高微第六週作業

**Read：Rudin** p178 ~p184 (The Exponential Function and Logarithmic Functions &
The Trignometric Functions)

**Rudin :** p.196　# 1, 2, 3 .

**Extra problems :**

1. Let $\{a_{ij}\}$ be a double sequence of real numbers. Show that if both $\displaystyle\lim_{i,j\to\infty} a_{ij}$ and

   $\displaystyle\lim_{j\to\infty}\lim_{i\to\infty} a_{ij}$ exist, then $\displaystyle\lim_{i,j\to\infty} a_{ij} = \lim_{j\to\infty}\lim_{i\to\infty} a_{ij}$ .

2. Let $a_{ij} = \dfrac{ij}{i^2 + j^2}$ for all $i, j \in N$ . Prove that $\displaystyle\lim_{j\to\infty}\lim_{i\to\infty} a_{ij}$ , $\displaystyle\lim_{i\to\infty}\lim_{j\to\infty} a_{ij}$ exist and

   $\displaystyle\lim_{j\to\infty}\lim_{i\to\infty} a_{ij} = \lim_{i\to\infty}\lim_{j\to\infty} a_{ij}$ , but $\displaystyle\lim_{i,j\to\infty} a_{ij}$ does not exist.

3. (a) Prove that $\ln 2 = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ .

   (b) Prove that $\ln 2 = \displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}$ . (Hint : $\ln 2 = -\ln\left(\dfrac{1}{2}\right)$ .)

   (c) When approximating $\ln 2$ with the series in (a) and (b), how many terms do
   we need to add, respectively, so that the error is to within $10^{-3}$ .