高微第十四週作業

<u>Rudin</u> : p.138 # 1, 2, 8.

Extra problems :

1. Suppose that $f: R \to R$ and $g: R \to R$ are differentiable, and that

$$\begin{cases} f'(x) = g(x) \text{ and } g'(x) = -f(x) & \text{for all } x \in R \\ f(0) = 0 \text{ and } g(0) = 1 \end{cases}$$

   Prove that $[f(x)]^2 + [g(x)]^2 = 1, \ \forall\, x \in R$.

2. Let $n \in N$ and $f:(a,b) \to R$. Suppose that $f^{(n+1)}$ exists and is continuous on (a,b) and suppose that $f^{(k)}(x_0) = 0$ where $x_0 \in (a,b)$, $\forall\ 1 \le k \le n$. Prove that

   (a) If $n+1$ is even and $f^{(n+1)}(x_0) > 0$, then $f(x_0)$ is a local minimum.

   (b) If $n+1$ is even and $f^{(n+1)}(x_0) < 0$, then $f(x_0)$ is a local maximum.

   (c) If $n+1$ is odd and $f^{(n+1)}(x_0) \ne 0$, then $f(x_0)$ is neither a local minimum nor a local maximum.

3. Let $f(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1, \\ 2, & \text{if } 1 < x \le 2. \end{cases}$,

$$\alpha_1(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1, \\ 2, & \text{if } 1 < x \le 2. \end{cases} \quad \text{and} \quad \alpha_2(x) = \begin{cases} 1, & \text{if } 0 \le x < 1, \\ 2, & \text{if } 1 \le x \le 2. \end{cases}$$

   (a) Is $f \in \mathscr{R}(\alpha_1)$ on $[0,2]$?
   (b) Is $f \in \mathscr{R}(\alpha_2)$ on $[0,2]$?