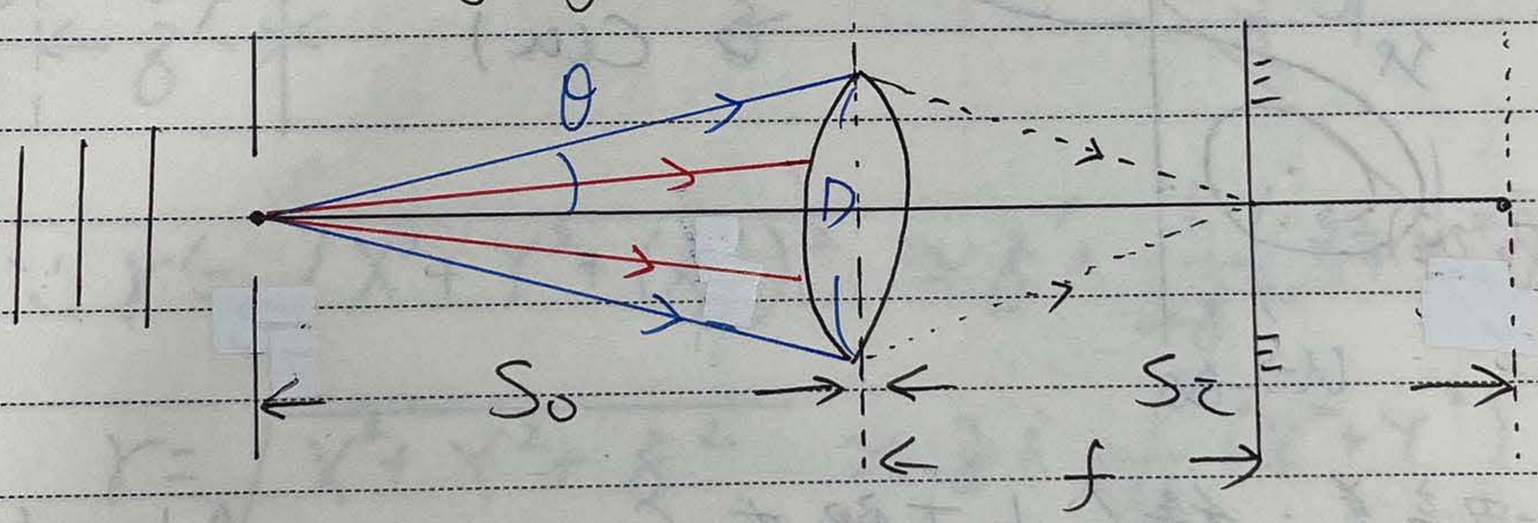
§1. Abbe's theory of image formation (10.2)

1. Geometric.



⇒ Object = $\sum_i$ (point i)

but there's always diffraction pattern!
→ 几何光学無法解釋!
→ 考慮 imaging = 2-step diffraction + interference



To be imaging, we require at one order of diffraction reach the image plane
⇒假設有 n 條狹縫間距為 d 之平板, 希望 image 也是 n 條

∵ Grating : $d\sin\theta = m\lambda$

∴ Consider $m=1 \Rightarrow d\sin\theta = \lambda$

∵ If d is too small s.t. the 1st order diffraction miss the lens, then no image can be format.

∴ $\sin\theta \geq \frac{D/2}{S_0} \approx \frac{1}{2}\frac{D}{f} = \frac{\lambda}{d_{min}}$

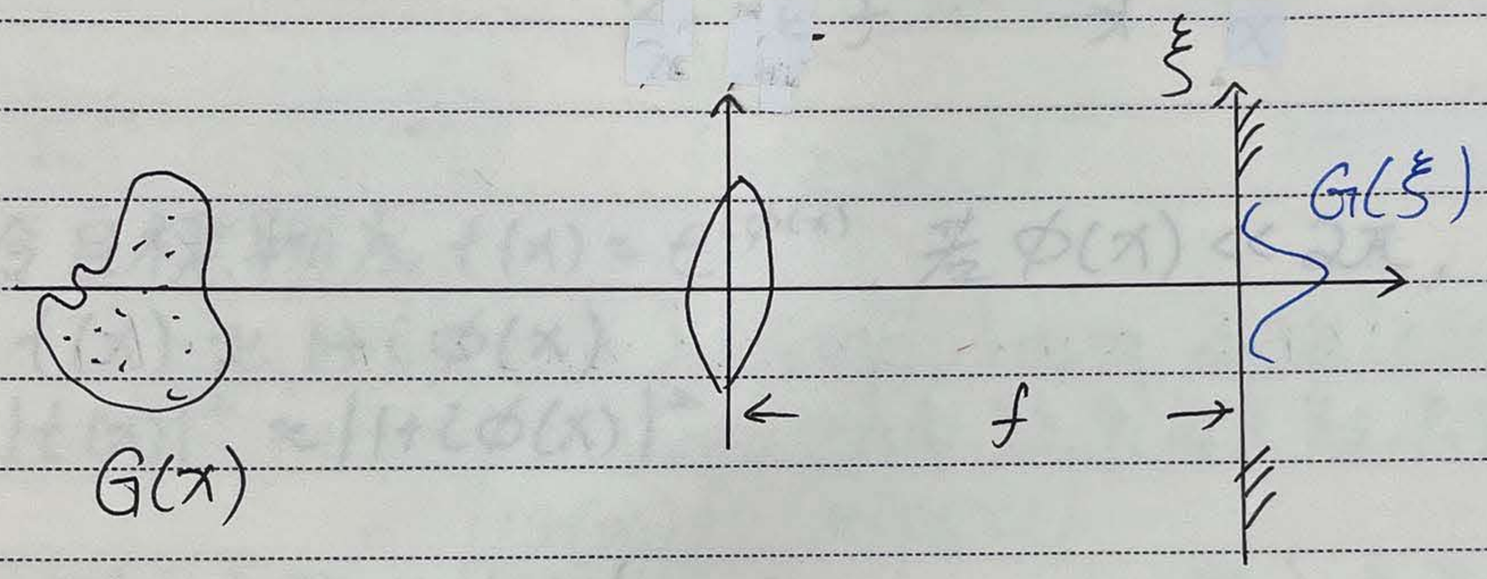
⇒ 对一个 lens D 来說，最大可容纳之角度为 θ，其中

$$\frac{1}{d_{min}} = \frac{\sin\theta}{\lambda} = \frac{1}{2}\frac{D}{\lambda f} \triangleq \frac{N.A.}{\lambda}, \text{ where}$$

N.A. = numerical apeture

⇒ resolving spot size $\delta_{min} = \frac{1}{2} d_{min} = \frac{1}{2}\frac{\lambda}{N.A.}$.

2. 以 Fraunhofer diffraction 的觀点来看：



$$\because G(\xi) = \int_{-\infty}^{\infty} G(x) e^{-i2\pi u x} dx, \quad u = \frac{\xi}{\lambda f}$$

∴ Focal plane = optical freq. plane.

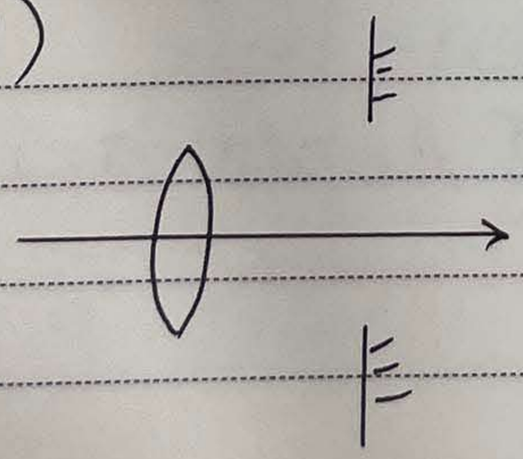
At focal plane,

$\xi = 0 \leftrightarrow u = 0$, unity

$\xi > 0 \leftrightarrow u > 0$, when $\xi \uparrow$, $u \uparrow$ → grating densitude ↓ (條紋更細！)

⇒ 在焦平面上放障礙有以下效果：

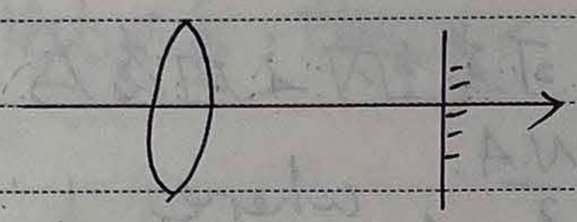
(1)



⇒ 把高頻的光擋住，影像較平滑

Date. Page.

(2)



⇒把低频光挡住

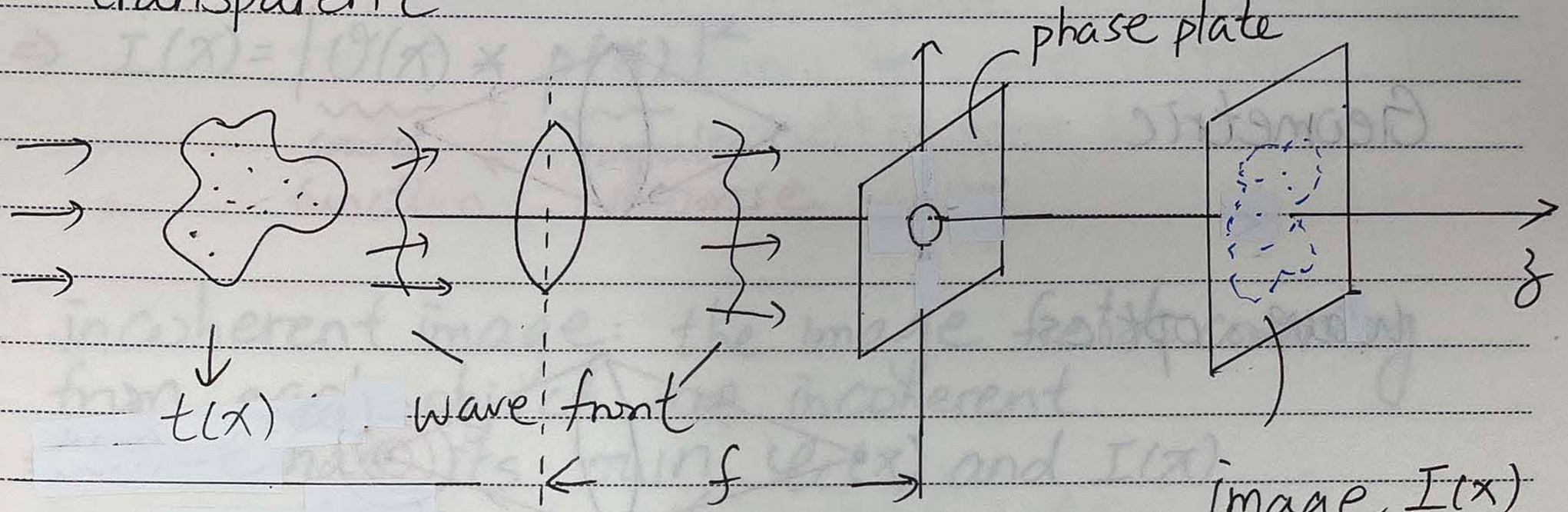
⇒ A freq. filtering can be performed at the focal plane to change the image characteristics

相　　襯　　顯微術

§2. Phase-contrast microscopy (10.2)

Date.　　Page.

1. Application: biology specimen, phase objects, transparent



∵令目標物為 $t(x)=e^{i\phi(x)}$，若 $\phi(x)\ll 2\pi$，則

$$\begin{cases} t(x)\approx 1+i\phi(x) \\ |t(x)|^2\approx|1+i\phi(x)|^2 \end{cases}$$

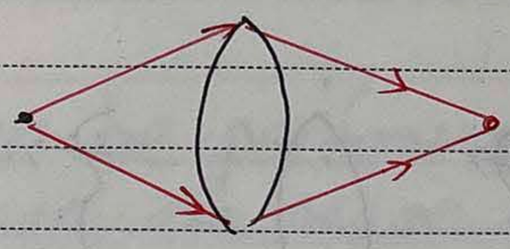
2. Phase contract microscopy:
A phase plate as a filter center with $\frac{\pi}{2}$-plane difference from other parts

$$\Rightarrow \text{image} \triangleq I(x)$$
$$= \left|e^{i\frac{\pi}{2}}+e^{i\phi(x)}\right|^2$$
$$= \sim$$

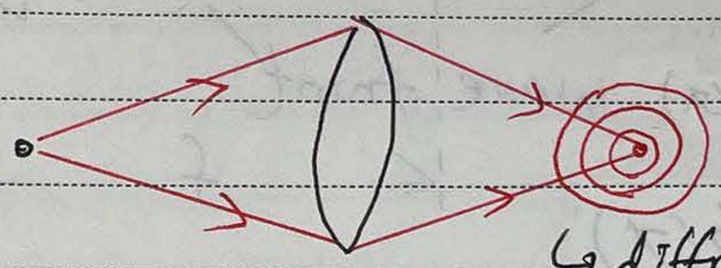
Date. Page. § 3. Point-spread function (10.3)

1. Linear system theory of imaging.

Geometric



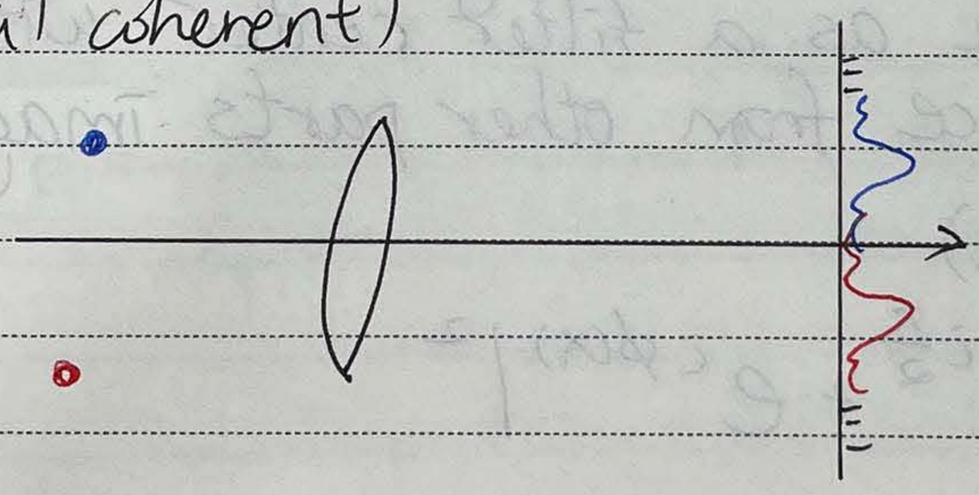
Wave optics



↳ diffraction pattern p(x) ≠ a point

⇒ 将p(x)称为 point spread function，如同电机系、訊号与系統中的impulse response

2. 空間不变（從time-invariant抄来的觀念）
(spatial coherent)



⇒ point spread function之形式不隨source之位置而改变。即：

$$\begin{cases} \delta(x) \to p(x) \\ \delta(x-a) \to p(x-a) \end{cases}$$

$$\Rightarrow \underbrace{\sum_i A_i \delta(x-a_i)}_{\text{taking limits,}} \triangleq o(x) \longrightarrow \text{Image}: \underbrace{\sum_i A_i p(x-a_i)}_{p(x)} \quad -(*)$$

Date. Page.

$$P(x) = \int_{-\infty}^{\infty} \mathcal{O}(X)\, p(x-X)\, dX \rightarrow \text{convolution!}$$

$$\Rightarrow I(x) = |\underbrace{\mathcal{O}(x)}_{\text{source function}} * \underbrace{p(x)}_{\text{impulse response}}|^2$$

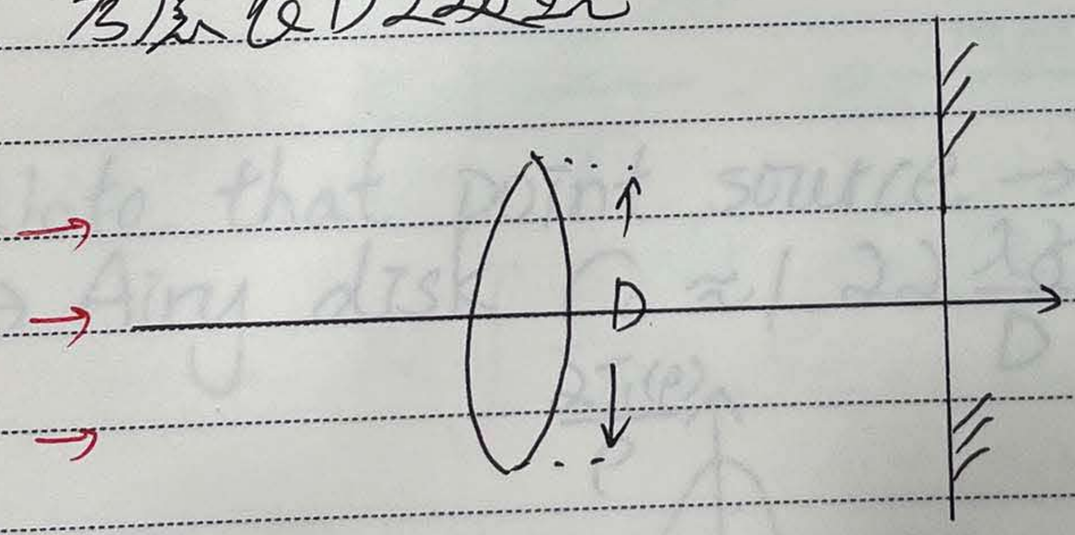
3. incoherent image: the image field comming from each object are incoherent.
⇒ Each has its own $\mathcal{O}_i(x)$ and $I(x)$

$$\Rightarrow \text{image} = \sum_i I_i |p(x-a_i)|^2$$

$$\Rightarrow I_{image}(x) = \int_{-\infty}^{\infty} |\mathcal{O}(x)|\, |p(x-X)|^2 dX$$

$$= |\mathcal{O}(x)| * |p(x)|^2$$

4. 考慮有D之透鏡



(1) D↑ → point spread function 越細

(2) 實際上 $p(x) = \mathcal{F}\{\text{pupil function}\}$, 比如

$\text{pupil} = \text{rect}(\xi/D) \longleftrightarrow p(x) = \text{sinc}\left(\frac{Dx}{\lambda f}\right)$.

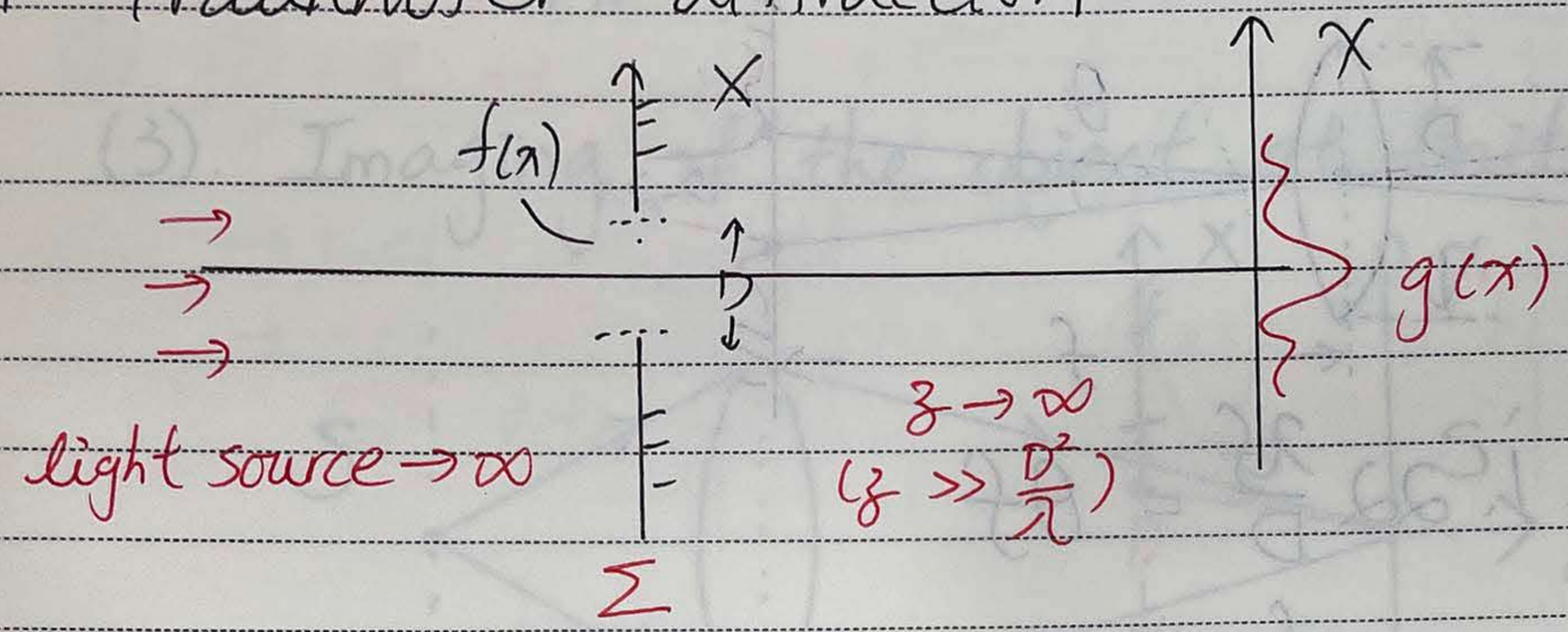
Date. Page. Lecture 10. Diffraction & Imaging.

* Abbe: Wave theory of imaging
  - resolution limit of instruments
  - Spatial freq. of waves
  - spatial freq. filtering

§1. Resolution limit of optical instruments (10.4)

Date. Page.

1. Fraunhofer diffraction



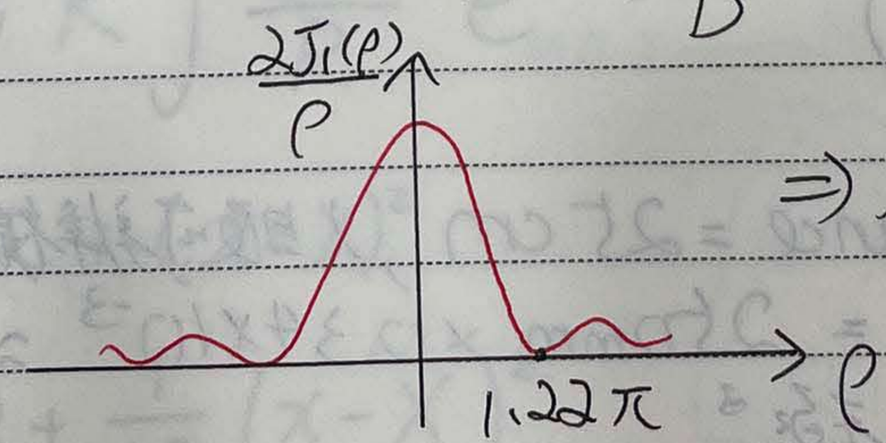
$$\Rightarrow g(x) = K\int_{\Sigma} f(x) e^{-i\frac{2\pi}{\lambda z}Xx}\,dX$$

$$= K\int_{-\infty}^{\infty} f(x) e^{-2\pi i u X}\,dX,\quad u = \frac{x}{\lambda z}$$

ex. $\mathrm{rect}\left(\frac{X}{D}\right) \longrightarrow \mathrm{sinc}\left(\frac{Dx}{\lambda z}\right)$

$$\mathrm{circ}\left(\frac{X}{D/2}\right) \rightarrow \frac{2J_1(\rho)}{\rho},\quad \rho = \frac{\pi D x}{\lambda z}$$

Note that point source → Fraunhofer diffraction
→ Airy disk $r_0 \approx 1.22\frac{\lambda z}{D}$

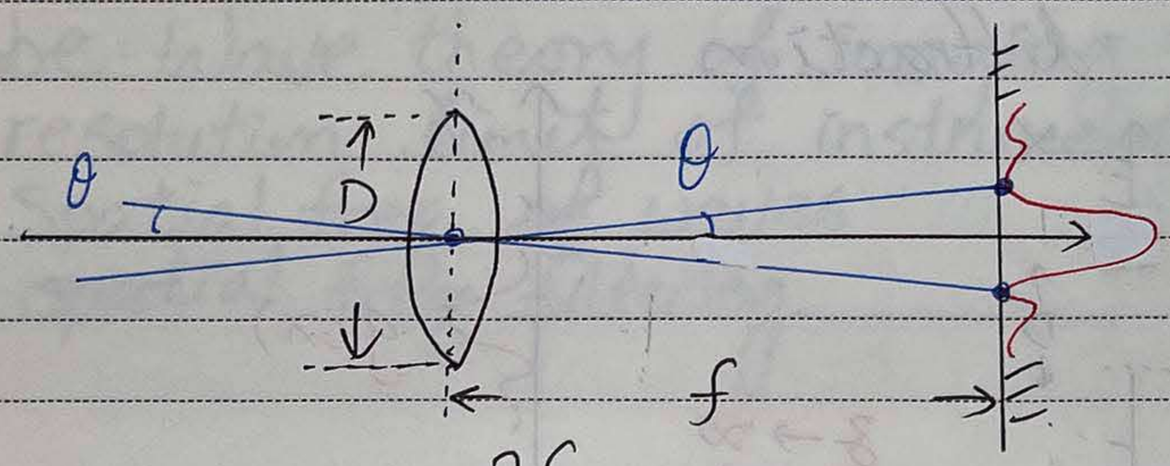


⇒ 解析极限为 Airy disk 之张角.

2. Examples.

(1) Telescope

Date. Page.



$\therefore r_0 = 1.22\frac{\lambda f}{D} = \theta f$

$\therefore \theta = 1.22\frac{\lambda}{D}$

(2) Human eyes.

若(1)可用同一弦图理解. 人数瞳孔直径约 $D \approx 2mm$, 取 $\lambda = 550nm$, 眼焦 18~25 mm, 得

$$\theta = 1.22\frac{\lambda}{D}$$

$$= 1.22 \times \frac{550nm}{2mm}$$

$$\approx 0.34 \times 10^{-3}\ rad$$

$$\approx 1'\ (1角分)$$

① When object distance = 25 cm, 人眼可辨视之最小间距为

$(\Delta X)_{min} \triangleq \delta_{eye} = 250mm \times 0.34 \times 10^{-3} \approx 80\mu m$,

约为一根头发之宽

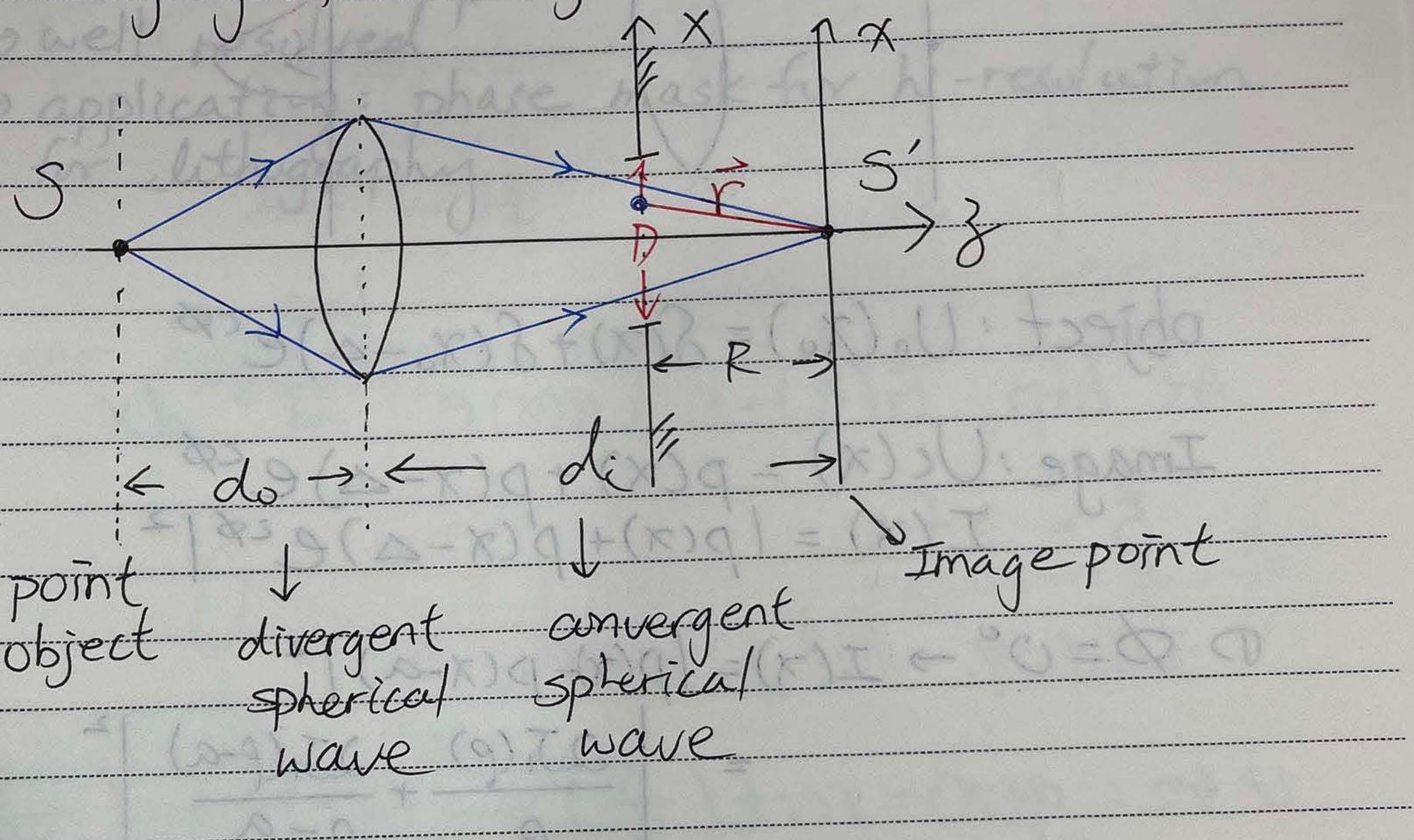
② 印表机解析度取 300 dpi (dots per inch), 即为①之应用

③ Telescope eye piece 放大倍率

Date. Page.

$$M = \frac{\delta_{eye}}{r_o}$$

(3). Imaging of the object at finite distance



Exit pupil at R.

$$g(x) = K \int \frac{e^{-ikr}}{r} \cdot e^{i\frac{\pi}{\lambda R}(x-X)^2} dx$$

$$\approx K \int \frac{e^{-ikr}}{R} e^{i\frac{\pi}{\lambda R}(x-X)^2} dX$$

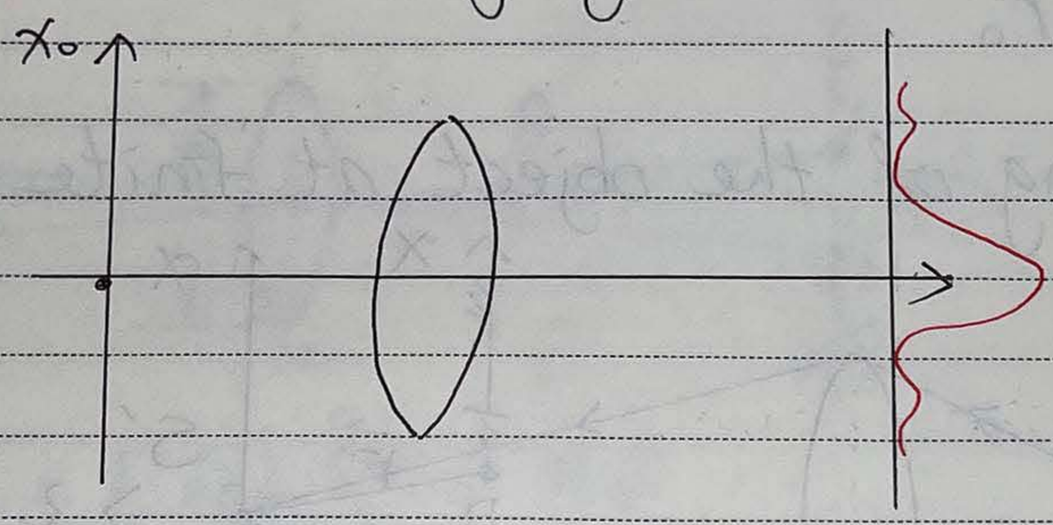
$$\therefore r = \sqrt{R^2 + (x-X)^2}$$

$$\approx R + \frac{1}{2R}(x-X)^2$$

$$\therefore g(x) \sim K \int e^{-i\frac{2\pi}{\lambda R} xX} dX$$

Date. Page.

(4) Coherent Imaging



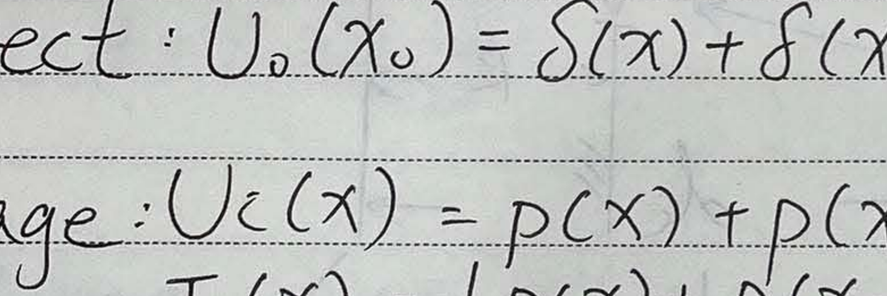
object : $U_0(x_0) = \delta(x) + \delta(x_0 - \Delta) e^{i\phi}$

Image : $U_i(x) = p(x) + p(x-\Delta) e^{i\phi}$

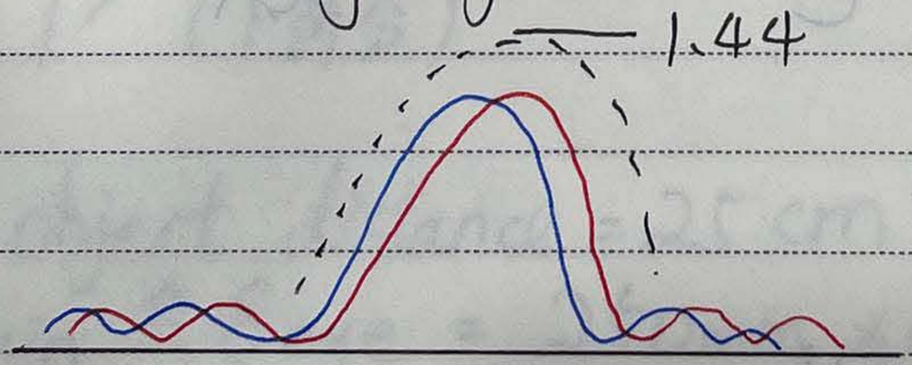
$$I(x) = |p(x) + p(x-\Delta) e^{i\phi}|^2$$

① $\phi = 0^\circ \rightarrow I(x) = |p(x) - p(x-a)|^2$

$$= \left| \frac{2J_1(\rho)}{\rho} + \frac{2J_1(\rho-a)}{\rho-a} \right|^2$$



at Rayleigh criterion



$a = 1.22\pi$

⇒ unsolvable!

* Incoherent : $I(x) = \left| \frac{2J_1(\rho)}{\rho} \right|^2 + \left| \frac{2J_1(\rho-a)}{\rho-a} \right|^2$

Date. Page.

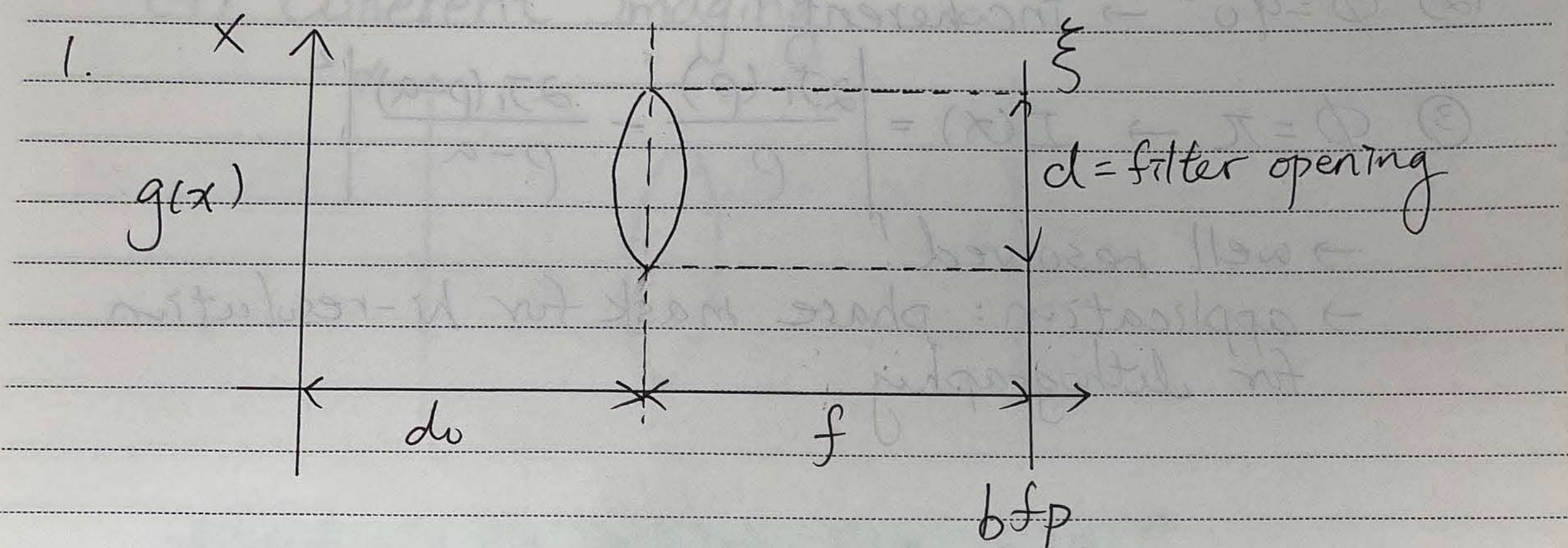
② $\phi = 90°$ → incoherent!

③ $\phi = \pi \rightarrow I(x) = \left| \frac{2J_1(\rho)}{\rho} - \frac{2J_1(\rho-a)}{\rho-a} \right|^2$

→ well resolved!

→ application: phase mask for hi-resolution for lithography.

Date. Page. §2 Spatial filtering (10-7)



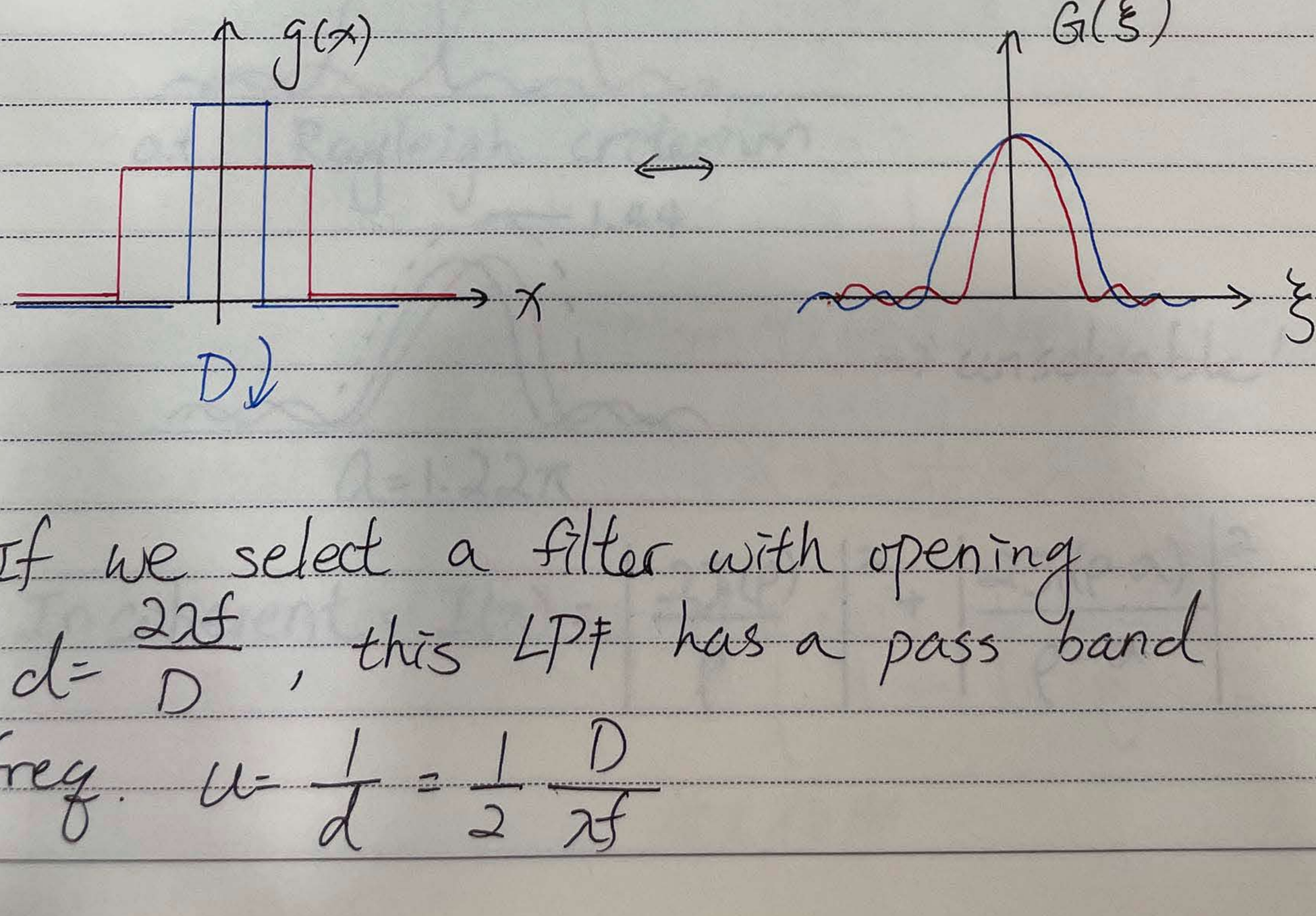
At bfp, $G(\xi) = K \int g(x) e^{-i \frac{2\pi}{\lambda f} \xi x} dx$,

$\xi = \lambda f u$

↑ 空間坐標　↓ 圖案 $g(x)$ 之空間頻率

ex: $g(x) = rect\left(\frac{x}{D}\right) \longleftrightarrow G(\xi) = sinc\left(\frac{D\xi}{\lambda f}\right)$

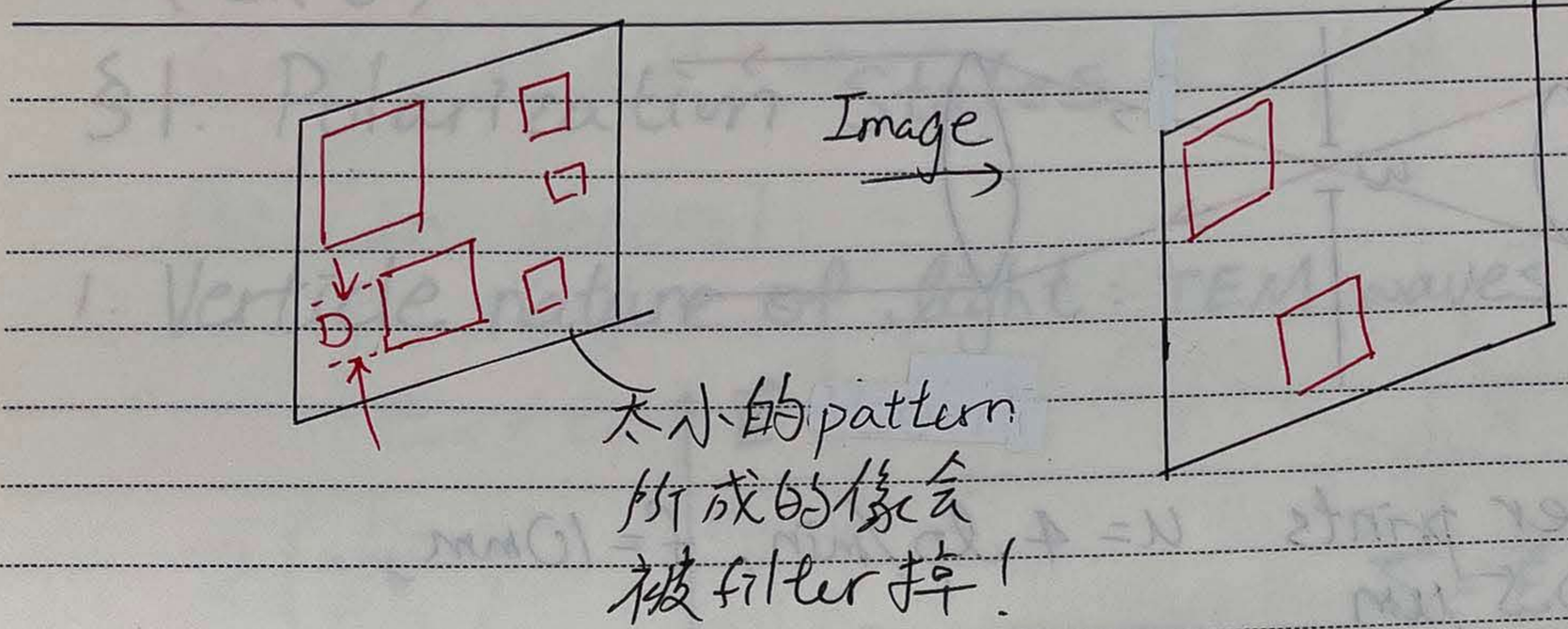
為一 LPF



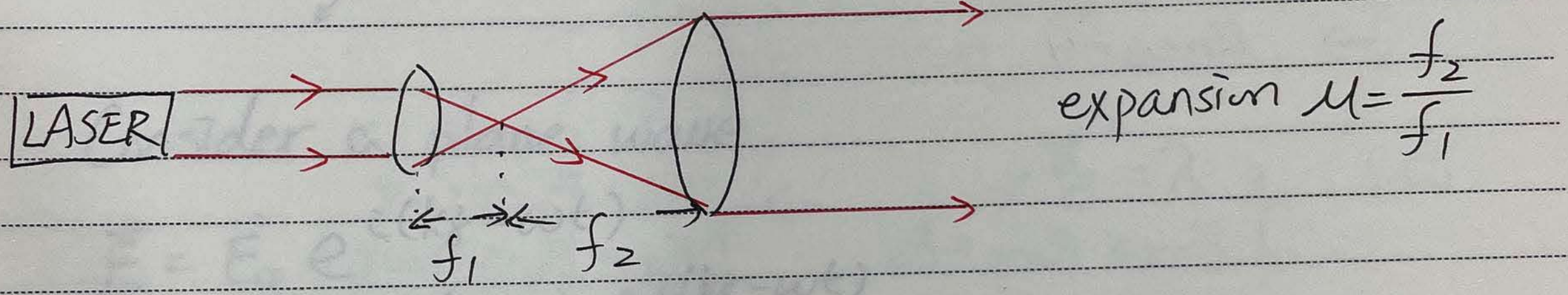
If we select a filter with opening $d = \frac{2\lambda f}{D}$, this LPF has a pass band freq. $u = \frac{1}{d} = \frac{1}{2} \frac{D}{\lambda f}$

L11. Polarization
(Ch 6)

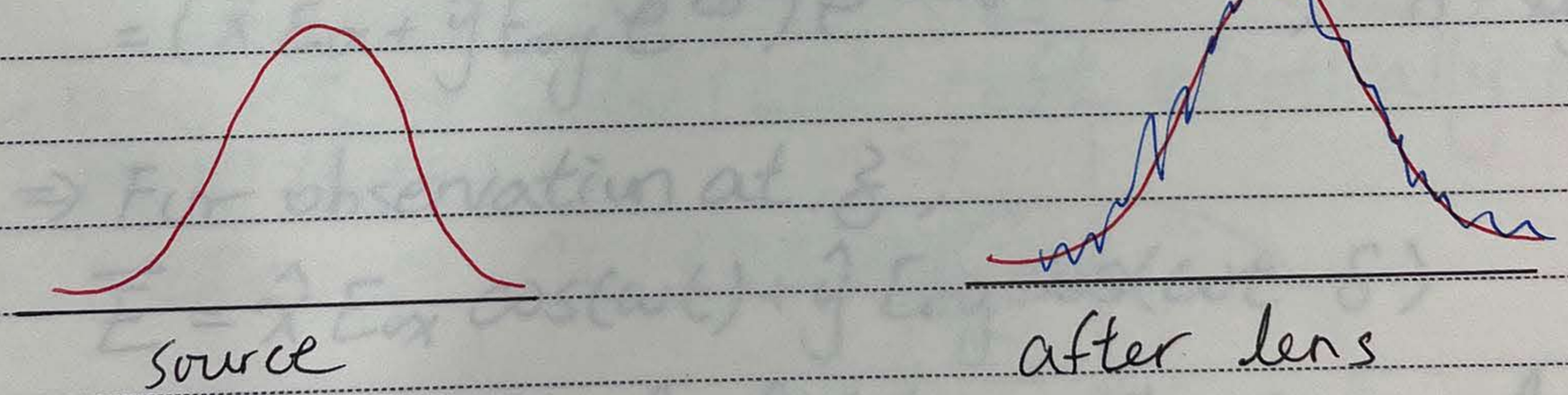
Date. Page.



2. Laser beam spatial filter



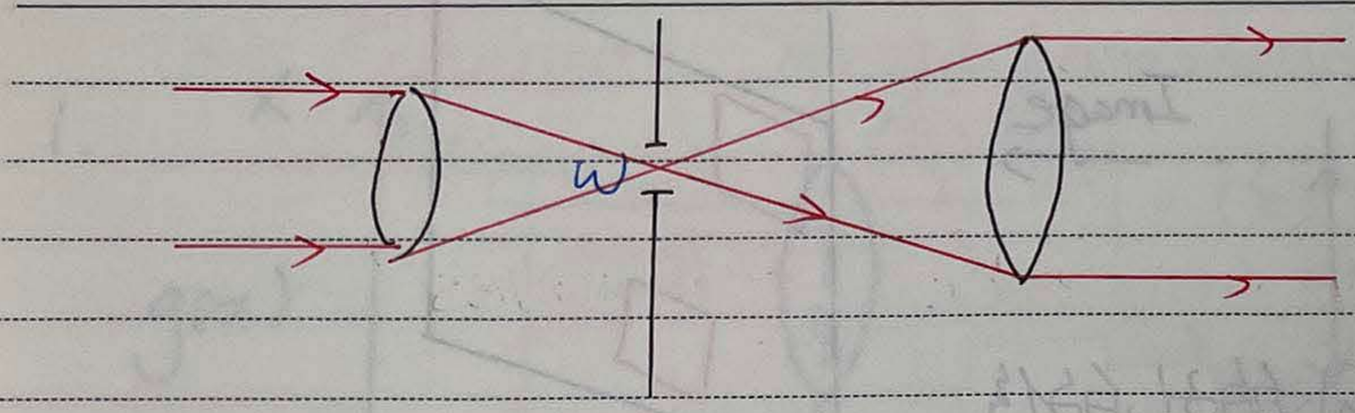
but 就算会变窄!



→ 这是因为 dust particles, finger prints 造成 high freq noises.

→ 解决方法：在焦点上置一 pinhole，形成 HPF，其中 pinhole 宽 $w > r$。

Date. Page.



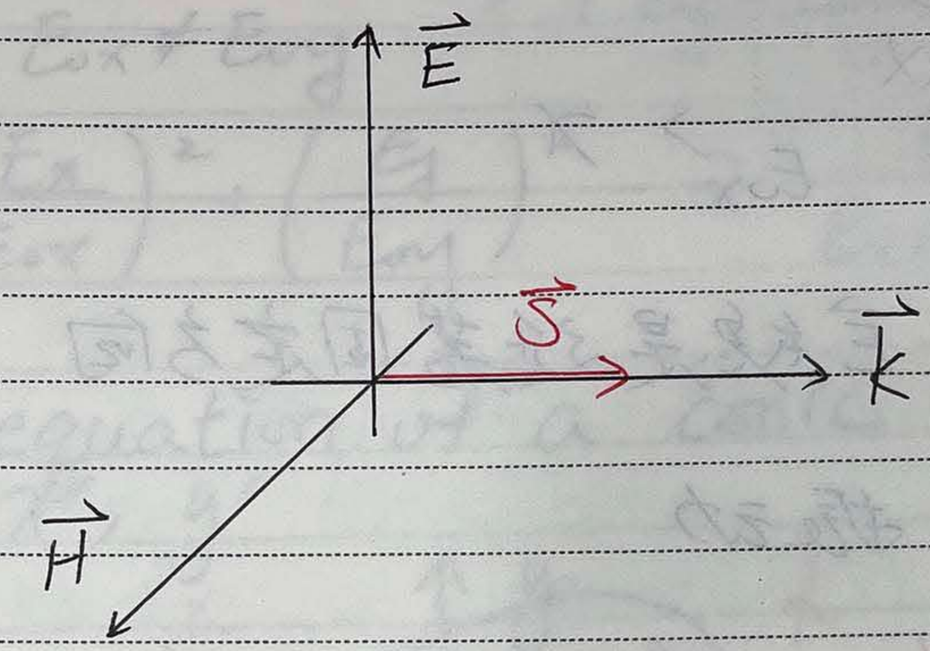
ex. finger prints $u = 4\ lp/mm$, $f = 10mm$,
$\lambda = 0.5\ \mu m$
$\Rightarrow w \leq 20\ \mu m$.

L11. Polarization (Ch 6)

Date. Page.

§1. Polarization States

1. Verticle nature of light : TEM waves



$$\vec{S} = \vec{E} \times \vec{H}$$

Consider a plane wave

$$\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$$
$$= (\hat{x}E_x + \hat{y}E_y) e^{i(kz-\omega t)}$$
$$= (\hat{x}E_{ox} e^{i\phi_x} + \hat{y}E_{oy} e^{i\phi}) e^{i(kz-\omega t)}$$
$$= (\hat{x}E_{ox} + \hat{y}E_{oy} e^{i\delta}) e^{i(kz-\omega t+\phi_x)}, \quad \delta = \phi_y - \phi_x$$

$\Rightarrow$ For observation at z,

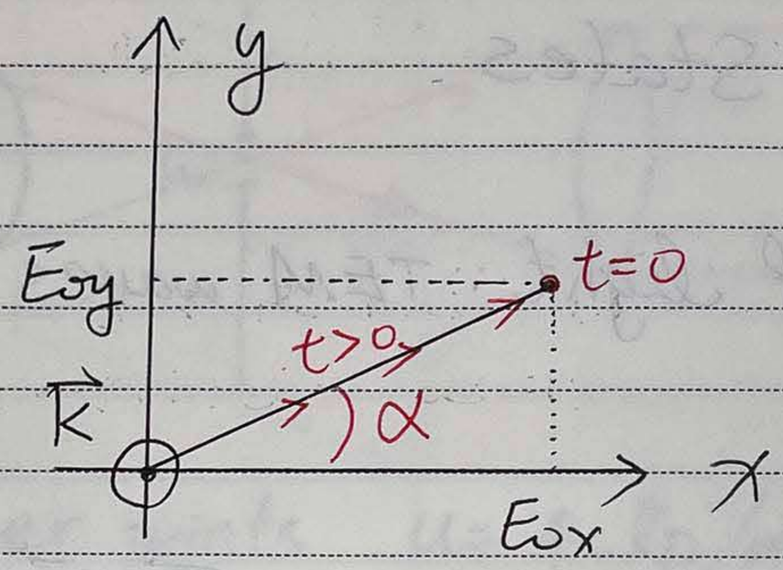
$$\vec{E} = \hat{x}E_{ox}\cos(\omega t) + \hat{y}E_{oy}\cos(\omega t - \delta)$$

* Polarization of light : vibration of $\vec{E}$ depends on $\frac{E_{ox}}{E_{oy}}$, δ.

(1) $\delta = 0 \rightarrow$
$$\begin{cases} E_x = E_{ox}\cos\omega t \\ E_y = E_{oy}\cos\omega t \end{cases}$$

$$\Rightarrow \frac{E_x}{E_y} = \frac{E_{ox}}{E_{oy}}$$

Date. Page.



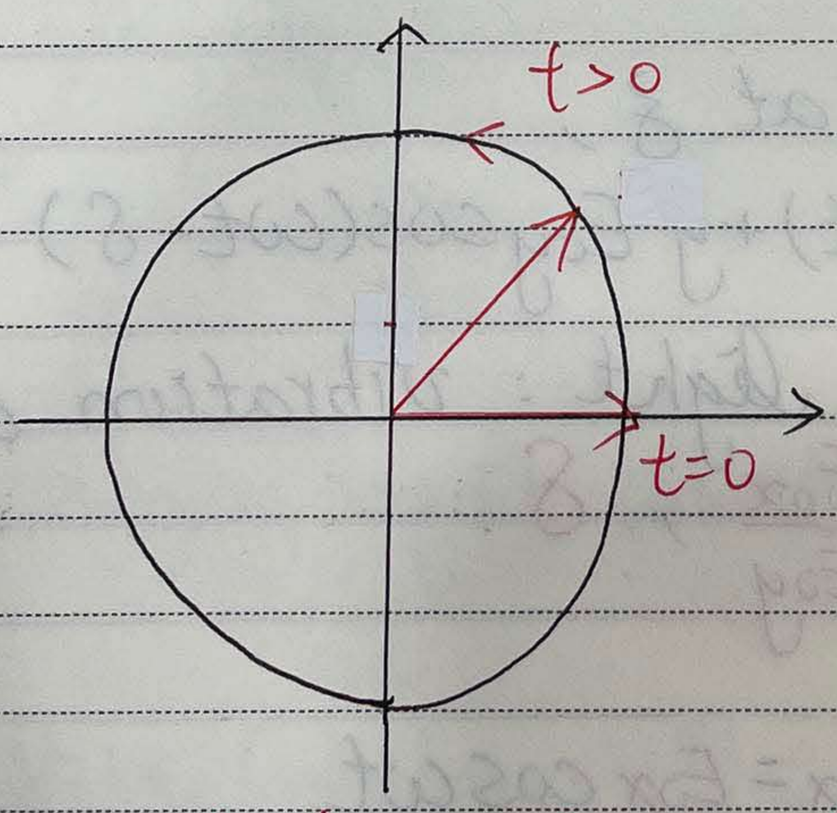
$\Rightarrow \frac{E_x}{E_y} = \frac{E_{ox}}{E_{oy}}$，$\vec{E}$總是沿著固定方向

$\tan\alpha = \frac{E_{oy}}{E_{ox}}$ 振動

$\Rightarrow$ linearly polarized

(2) $\begin{cases} \delta = \frac{\pi}{2} \\ E_{ox} = E_{oy} = E_o \end{cases} \Rightarrow \begin{cases} E_x = E_o \cos\omega t \\ E_y = E_o \sin\omega t \end{cases}$

$\Rightarrow \left(\frac{E_x}{E_o}\right)^2 + \left(\frac{E_y}{E_o}\right)^2 = 1.$



$\Rightarrow$ left circularly-polarized (LCP，左圓偏極化)

application：①有些癌細胞只吸收某种圓偏振光
$\Rightarrow$可用於治療

Date. Page.

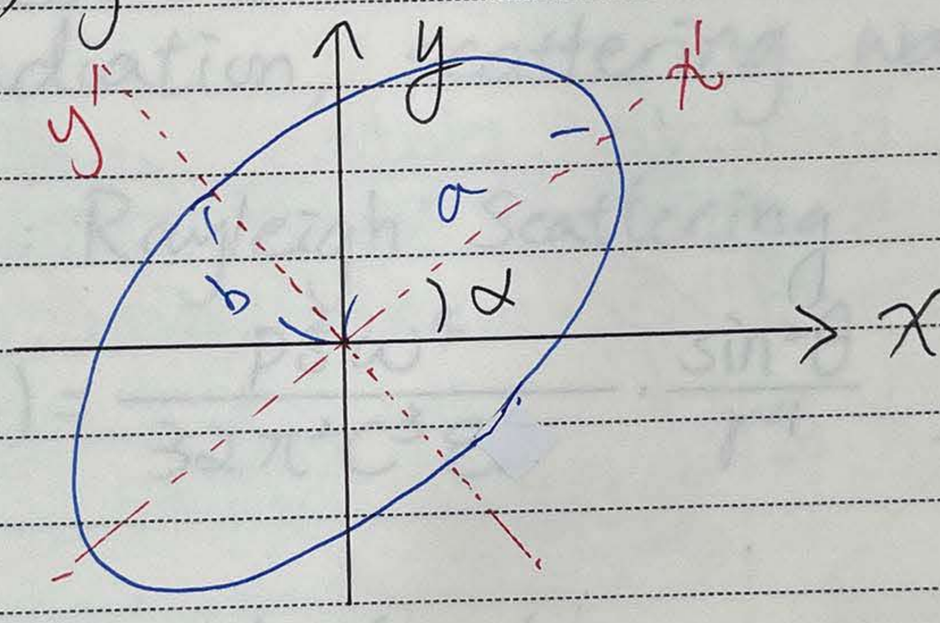
② 葡萄糖有些只吸收左/右旋偏振光

(3). In general,

$$\begin{cases} \delta \neq \pm\frac{\pi}{2} \\ E_{0x} \neq E_{0y} \end{cases} \Rightarrow \begin{cases} E_x = E_{0x}\cos\omega t \\ E_y = E_{0y}\cos(\omega t - \delta) \end{cases}$$

$$\Rightarrow \left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\frac{E_x E_y}{E_{0x}E_{0y}}\cos\delta = \sin^2\delta$$

→ equation of a conic in principal axis x', y'

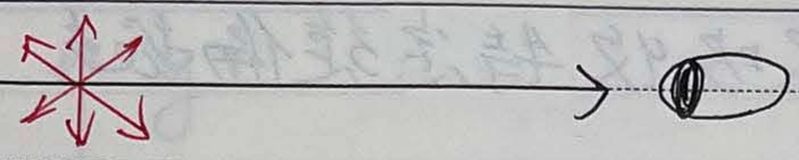


$$\Rightarrow \begin{cases} 0 < \delta < \pi \rightarrow \text{CCW (LEP, Left elliptically polarized, 橢圓偏振)} \\ -\pi < \delta < 0 \rightarrow \text{CW (REP, Right "}) \end{cases}$$

其中, $\begin{cases} a^2 = E_{0x}^2\cos^2\phi + E_{0y}^2\sin^2\phi + 2E_{0x}E_{0y}\cos\delta\cos\phi\sin\phi \\ b^2 = \cdots \\ \tan 2\phi = \dfrac{2E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta \end{cases}$

(4). Natural light → indep. emitters

Date. Page.



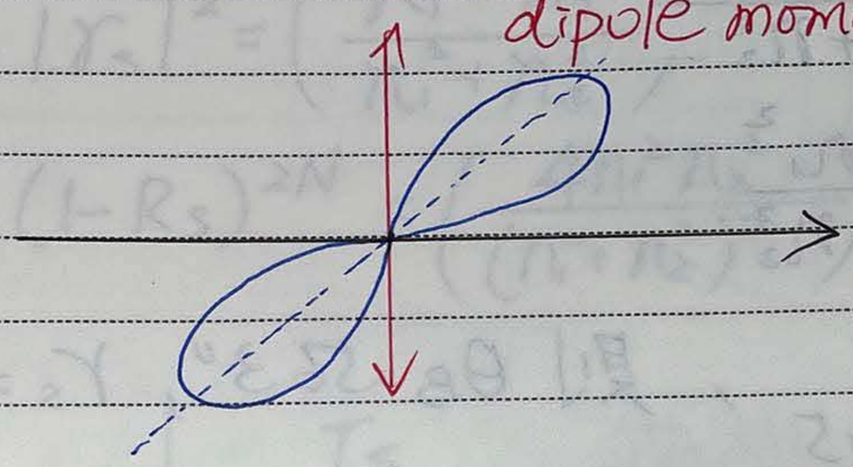
⇒ $\vec{E}$'s initial phase vibration is randomly distributed equal prob. for any direction.

⇒ unpolarized light.

§2. Generating Polarized Light

Date/ Page.

1. Scattering



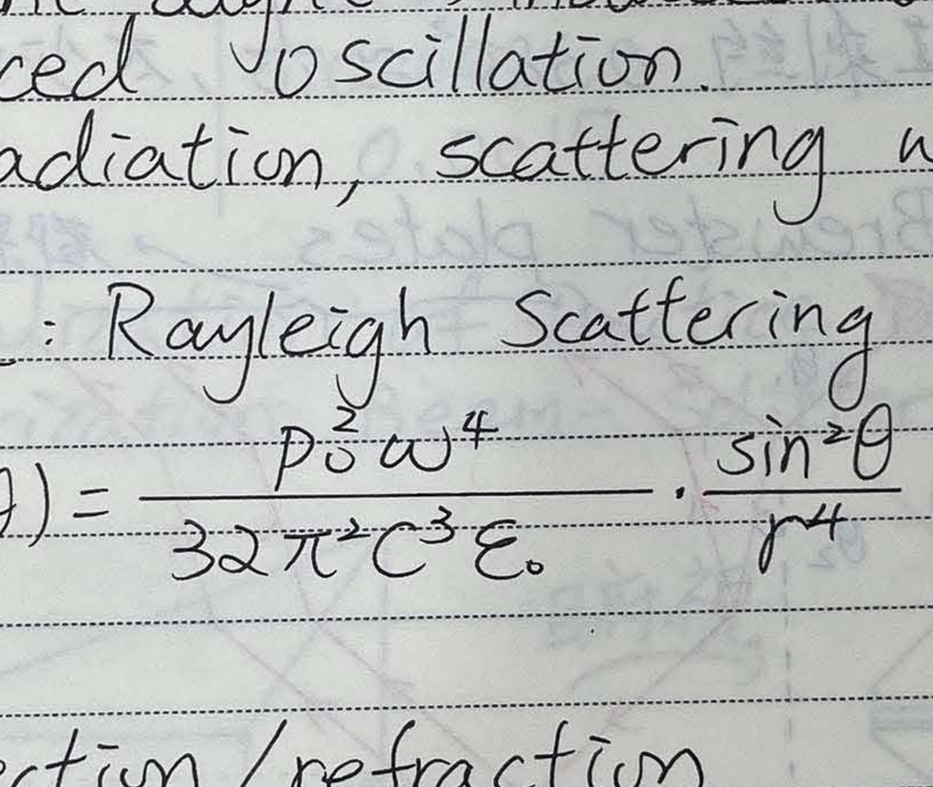
incident light ⇒ induced dipole vibration, forced oscillation.

⇒ re-radiation, scattering wave.

補充: Rayleigh Scattering

$$I(\theta) = \frac{p_0^2 \omega^4}{32\pi^2 C^3 \varepsilon_0} \cdot \frac{\sin^2\theta}{r^4}, \quad \omega = \frac{2\pi}{\lambda} C$$

2. Reflection / refraction



At $\theta_1 = \theta_B$, the reflected wave is s-polarized (linearly polarized).

⇒ $r_p = 0$, $t_p = 1$, where

Date. Page.

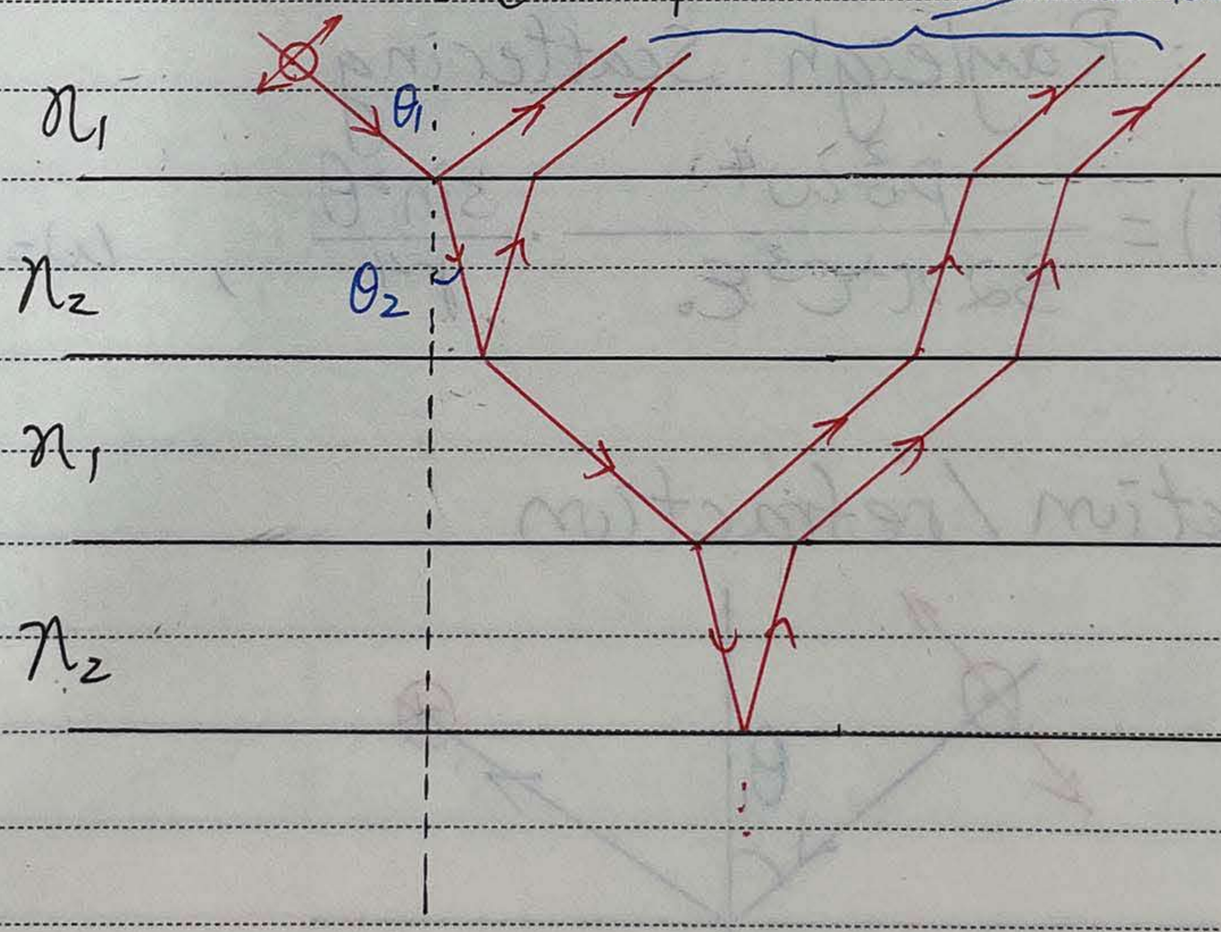
$$\begin{cases} \tan\theta_B = \dfrac{n_2}{n_1} \\ r_S = \dfrac{n_1^2 - n_2^2}{n_1^2 + n_2^2} \\ t_S = \dfrac{2n_1^2}{n_1^2 + n_2^2} \end{cases}$$

比如：当 $\begin{cases} n_1 = 1 \\ n_2 = 1.5 \end{cases}$，则 $\theta_B = 56.3°$，$r_S = 0.385$，

$t_S = 0.615$

⇒ 反射光强度仅剩约 $0.385^2 \approx \frac{1}{10}$，太少了！

(1). Piles of Brewster plates → 都是 S-wave!



At $\theta_1 = \theta_B$, $\because \theta_1 + \theta_2 = \frac{\pi}{2}$ and $\tan\theta_B' = \frac{n_1}{n_2}$

$\therefore \theta_2 = \theta_B'$

$$\Rightarrow r_S' = \frac{n_2^2 - n_1^2}{n_2^2 + n_1^2}$$

$$\Rightarrow R_S' = R_S$$

$$\Rightarrow T_S' = 1 - R_S'$$

$$= 1 - R_S$$

$$= T_S$$

⇒ Net transmission from one-plate $= T_S = (1-R_S)^2$

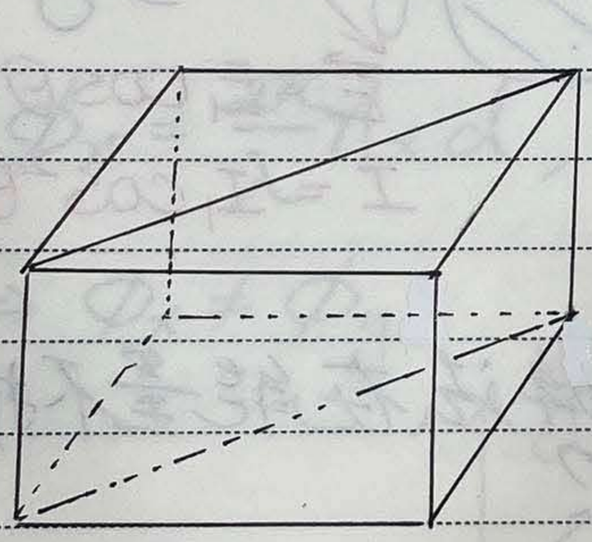
〈 " N-" $T_S = (1-R_S)^{2N}$,

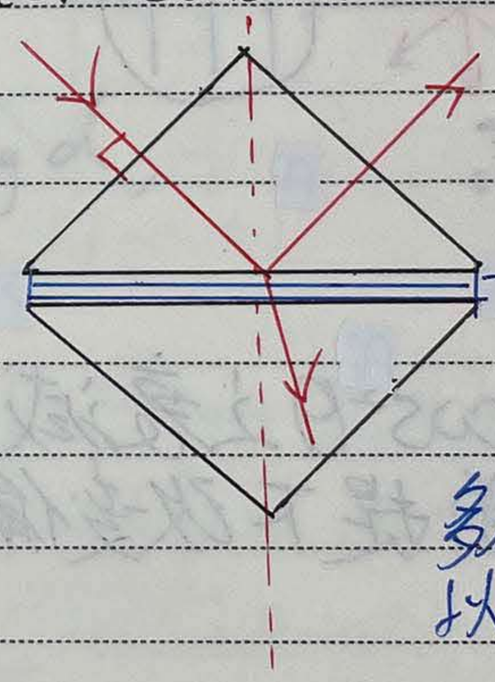
where $R_S = |r_S|^2 = \left(\frac{n_1^2 - n_2^2}{n_1^2 + n_2^2}\right)$

⇒ $T_S = (1-R_S)^{2N} = \left(\frac{4n_1^2 n_2^2}{(n_1^2+n_2^2)^2}\right)^{2N}$

N	T_S
1	0.852
10	0.04
20	0.0017

(2). Realization of Practice Device:
Polarization Beam-Splitter Cube.



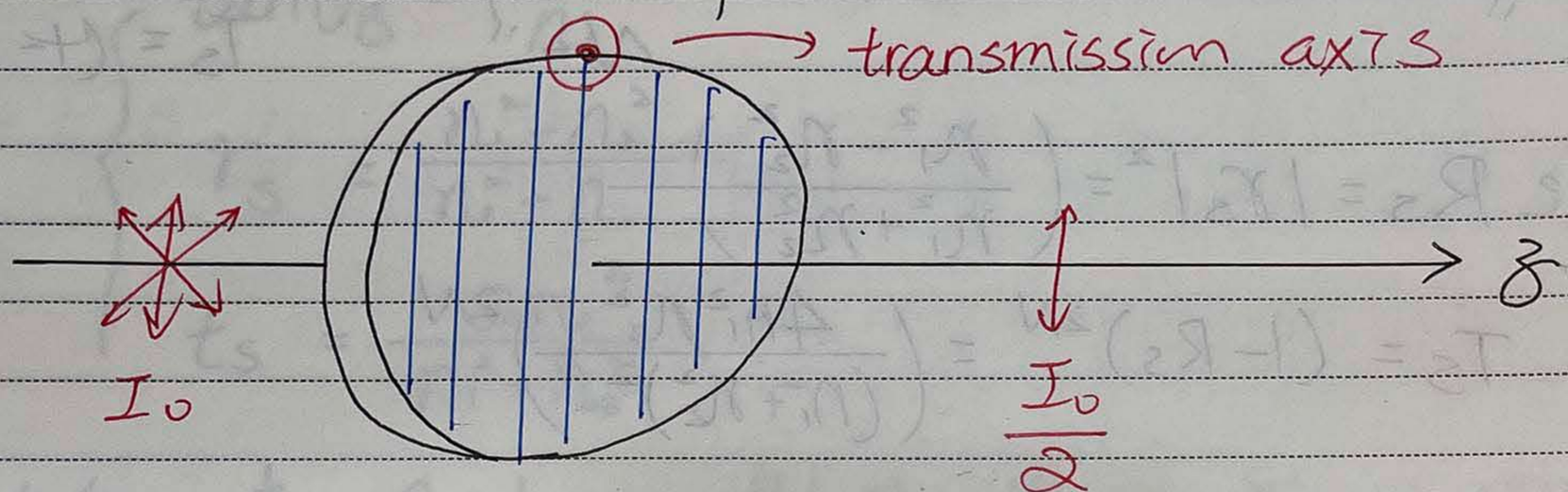


$$\Rightarrow n = \frac{\sqrt{2}\, n_1 n_2}{\sqrt{n_1^2 + n_2^2}}$$

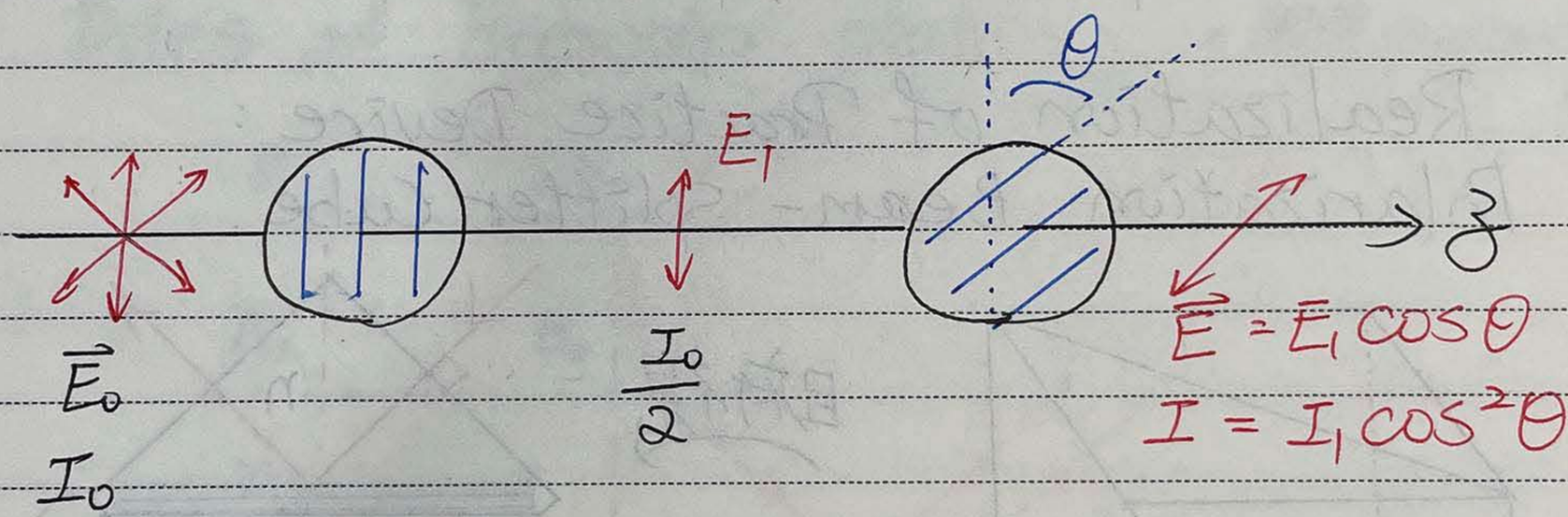
e.g. $n_2 = 2.10\ (Ta_2O_5)$, $n_1 = 1.38\ (MgF_2)$
$\rightarrow n = 1.63$

Date. Page.

3. Polaroid H-plate



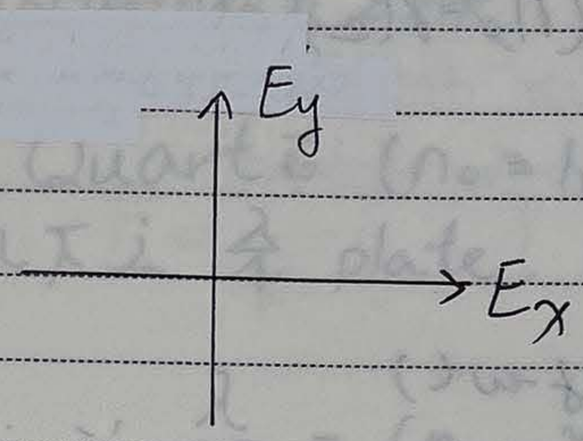
∵ 各方向均勻隨机
∴ 投影到 transmission axis 方向的分量加總約為 $\frac{1}{2}I_0$



⇒ $\cos^2\theta$ 之衰減太大了，有辦法在能量不損失的前提下改變偏振角度嗎？

§3 Adjusting the Polarizing States

Date Page.



$$\therefore \begin{cases} E_x = E_{x0} e^{i(kz-\omega t)} \\ E_y = E_{y0} e^{i\delta} \cdot e^{i(kz-\omega t)} \end{cases}$$

∴ Change relative plane → change polarization state,

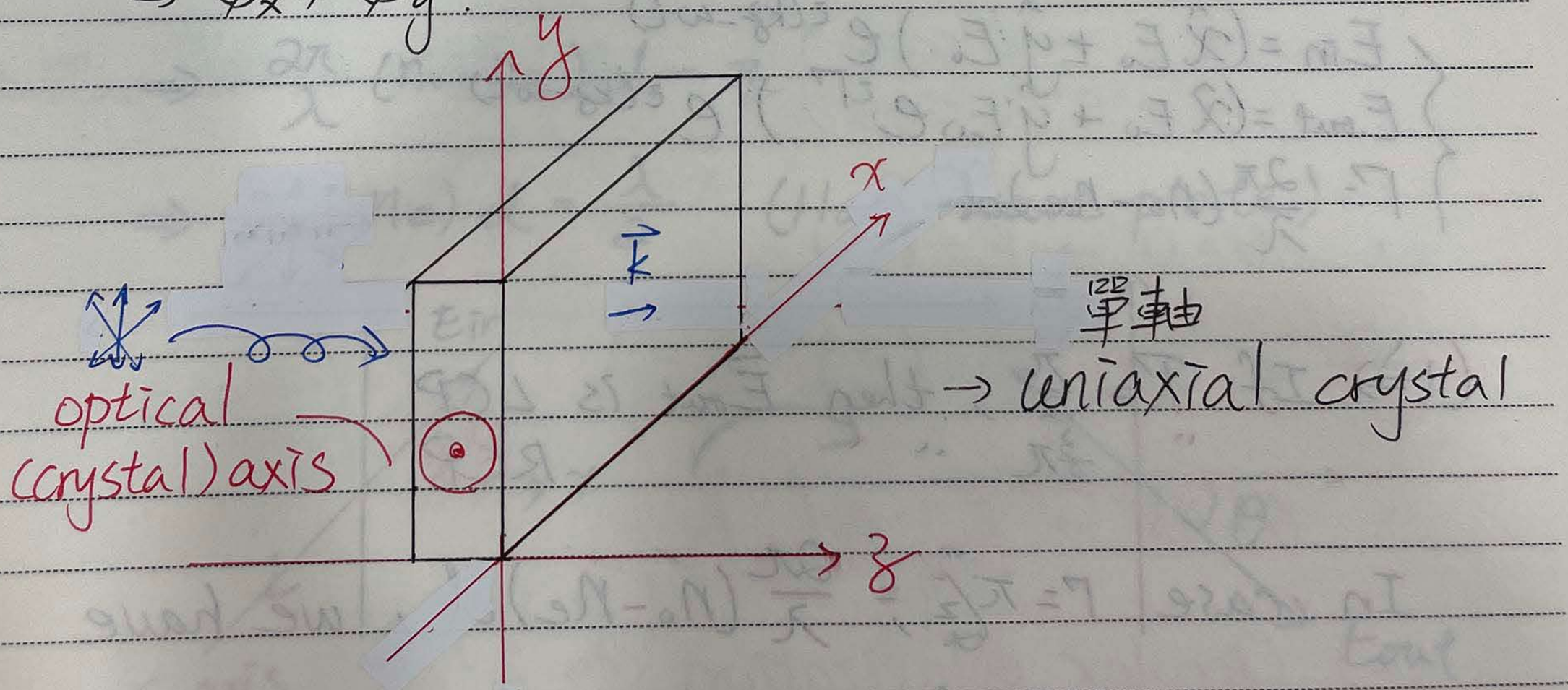
$$\text{phase} = kd = \frac{2\pi}{\lambda/m} d = \frac{2\pi}{\lambda} nd$$

延不同方向之折射率

If $n_x \neq n_y$ (anisotropic material, 各向異性),

$$\phi_x = \frac{2\pi}{\lambda} n_x d, \quad \phi_y = \frac{2\pi}{\lambda} n_y d,$$

$$\Rightarrow \phi_x \neq \phi_y.$$



單軸 → uniaxial crystal

$$\Rightarrow \text{If} \begin{cases} \vec{E} \parallel \text{crystal axis}, & \text{then } n = n_e \text{ (extra-ordinary)} \\ \vec{E} \perp " & n = n_o \text{ (ordinary)} \end{cases}$$

Date. Page.

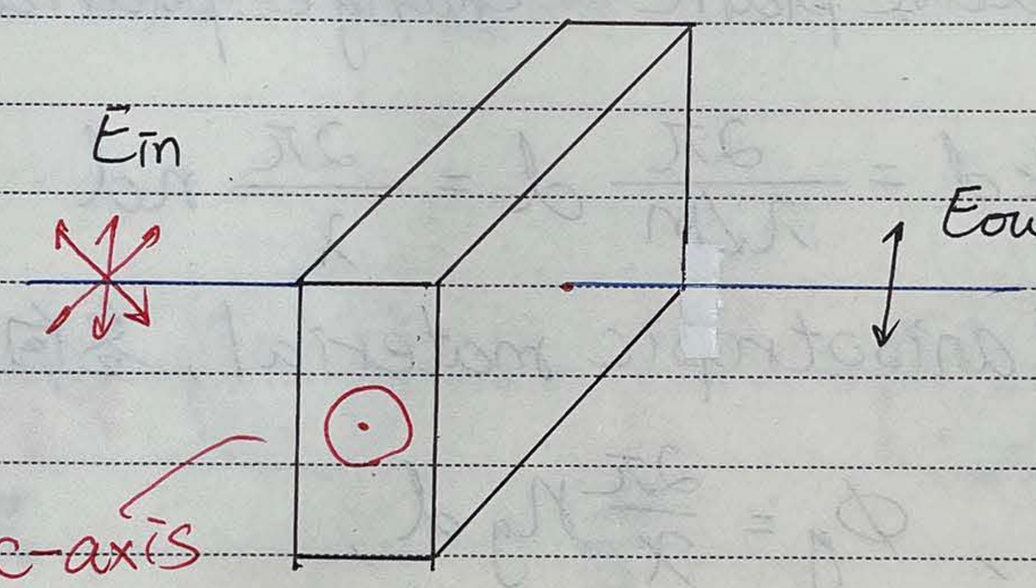
$$\Rightarrow \Delta\phi = \phi_y - \phi_x = \frac{2\pi}{\lambda}(n_o - n_e)d$$

$\Rightarrow$ transmitted wave

$$\begin{cases} E_x = E_{xo}\, e^{i(kz-\omega t)} \\ E_y = E_{yo}\, e^{i\delta} e^{\Delta\phi} e^{i(kz-\omega t)} \end{cases}$$

phase retardation $\Delta\phi$ 之下，物理意義為光跑過該塊材料後之相位差.

ex.



$$\begin{cases} \vec{E}_{in} = (\hat{x}E_o + \hat{y}E_o)\, e^{i(kz-\omega t)} \\ E_{out} = (\hat{x}E_o + \hat{y}E_o e^{i\Gamma})\, e^{i(kz-\omega t)} \\ \Gamma = \frac{2\pi}{\lambda}(n_o - n_e)d \end{cases}$$

(1) If $\Gamma = \frac{\pi}{2}$, then $\vec{E}_{out}$ is LCP

" $\frac{3}{2}\pi$ " RCP

In case $\Gamma = \frac{\pi}{2} = \frac{2\pi}{\lambda}(n_o - n_e)d$, we have

$(n_o - n_e)d = \frac{\lambda}{4}$ for suitable d

$\Rightarrow$ Called $\frac{\lambda}{4}$ wave plate，可將線性偏極光

Date. Page.

旋圓偏振元互換

ex. 以 Quartz ($n_o = 1.5442$, $n_e = 1.5533$) 製成 $\lambda = 589$ nm 下之 $\frac{\lambda}{4}$ plate, 問厚度 d 需再多少?

-sol- $\because \frac{\lambda}{4} = (n_o - n_e)\, d$

$$\therefore d \approx \frac{589\ \text{nm}}{4 \times (1.5442 - 1.5533)} \approx 16.2\ \mu\text{m}$$ ※

推廣: multiple order:

$$m \cdot \frac{\pi}{2} + \frac{\pi}{2} = \frac{2\pi}{\lambda}(n_o - n_e)\, d$$

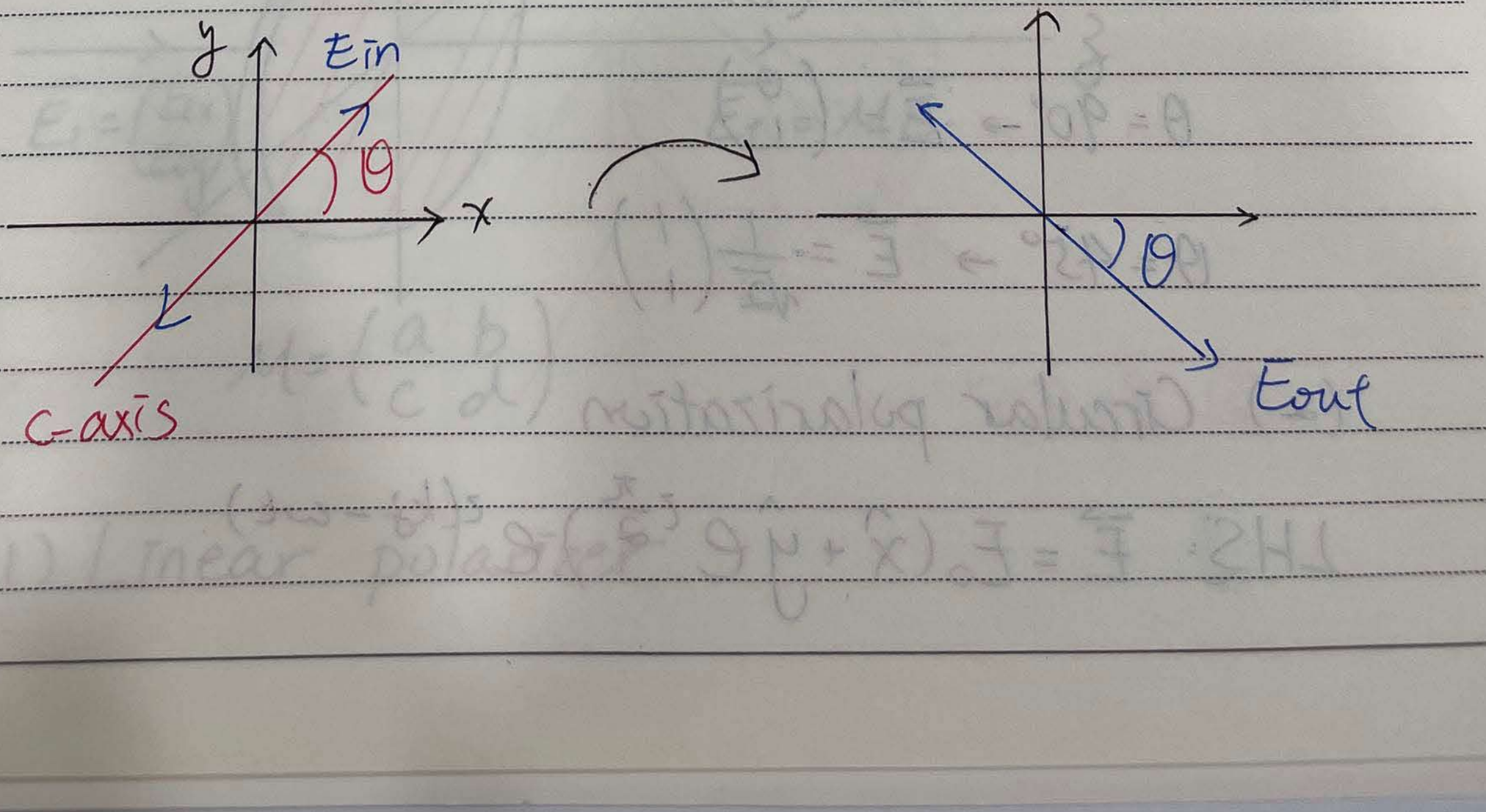
$\Rightarrow m \in \mathbb{N}$, 做出來都是 $\lambda/4$-plate

(2). If $\Gamma = \pi$

$$\Rightarrow \vec{E}_{out} = (\hat{x}E_o - \hat{y}E_o)\, e^{i(kz - \omega t)}$$

$$\Rightarrow \frac{2\pi}{\lambda}(n_o - n_e)\, d = \pi$$

$$\Rightarrow (n_o - n_e)\, d = \frac{\lambda}{2} \quad \text{(Half-wave plate)}$$

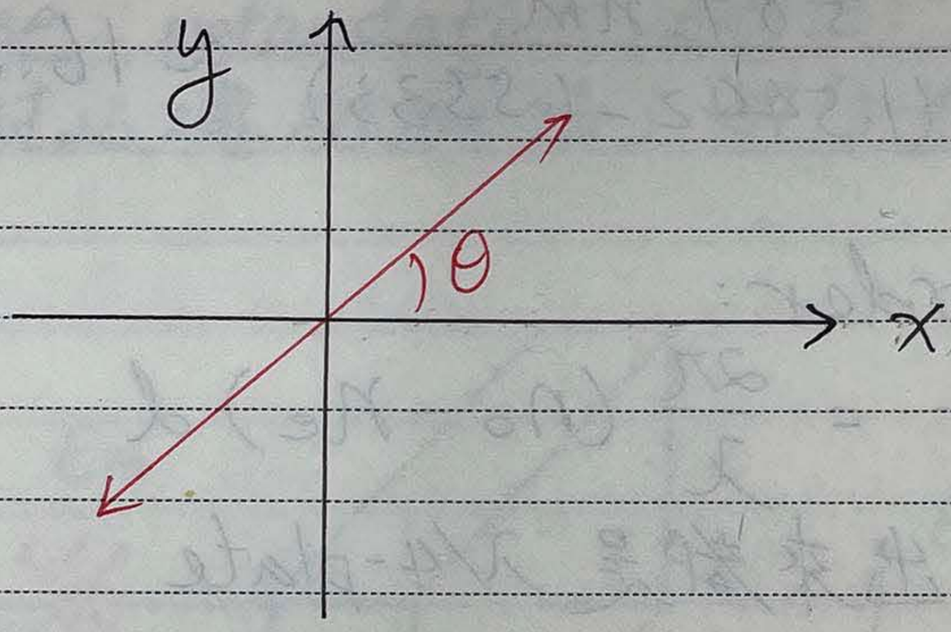


Date. Page. # §4. Jones Calculus

Polarization states → vector form
Polarization devices → matrix form

1. Jones vectors.

(1) Linearly polarization



$$\vec{E} = (\hat{x}E_{0x} + \hat{y}E_{0y})e^{i(kz-\omega t)}$$
$$= E_0(\hat{x}\cos\theta + \hat{y}\sin\theta)\, e^{i(kz-\omega t)}$$

vector form:

$$\vec{E} = E_0\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, \text{ take } E_0 = 1 \text{ for normalization}$$

ex. $\theta = 0^\circ \rightarrow \vec{E} = \begin{pmatrix}1\\0\end{pmatrix}$

$\theta = 90^\circ \rightarrow \vec{E} = \begin{pmatrix}0\\1\end{pmatrix}$

$\theta = 45^\circ \rightarrow \vec{E} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

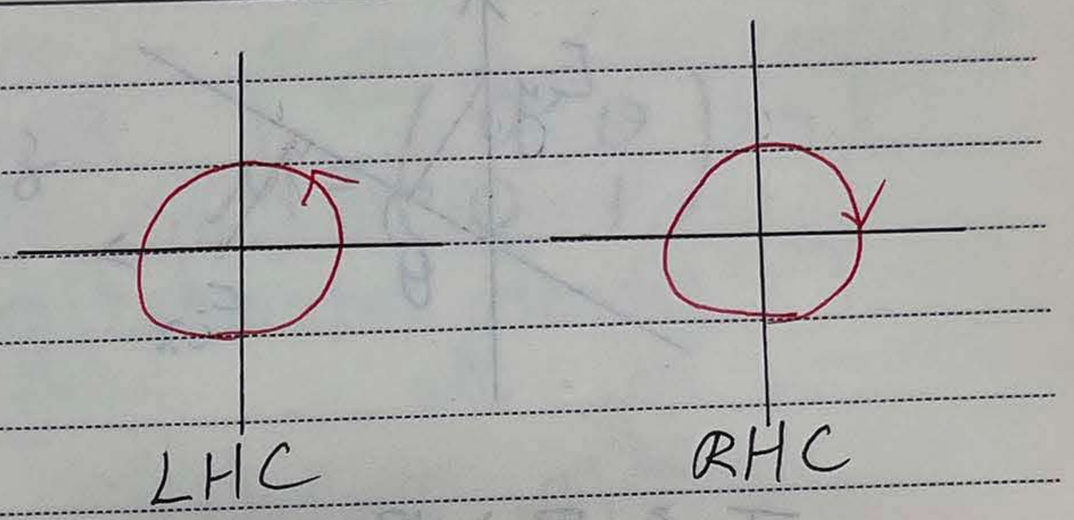
(2) Circular polarization

LHC: $\vec{E} = E_0(\hat{x} + \hat{y}e^{i\frac{\pi}{2}})e^{i(kz-\omega t)}$

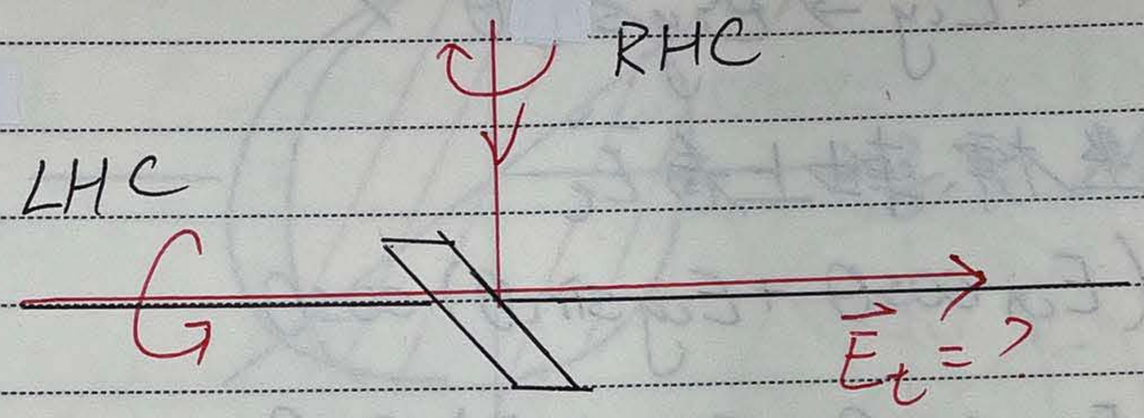
Date. Page.

$$\Rightarrow \vec{E} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} = \hat{e}_L$$

RHC: $\vec{E} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$



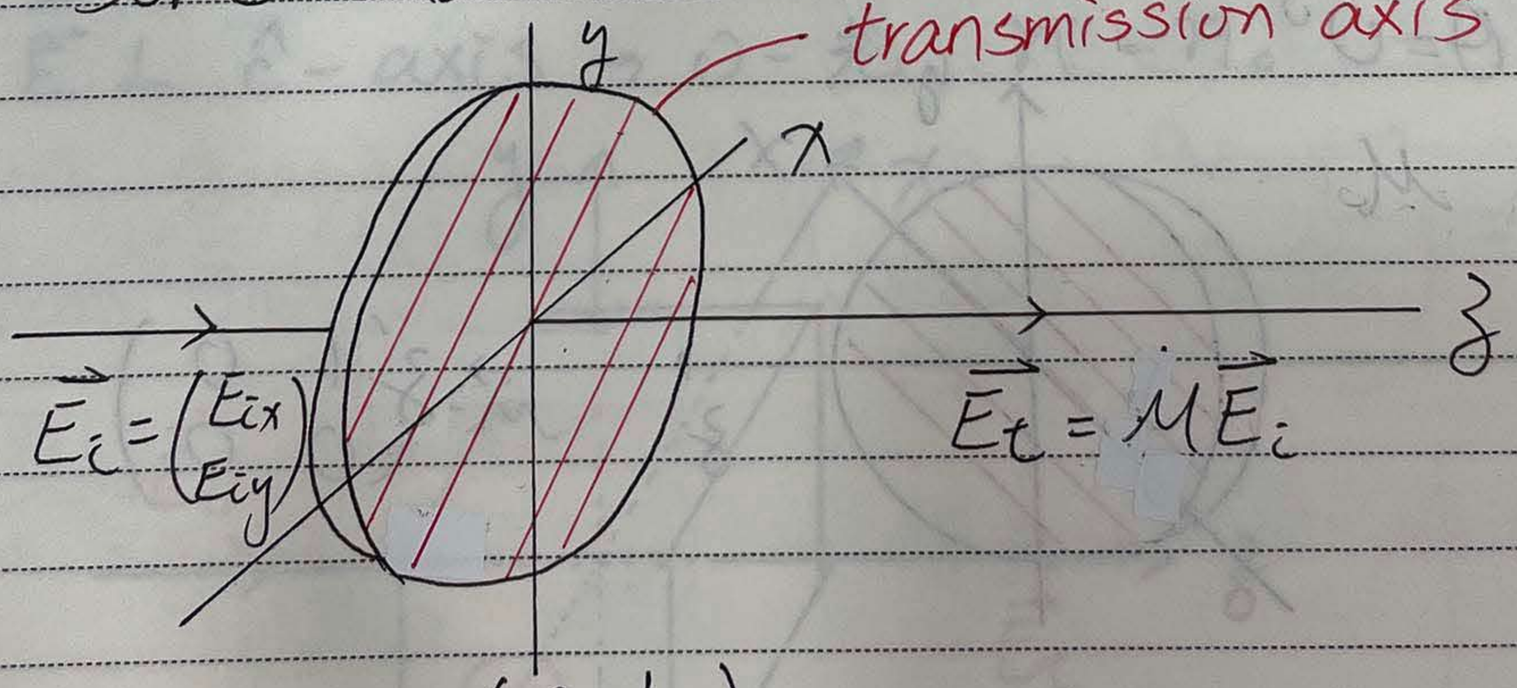
(3)



$$\vec{E}_t = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\ i\end{pmatrix} + \begin{pmatrix}1\\ -i\end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\end{pmatrix}$$

⇒ 2个圆偏振光(一左一右)相加会变成水平偏振光!

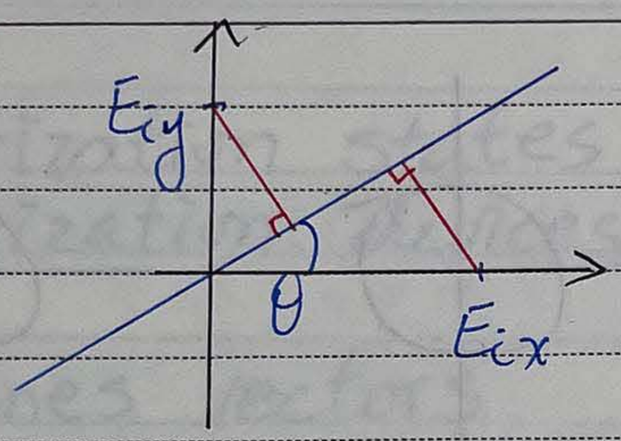
2. Jone Matrices



$$M = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$$

(1) Linear polarizer

Date. Page.



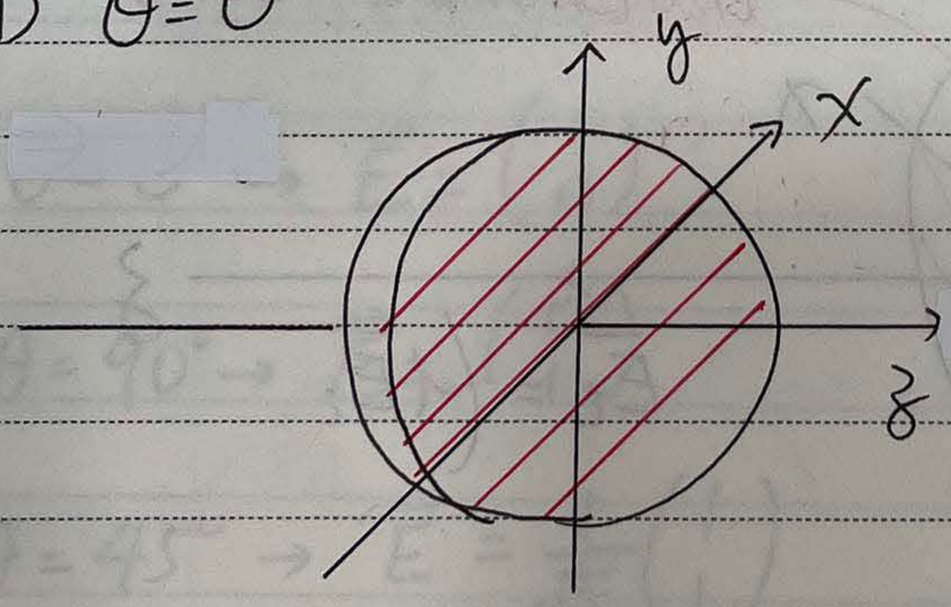
可通過之量：$\langle \begin{matrix} E_{ix} \to E_{ix}\cos\theta \\ E_{iy} \to E_{iy}\sin\theta \end{matrix} \rangle$ 組合：$\vec{E}_t$

→ 在 (x, y) 坐標軸上看 $\vec{E}_t$：

$$\begin{cases} \vec{E}_{tx} = (E_{ix}\cos\theta + E_{iy}\sin\theta)\cos\theta \\ \vec{E}_{ty} = (E_{ix}\cos\theta + E_{iy}\sin\theta)\sin\theta \end{cases}$$

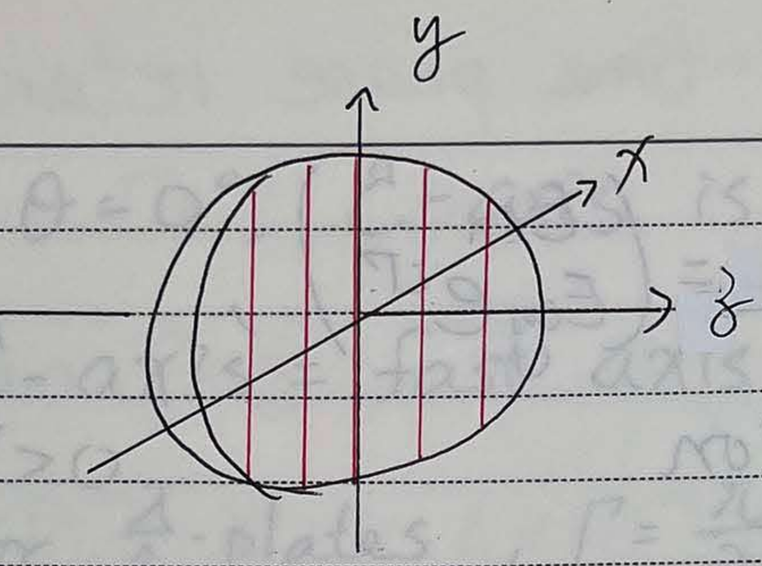
$$\Rightarrow \begin{pmatrix} E_{tx} \\ E_{ty} \end{pmatrix} = \underbrace{\begin{pmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta \\ \frac{1}{2}\sin 2\theta & \sin^2\theta \end{pmatrix}}_{M\ \text{polarizer}} \begin{pmatrix} E_{ix} \\ E_{iy} \end{pmatrix}$$

ex. ① $\theta = 0°$



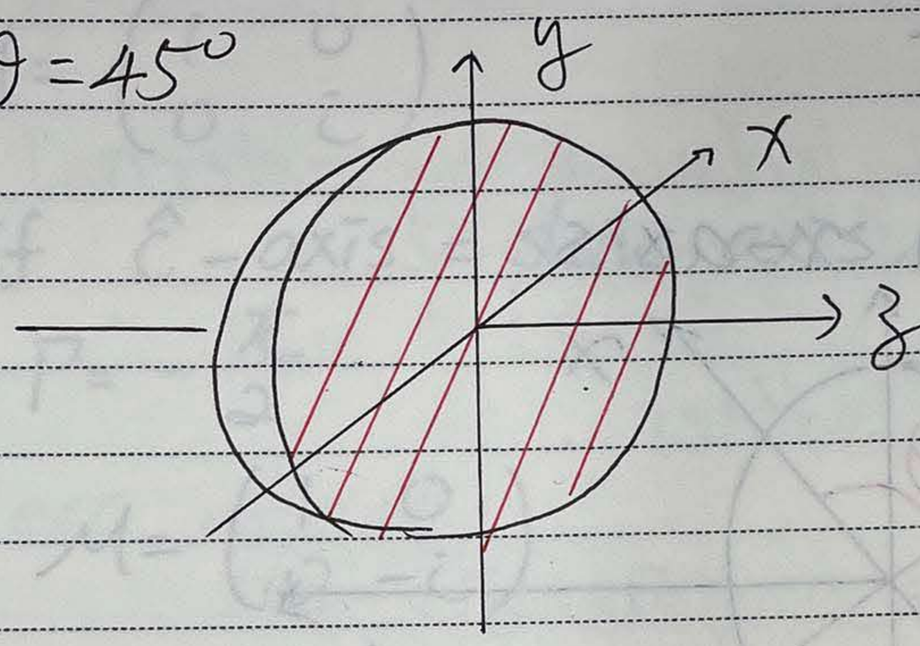
$$M = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

② $\theta = 90°$



$$M = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

③ $\theta = 45°$



$$M = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

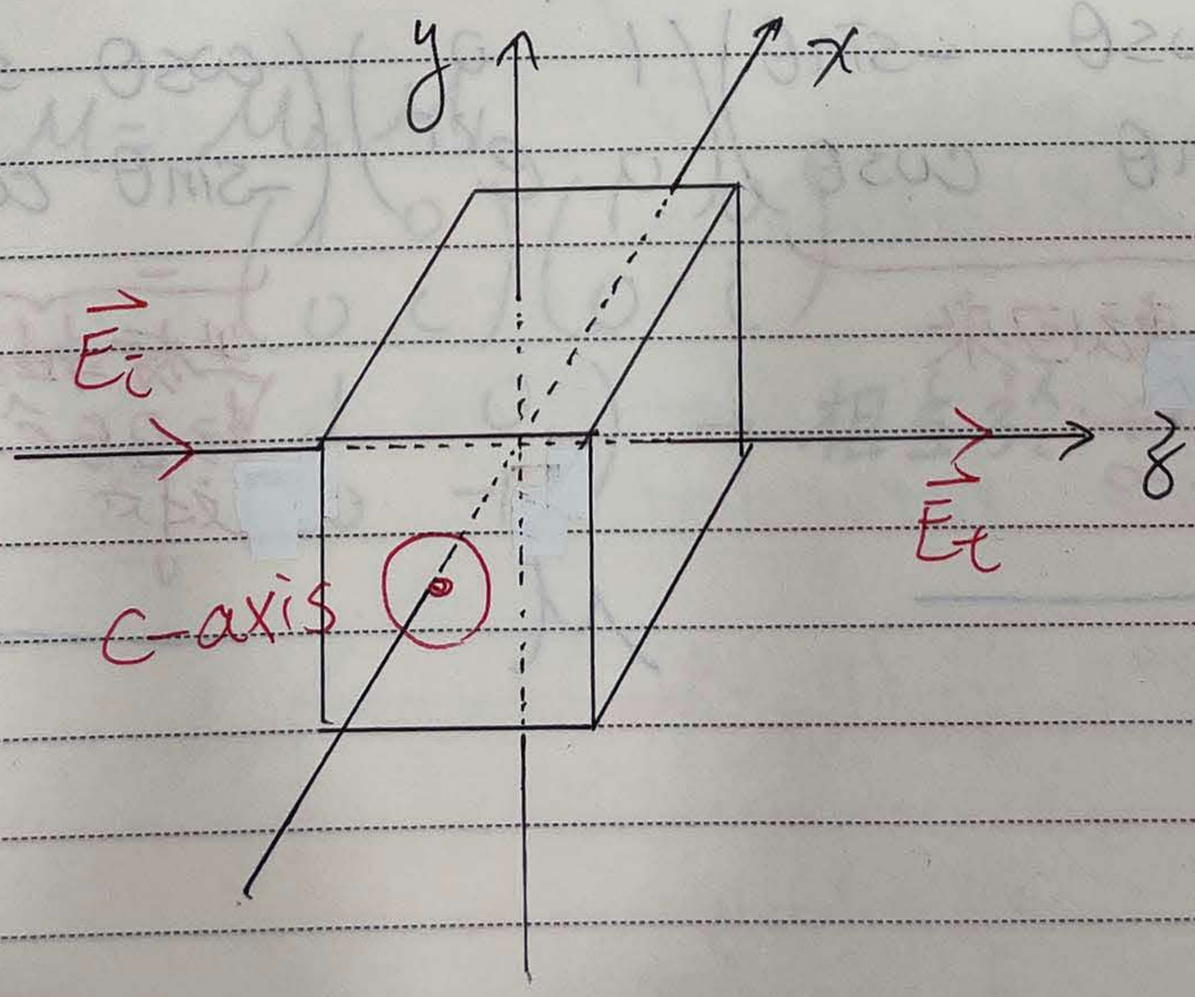
(2) Wave plates

① uniaxial (單軸) crystals

→ 有一組主軸坐標系 (optic axis, crystal axis)

$\hat{c}$-axis s.t.

$$\begin{cases} \vec{E} \parallel \hat{c}\text{-axis} \rightarrow e\text{-光}, & n = n_e \\ \vec{E} \perp \hat{c}\text{-axis} \rightarrow o\text{-光}, & n = n_o \end{cases}$$



Date. Page.

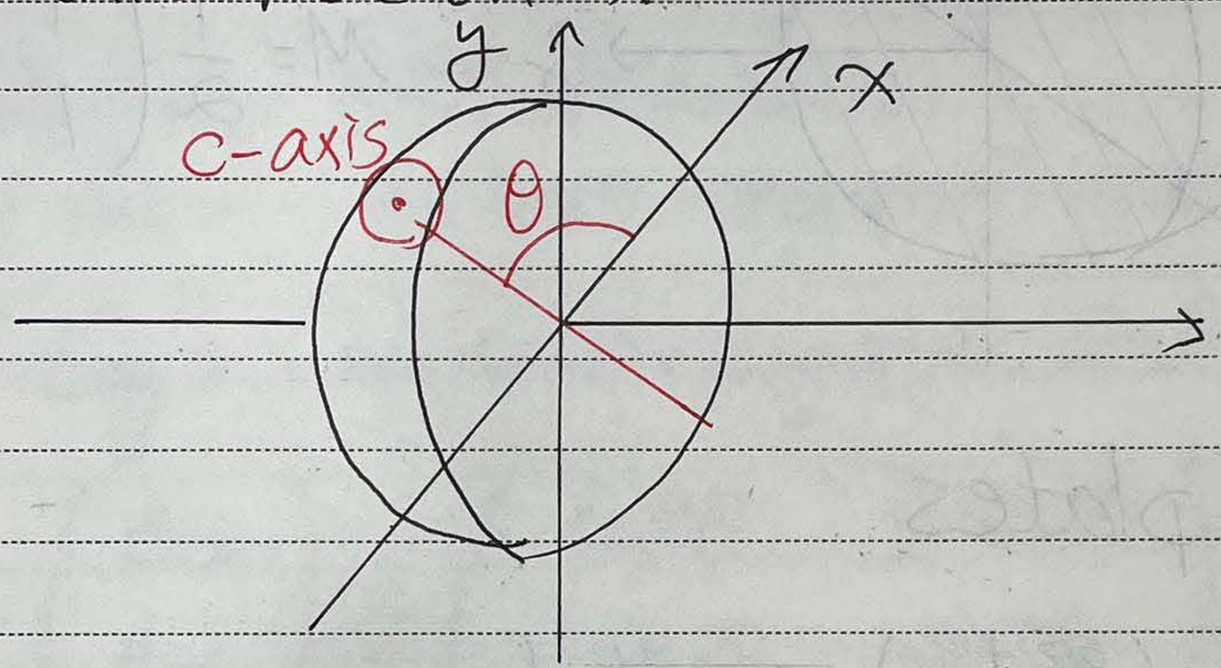
Let $E_i = \begin{pmatrix} E_{ix} \\ E_{iy} \end{pmatrix}$, $E_t = \begin{pmatrix} E_{ix} \\ E_{iy} e^{i\Gamma} \end{pmatrix}$,

Γ = phase retardation

$= \phi_y - \phi_x$

$= \frac{2\pi}{\lambda}(n_o - n_e)d$

② $\hat{c}$-axis not on x-axis



$\Rightarrow \vec{E}_t = \begin{pmatrix} ? \end{pmatrix} \begin{pmatrix} E_{ix} \\ E_{iy} \end{pmatrix}$

⇒ 考慮坐標軸的旋轉，

$$\vec{E}_t = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{\text{轉回來}} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Gamma} \end{pmatrix} \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{坐標軸旋轉 } \theta \text{ 與 } \hat{c}\text{-axis align}} \vec{E}_i$$

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(i) When $\theta = 0°$, $\hat{c}$-axis is on x-axis (horizontal)

(ii) If $\hat{c}$-axis = fast axis ($n_e < n_o$) $\rightarrow \Gamma > 0$

$\because \Gamma > 0$

$\therefore$ For $\frac{\lambda}{4}$-plates, $\Gamma = \frac{\pi}{2}$

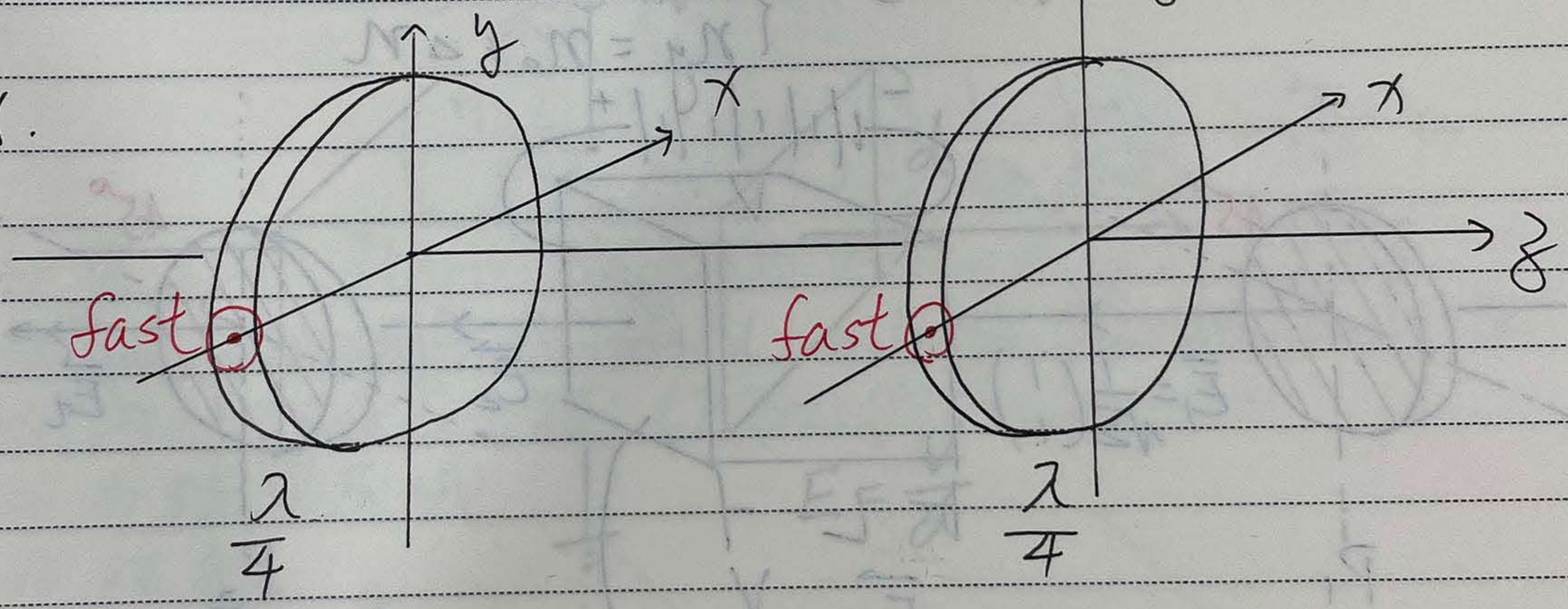
$$\Rightarrow M = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

(iii) If $\hat{c}$-axis = slow axis, $n_e > n_o$, $\Gamma < 0$.

$$\Rightarrow \Gamma = -\frac{\pi}{2}$$

$$\Rightarrow M = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

ex.



$$\Rightarrow M = M_2 M_1$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow$$ 相当於 $\frac{\lambda}{2}$ plate!

Date. Page. # §5. Real-time phase retarders

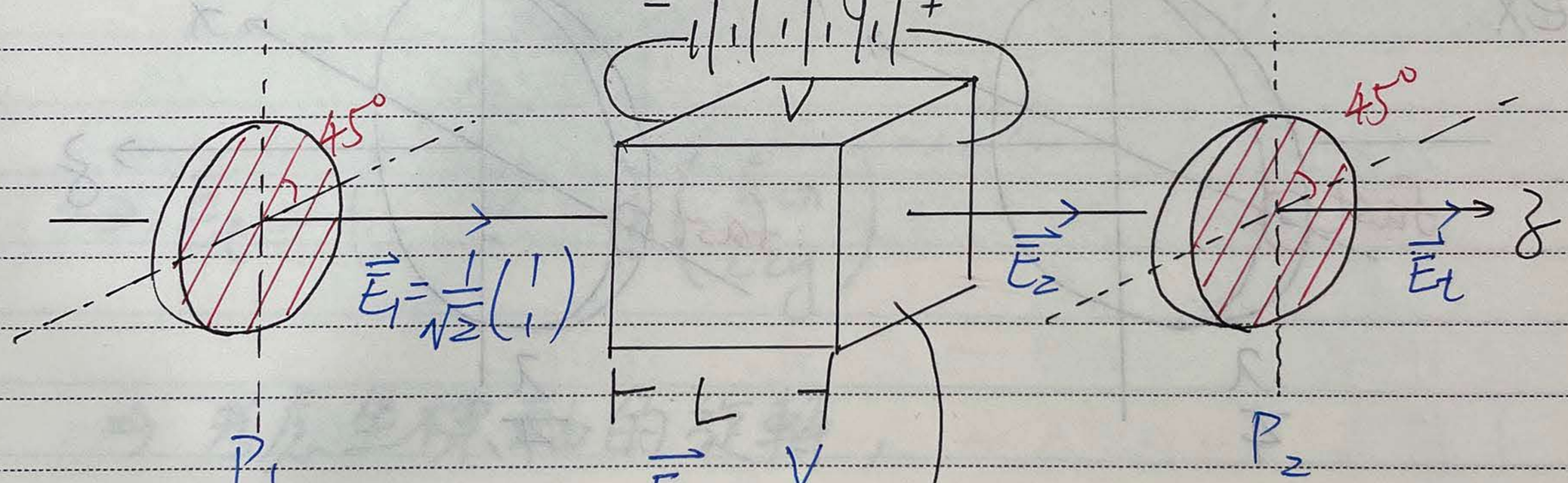
- Electro optics
- Magneto optics
- Acousto optics

1. Electro optic effect (KDP, AIP晶体)

- birefringence induced by external E field (by applied voltage)

(1) Linear E-O effect

Pockels effect: $\begin{cases} n_x = n_o - \Delta n \\ n_y = n_o + \Delta n \end{cases}$



$$\Delta n = \frac{1}{2} n_o^3 \gamma E_3$$

① $V = 0 \rightarrow \Delta n = 0,\ n_x = n_y$

⇒ no phase retardation!

② $V \neq 0 \rightarrow \Gamma = \frac{2\pi}{\lambda}(n_y - n_x)L$

$= \cdots$

$= \frac{2\pi}{\lambda} n_o^3 \gamma V$

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When $\Gamma=\pi$, $I_t=1$

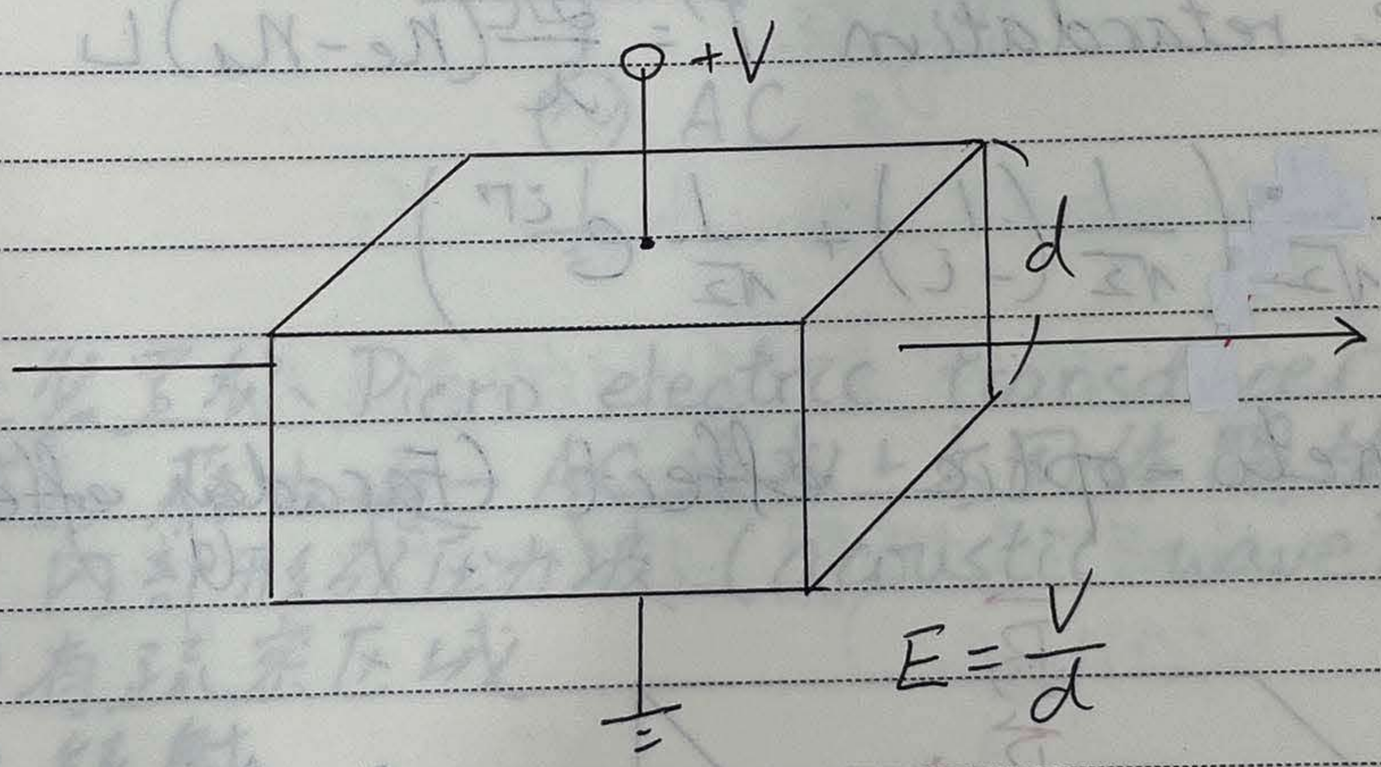
When $\Gamma=\frac{\pi}{2}$, $\vec{E_t}$ is circularly polarized.

ex. KPP, $\gamma=10.6\times10^{-12}\,m/V$, $n_o=1.51$, $\lambda=550$ nm, $\Gamma=\pi$, find V necessary

-sol- $\because \pi=\frac{2\pi}{550\,nm}\times1.51^3\times(10.6\times10^{-12}\,m/V)\times V$

$\therefore V\approx 7.54\times10^3\,V$ #

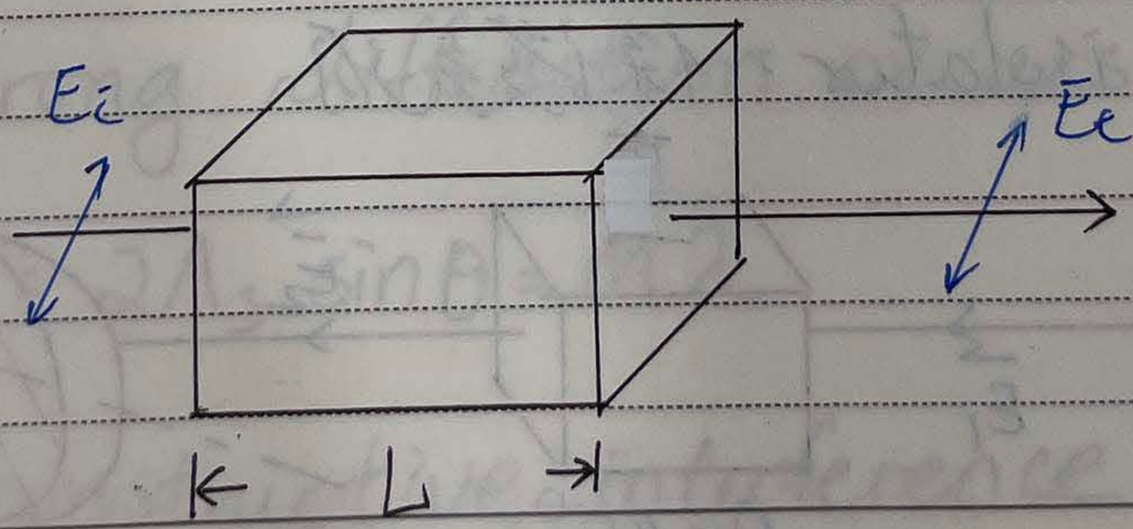
(2) Kerr effect



$\Rightarrow \Delta n=\lambda K E^2$, $\Gamma=\frac{2\pi}{\lambda}(n_y-n_x)L=2\pi K E^2 L$

2. Optical activity
→ Rotation of linearly polarized light

(1)



Date. Page.

$$E_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\left(\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}}_{RCW} + \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}}_{LCW} \right)$$

→ 將線性偏振拆成圓偏振之組合

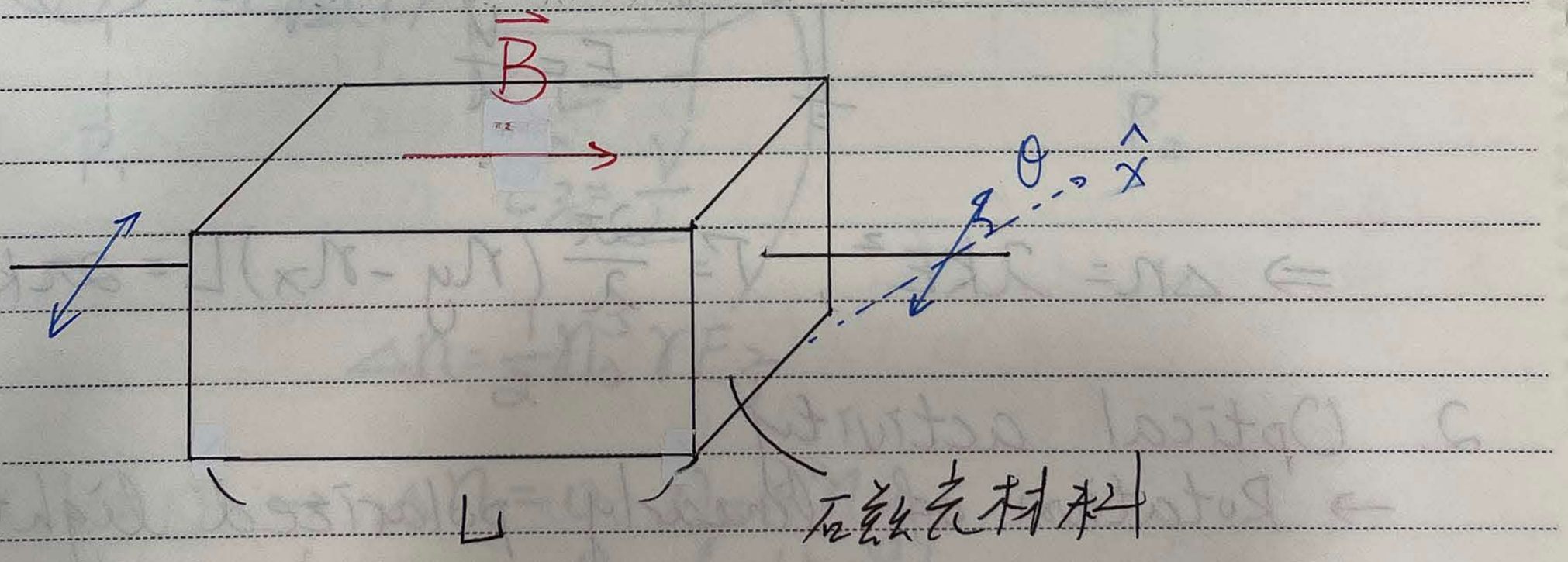
(2) Optical active materials:

refractive index for RCW (n_d), LCW (n_e) are different

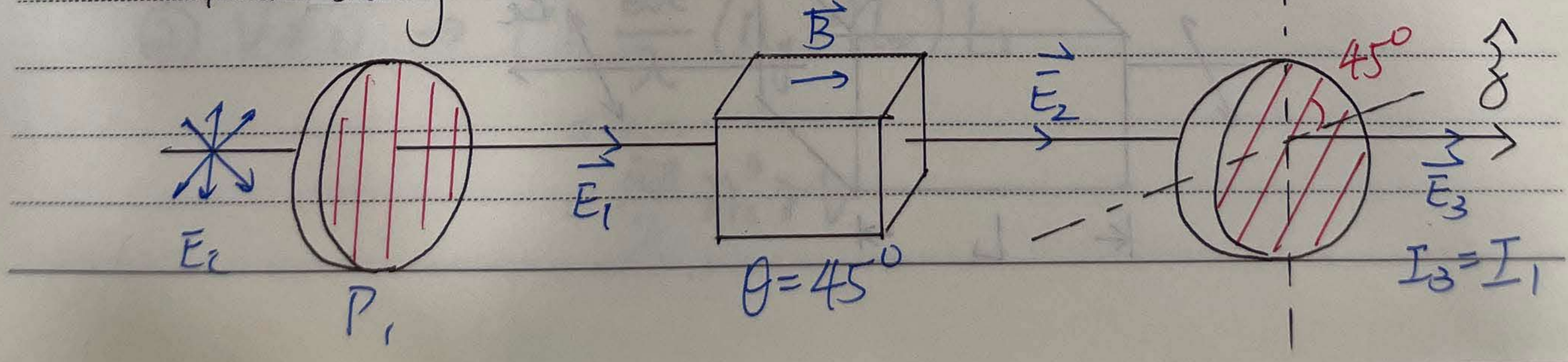
⇒ phase retardation $\Gamma = \frac{2\pi}{\lambda}(n_e - n_d)L$

$$\Rightarrow \vec{E}_t = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} + \frac{1}{\sqrt{2}} e^{i\Gamma} \right)$$

3. Magneto-optic effect (Faradey effect)

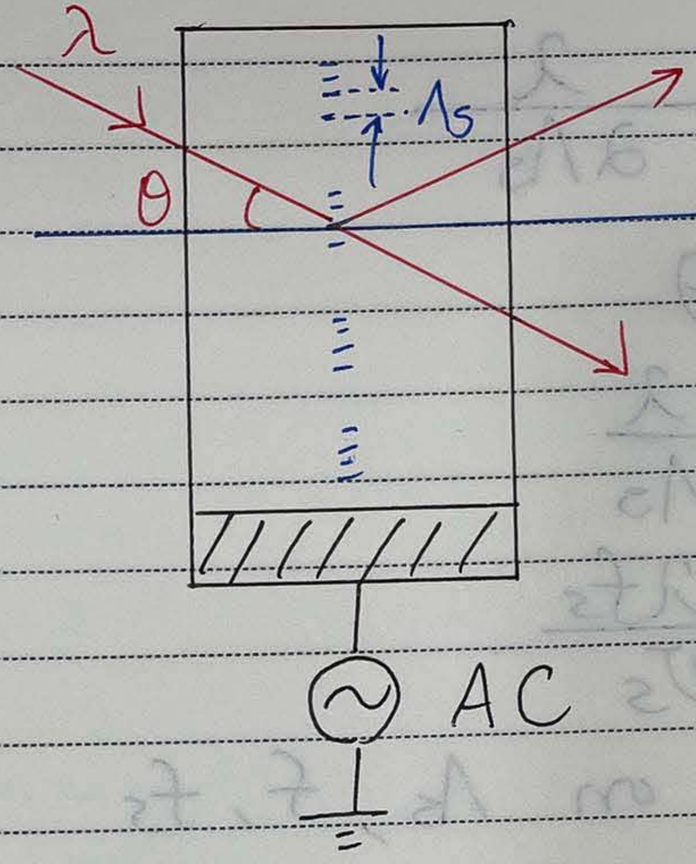


→ Faraday isolator: 法拉第旋光器



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4. Acousto optics



此装置称 Piero electric transducer (PZT, 压电体), 藉由两-AC造成上方固体间的振动(縱波)
⇒ 内部形成压力波 (acoustic wave)
⇒ 有疏密区域
⇒ 绕射

Acoustic wave $v_s = \Lambda_s f_s$, 常取 $f_s = 40\,MHz$,
$v_s = 4\,km/s$

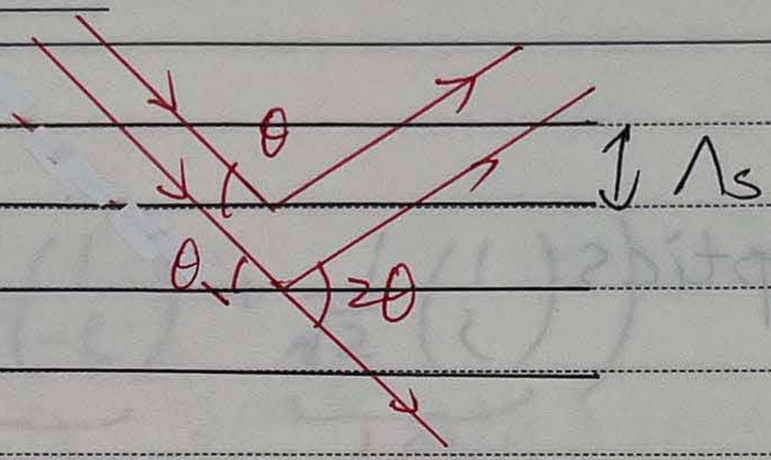
$$\Rightarrow \Lambda_s = \frac{v_s}{f_s} = 100\,\mu m$$

⇒ Strong diffraction occurs at

$$\delta = 2\Lambda_s \sin\theta = m\lambda$$

→ constructive interference

Date. Page.



$\Rightarrow$ 此时 $\sin\theta = m\frac{\lambda}{2\Lambda_s}$

$\Rightarrow$ 绕射角 $\theta_d = 2\theta$

$$= m\frac{\lambda}{\Lambda_s}$$

$$= \frac{m\lambda f_s}{v_s}$$

$\Rightarrow$ θ_d depends on Λ_s, f, f_s

$\Rightarrow$ 不同 f_s 可以控制光绕射的角度!

application: 声波探测器 (acoustic optic scanner)