高微第六週作業

Rudin : p.43 # 20, 21; p.44 # 22; p.78 # 1, 2.

Optional : p.44 # 23, 24, 26, 30.

Extra problems :

1. Let $\mathbf{x}_n \in R^k$, $\forall\, n \in N$ and let $S = \{\mathbf{x}_n \mid n \in N\}$. Suppose that

   $d(\mathbf{x}_n, \mathbf{x}) = |\mathbf{x} - \mathbf{x}_n| < \dfrac{1}{n}$, where $\mathbf{x} \in R^k$, show that $\mathbf{x}$ is the only limit point of S.

   Is this true in general metric space ?

2. Let E be a nonempty set of real numbers which is bounded above. Let $y = \sup E$. Show that $y \in \overline{E}$.

3. Let $E \subseteq R^k$. Show that if E is bounded, then $\overline{E}$ is also bounded. Is this true in general metric space ?

4. Let X be a metric space and $\{x_n\}$ be a sequence in X.
   (a) Show that $\{x_n\}$ converges if and only if $\{x_{2n}\}$ and $\{x_{2n-1}\}$ both converge to the same limit.
   (b) Show that $\{x_n\}$ converges if and only if the subsequences $\{x_{2n}\}$, $\{x_{2n-1}\}$, and $\{x_{3n}\}$ all converge.

5. Prove Theorem 3.3 (a), (b) and (c).