高微第十四週作業

1. Use Lagrange multipliers method to show the followings :

(a) Let $x_1,\dots,x_n$ be nonnegative. Show that

$$(x_1\cdots x_n)^{1/n}\le\frac{x_1+\cdots+x_n}{n}.$$

Hint : Let $x_1+\cdots+x_n=c$, and find the maximum value of $(x_1\cdots x_n)^{1/n}$.

(b) Let A be a $n\times n$ symmetric matrix. In linear algebra, we have known that there are real eigenvalues $\lambda_1,\dots,\lambda_n$ with $\lambda_1\le\cdots\le\lambda_n$. Show that

$$\min\{\langle Ax,x\rangle:\|x\|=1\}=\lambda_1,\quad\text{and}\quad\max\{\langle Ax,x\rangle:\|x\|=1\}=\lambda_n.$$

(c) For numbers a,b,c, find the minimum of $\{ax+by+cz \mid x^2+y^2+z^2\le1\}$. Give a geometric interpretation of the answer by viewing $ax+by+cz$ as the inner product of (x,y,z) and (a,b,c).

2. Define $f:R^2\to R$ by $f(x,y)=xy$ for all (x,y) in $\mathbb{R}^2$.

(a) Show that for all the critical points (x,y) of f, $\exists\,\mathbf{u},\mathbf{v}\in R^2$ at which $H\mathbf{u}\cdot\mathbf{u}>0$ and $H\mathbf{v}\cdot\mathbf{v}<0$, where $H=(D^2f)(x,y)$.

(b) Use (a) to explain why the function $f:R^2\to R$ has no local extreme points.

3. Consider the problem of finding the extreme value of $f(x,y)=x$ on the curve $g(x,y)=y^2+x^5+x^3=0$.

(a) Try using Lagrange multipliers Method to solve the problem.

(b) Show that the maximum value is $f(0,0)=0$ but $\nabla f(0,0)=\lambda\nabla g(0,0)$ is not satisfied for any value of λ.

(c) Explain why Lagrange multipliers Method fails to find the maximum value in this case.

(d) Does Lagrange multipliers Method succeed to answer the minimum value problem ?

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