高微第十週作業

Rudin : p.98 # 1, 2, 3, 4.

Extra problems

1. Let X and Y be metric spaces; suppose $E \subset X$, $f : E \to Y$, and p is a limit point of E.

   (a). Prove that If f has a limit at p, then this limit is unique.

   (b). Prove that if $\lim_{x \to p} f(x)$ exists, then $\forall\, \varepsilon > 0,\ \exists\, \delta > 0$ such that

$$d_Y(f(p_1), f(p_2)) < \varepsilon,$$

   whenever $p_1, p_2 \in E$ and $0 < d_X(p_1, p) < \delta,\ 0 < d_X(p_2, p) < \delta$.

2. Use 1 (b) to prove that $\lim_{x \to 0} \frac{x}{|x|}$ does not exist.

3. Prove theorem 4.4 by the definition of limit.