高微第十二週作業

Rudin : p.241 # 16, 17, 19, 21, 23, 27.

Extra Problems :

1. Find the values of a,b,c,d,e,f such that

$$\lim_{(x,y)\to(0,0)} \frac{\cos(x-y+xy)-\left[a+bx+cy+dx^2+exy+fy^2\right]}{x^2+y^2}=0.$$

2. Take $n=m=1$ in the implicit function theorem, and interpret the theorem graphically and prove it.
3. Let $E\subseteq R^2$ be open, $(a,b)\in E$, and suppose that $f:E\to R$ is continuously differentiable. Suppose that $f(a,b)=0$ and $\nabla f(a,b)\neq(0,0)$. Prove that $\nabla f(a,b)$ is orthogonal to the level curve $f(x,y)=0$ at (a,b) (that is, $\nabla f(a,b)$ is orthogonal to the tangent line at (a,b) of the implicit defined function.)