NCTU Open Course Ware
國立交通大學開放式課程

高微第一週作業

Rudin : p.142 # 19.

Extra problems :

1. Let $f:[-1,1]\to R$ defined by $f(x)=\begin{cases} x\sin\left(\dfrac{\pi}{2x}\right), & \text{if } x\neq 0. \\ 1, & \text{if } x=0. \end{cases}$

   Define $F(x)=\int_0^{x^2} f(t)dt$ for all $x\in[-1,1]$, is F differentiable ?

   If so, find F'. Justify your answer.

2. Is the following statement true ?

   " If $f\in\mathscr{R}$ *on* $[a,b]$, then $F(x)=\int_a^x f(t)dt$ is differentiable *on* $[a,b]$. " ?

   Justify your answer.

3. Let $f:[0,1]\to R$ defined by $f(x)=\begin{cases} x\sin\left(\dfrac{\pi}{2x}\right), & \text{if } x\neq 0. \\ 0, & \text{if } x=0. \end{cases}$

   Prove that f is not rectifiable.

4. Let $f:[0,1]\to R$ defined by $f(x)=\begin{cases} x^2\sin\left(\dfrac{\pi}{2x}\right), & \text{if } x\neq 0. \\ 0, & \text{if } x=0. \end{cases}$

   Prove that f is rectifiable.

5. Let $\gamma:[a,b]\to R^k$ be a curve on $[a,b]$. If $c\in(a,b)$, let γ_1 and γ_2 be the restriction of γ on $[a,c]$ *and* $[c,b]$, respectively, then

$$\Lambda(\gamma)=\Lambda(\gamma_1)+\Lambda(\gamma_2).$$