高微第十三週作業

Rudin : p.115 # 9, 11, 13.

(Optional) p.118 # 25, 27.

Extra problems

1. Let $n \in N$ and $p: R \to R$ be a polynomial of degree less than or equal to n. Suppose that at some point $a \in R$,
$$p(a) = p'(a) = \ldots = p^{(n)}(a) = 0.$$
Prove that $p(x) = 0,\ \forall\, x \in R$.
2. Suppose that the function $f:(-1,1) \to R$ has n derivatives (that is $f^{(n)}(x)$ exists $\forall\, x \in (-1,1)$). Assume that there is a positive number M such that
$$|f(x)| \le M|x|^n,\ \ \forall\, x \in (-1,1).$$
Prove that $f(0) = f'(0) = \ldots = f^{(n-1)}(0) = 0$
3. Suppose that the function $f:(-1,1) \to R$ has n derivatives, and $f^{(n)}:(-1,1) \to R$ is bounded. Assume also that $f(0) = f'(0) = \ldots = f^{(n-1)}(0) = 0$. Prove that
$$|f(x)| \le M|x|^n,\ \ \forall\, x \in (-1,1).$$