高微第八週作業

Rudin : p.78 # 6, 7, 8.

Read the proof of Theorem 3.20, 3.27, 3.28, 3.29.

Extra problems

1. Let $a_n \geq 0$ and $b_n > 0 \quad \forall\, n$, and suppose $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ ($L \in R$, or $L = \infty$).

   Prove that

   (a) If $L \in R,\ L \neq 0$, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges.

   (b) If $L = 0$, then if $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

   (c) If $L = \infty$, then if $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ converges.

2. Prove that if $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} {a_n}^2$ converges.

3. (optional)

   Suppose that $\{a_n\}$ is decreasing and $\sum_{n=1}^{\infty} a_n$ converges, prove that $\lim_{n\to\infty} n a_n = 0$.