高微第四週作業

Rudin : p.45 # 25.

Extra problems :

1. Let $f_n(x)=\dfrac{x}{nx+1}, \forall x\in[0,1],\ \forall n$. Does $\{f_n\}$ converge uniformly on $[0,1]$?

2. Let $f_n(x)=\dfrac{x}{1+nx^2}, \forall x\in[-1,1],\ \forall n$.

   (a) Find the limit function f of the sequence $\{f_n\}$ and the limit function g of the sequence $\{f_n'\}$.

   (b) Does $f'(x)=g(x),\ \forall x\in[-1,1]$?

   (c) Does $\{f_n\}$ converge uniformly to f on $[-1,1]$?

   (d) Does $\{f_n'\}$ converge uniformly to g on $[-1,1]$?

3. (a) Does $\sum_{n=1}^{\infty} x^n(1-x)$ converge uniformly on $[0,1]$?

   (b) Does $\sum_{n=1}^{\infty} (-1)^n x^n(1-x)$ converge uniformly on $[0,1]$?

   (c) Let $\{f_n\}$ be a sequence of functions on a metric space X. Suppose that $\sum_{n=1}^{\infty} f_n$ converges uniformly and $\sum_{n=1}^{\infty} |f_n|$ converges pointwise on X, does $\sum_{n=1}^{\infty} |f_n|$ necessarily converge uniformly on X ?