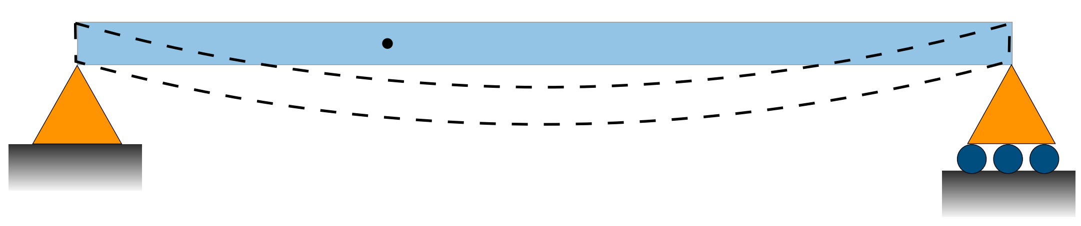
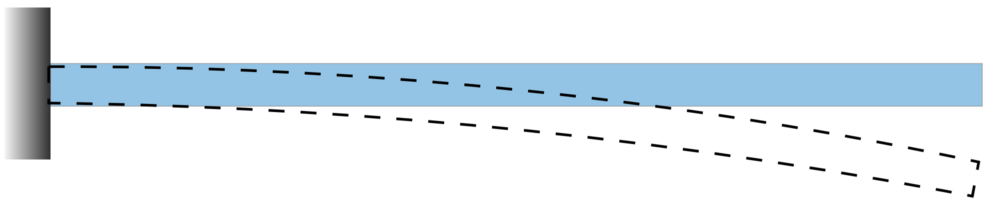
4. Shear forces and bending moments

剪力與彎矩

Introduction

- Beam：梁，桿件主要承受橫向載荷的作用。梁上的力和力矩向量會垂直於桿件的軸線。
- 負載 (loading)、剪力(shear force)和彎矩(bending moment) 的關係。
- **剪力圖和彎矩圖的繪製**。





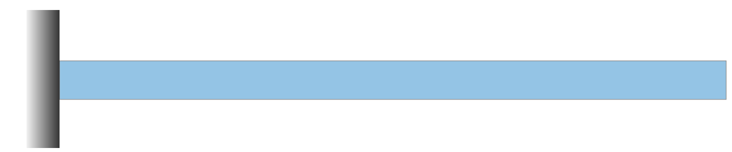
Types of beams, loads, and reactions

樑、負載、反力的種類

支承種類

簡支樑

懸臂樑



pin support
銷支承

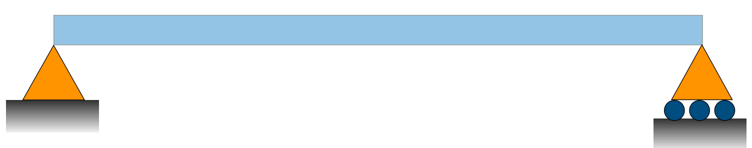
roller support
滾子支承

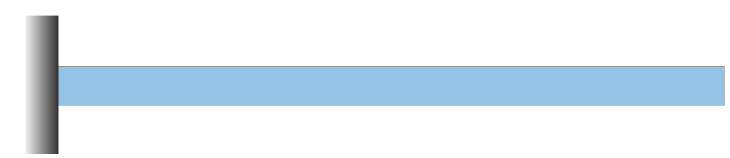
fixed support
固定支承

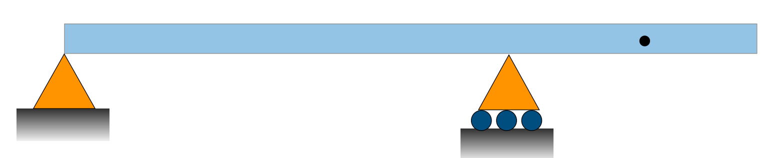
外伸樑

負載的種類

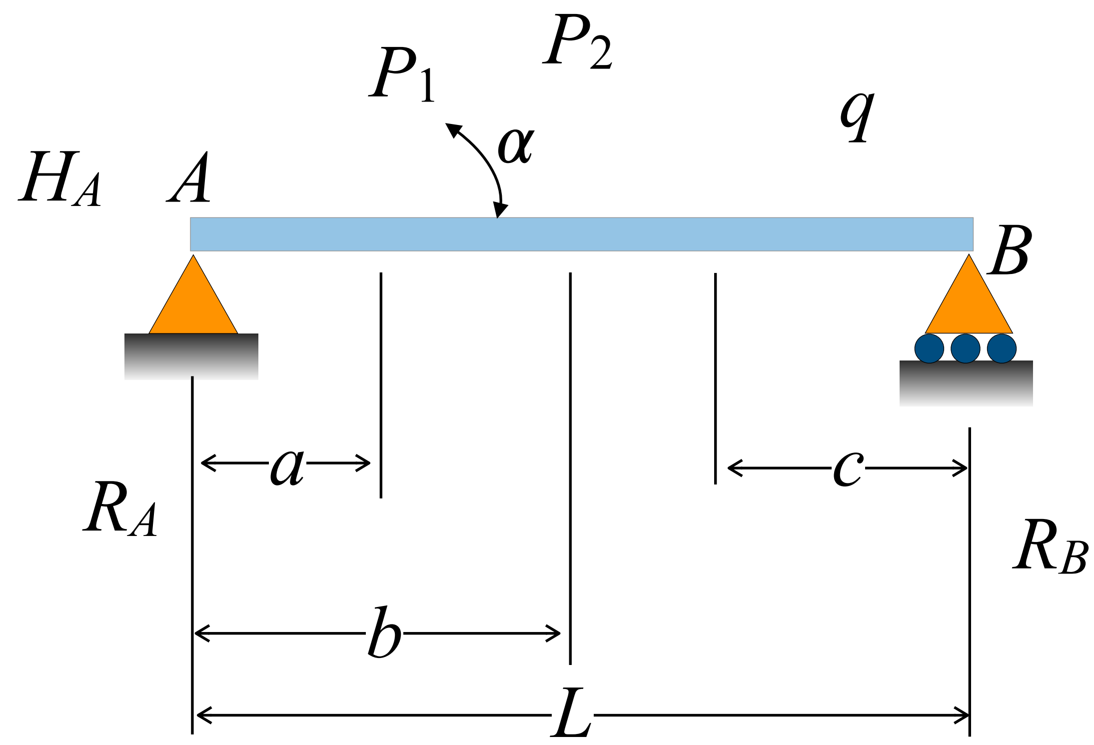
- 集中負載 P_1、 P_2、 P_3、 P_4
- 分佈負載 q
- 力偶矩 M_1







反力



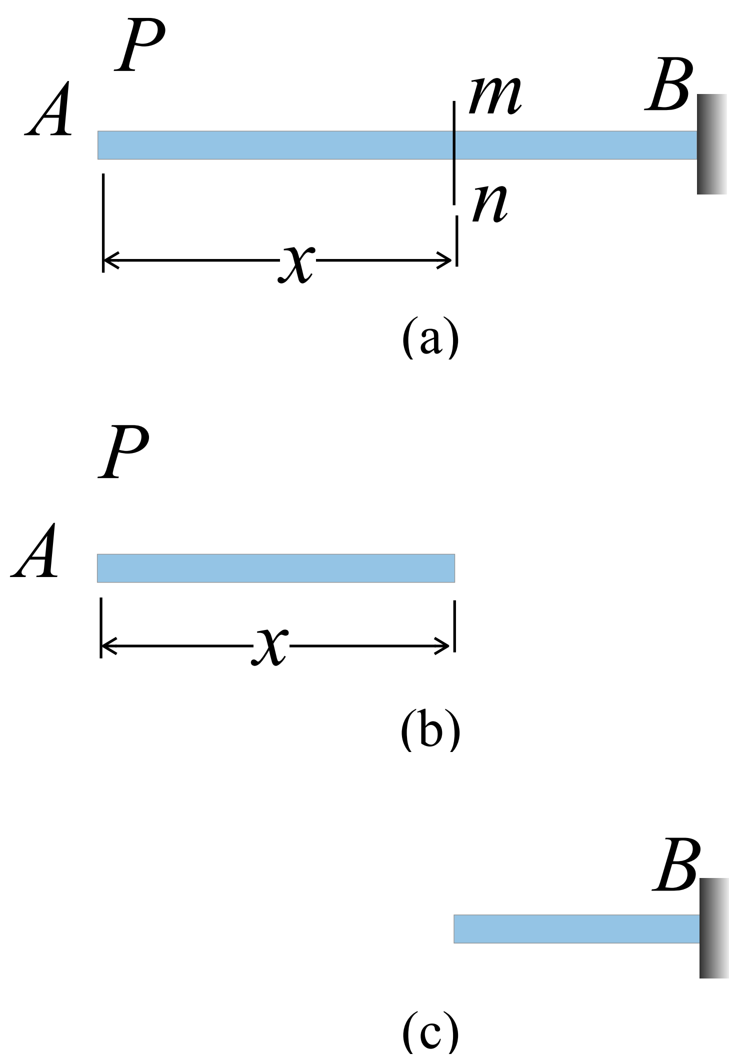
$$\sum F_x = 0 \Rightarrow H_A - P_1\cos\alpha = 0$$
$$\Rightarrow H_A = P_1\cos\alpha$$

$$\sum M_B = 0 \Rightarrow -R_A L + (P_1\sin\alpha)(L-a) + P_2(L-b) + qc^2/2 = 0$$
$$\Rightarrow R_A = \frac{(P_1\sin\alpha)(L-a)}{L} + \frac{P_2(L-b)}{L} + \frac{qc^2}{2L}$$

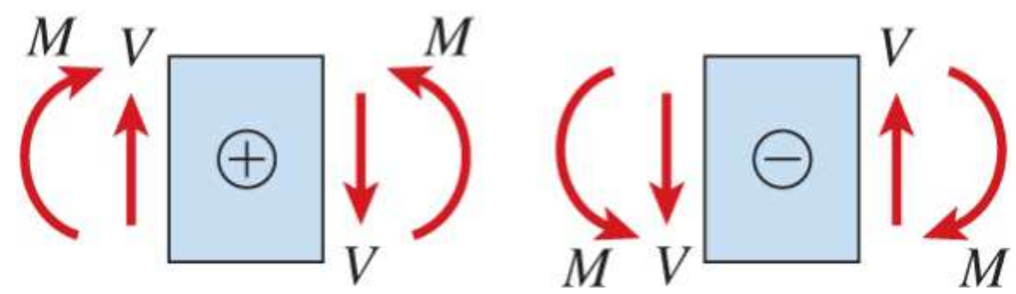
$$\sum M_A = 0 \Rightarrow R_B L - (P_1\sin\alpha)(a) - P_2(b) - qc(L-c/2) = 0$$
$$\Rightarrow R_B = \frac{(P_1\sin\alpha)(a)}{L} + \frac{P_2(b)}{L} + \frac{qc(L-c/2)}{L}$$

Shear forces
and bending moments
剪力與彎矩

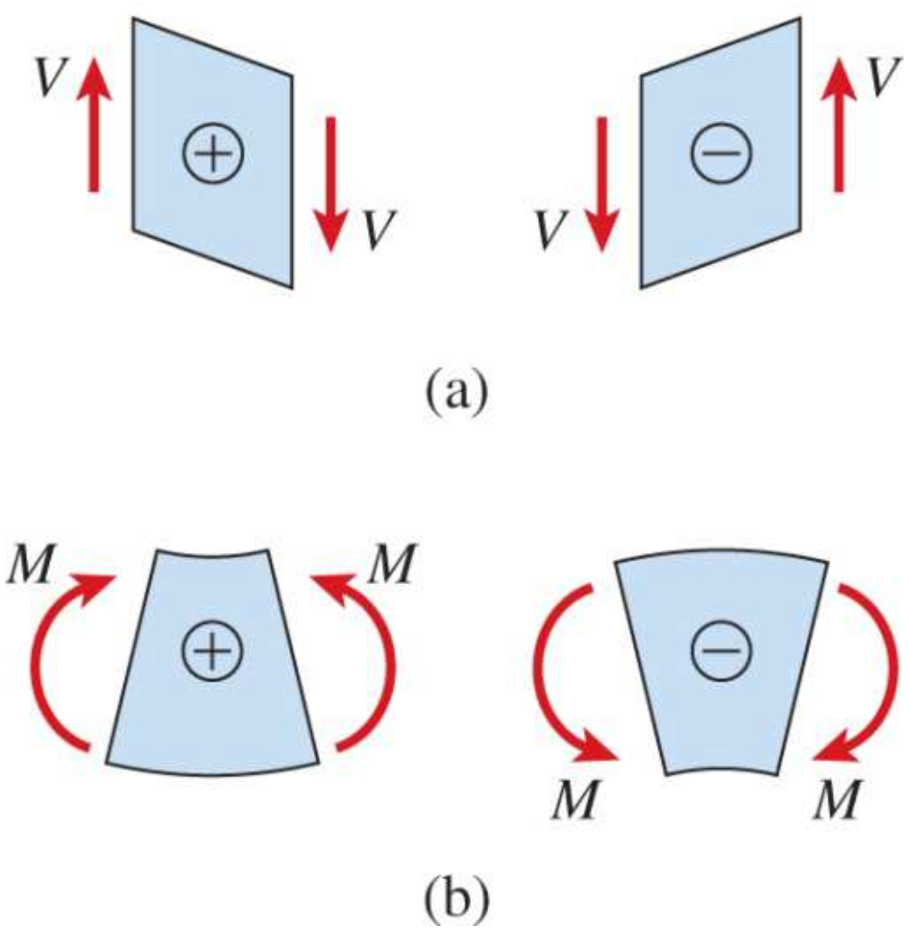
正負號規則



- 剪力彎矩



- 所造成之變形

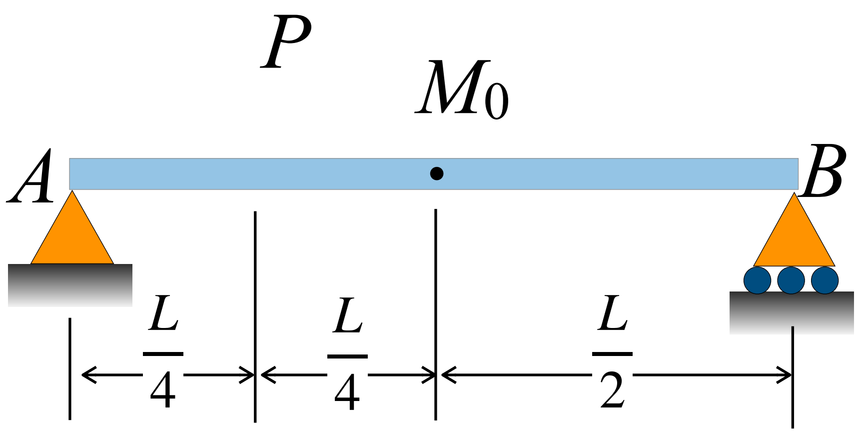


例：

Find：V and M

(1) a small distance to the left of the midpoint of the beam.

(2) a small distance to the right of the midpoint of the beam.



Relationships between loads, shear forces, and bending moments
負載、剪力與彎矩間的關係

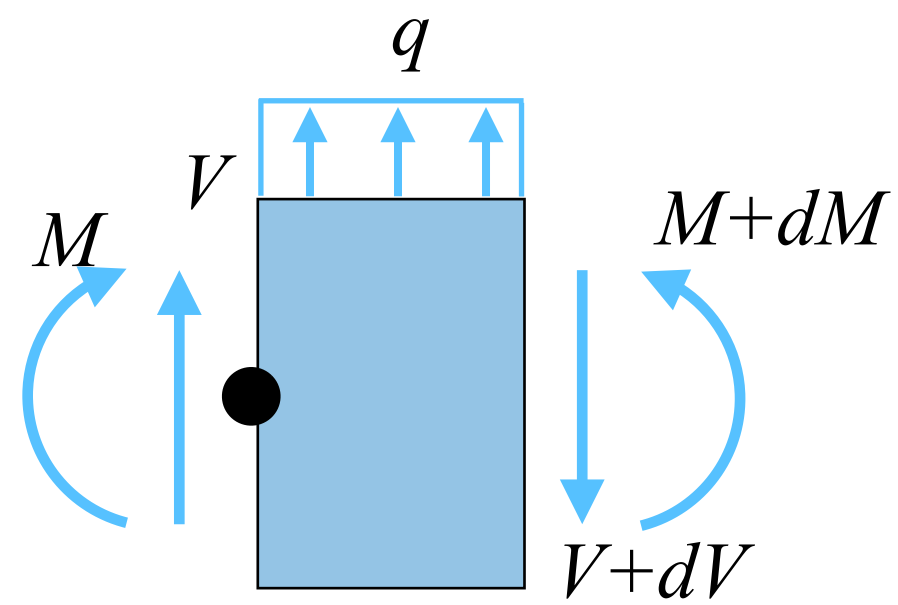
分佈負載

- $\sum F_y = 0 \Rightarrow V + qdx - (V + dV) = 0$

$$\Rightarrow \frac{dV}{dx} = q$$

兩邊積分 $\Rightarrow \int_A^B dV = \int_A^B qdx$

$$\Rightarrow V_B - V_A = \int_A^B qdx$$



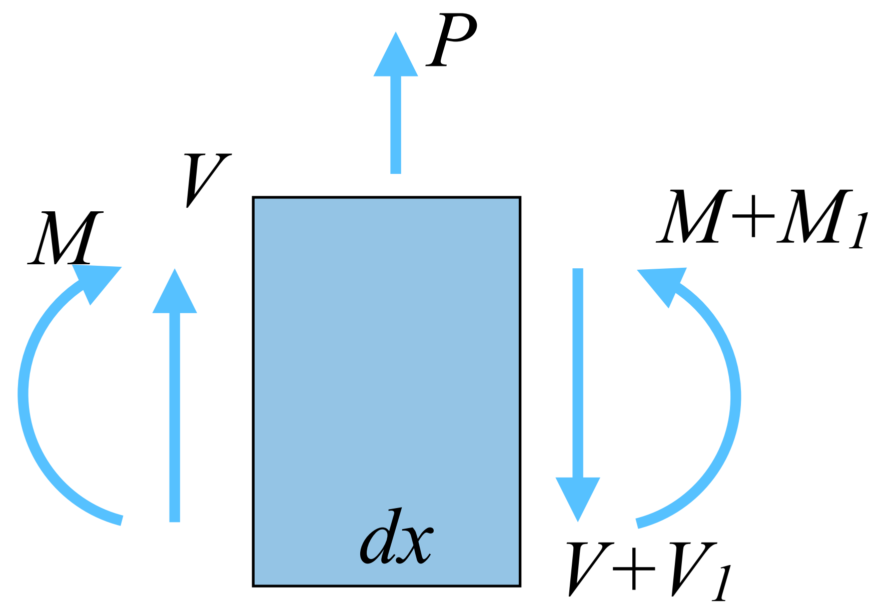
- $\sum M = 0 \Rightarrow -M + qdx\left(\frac{dx}{2}\right) - (V + dV)dx + M + dM = 0$

假若忽略微小項 $\Rightarrow \frac{dM}{dx} = V$

兩邊積分 $\Rightarrow \int_A^B dM = \int_A^B Vdx \quad \Rightarrow M_B - M_A = \int_A^B Vdx$

彎矩差　　剪力圖的面積

集中負載



- $\sum F_y = 0 \Rightarrow V + P - (V + V_1) = 0$

  $\Rightarrow V_1 = P$

- $\sum M = 0 \Rightarrow -M + P\left(\frac{dx}{2}\right) - (V + V_1)dx + M + M_1 = 0$

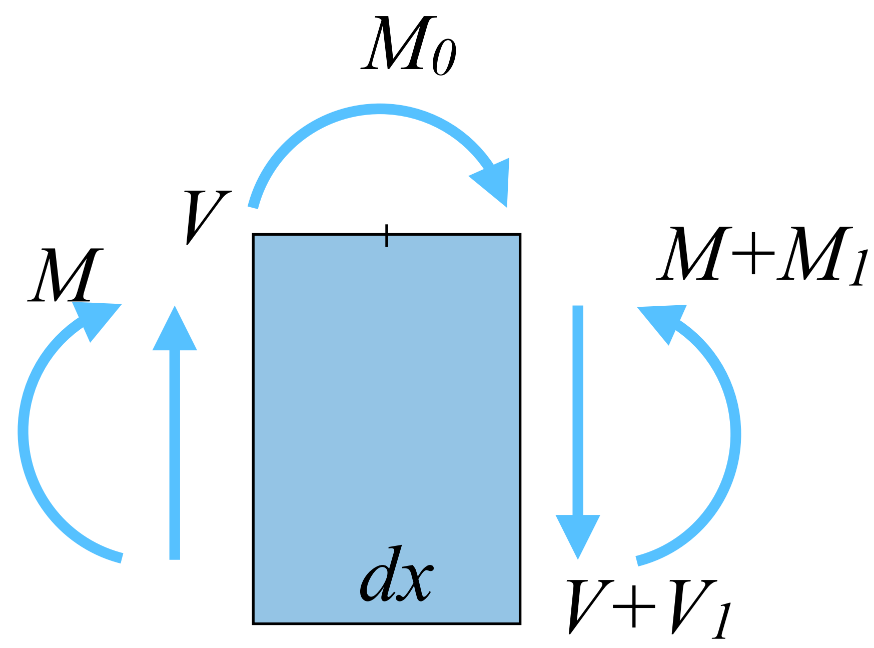
  $\Rightarrow M_1 = -P\left(\frac{dx}{2}\right) + Vdx + V_1 dx$

  由於 $dx \to 0$，所以增量 $M_1 \to 0$。

- 左側彎矩改變率 $\frac{dM}{dx} = V$

  右側彎矩改變率 $\frac{dM}{dx} = V + V_1 = V + P$

彎矩負載



- $\sum F_y = 0 \Rightarrow V - (V + V_1) = 0$

  $\Rightarrow V_1 = 0$

- $\sum M = 0 \Rightarrow -M - M_0 - (V + V_1)dx + M + M_1 = 0$

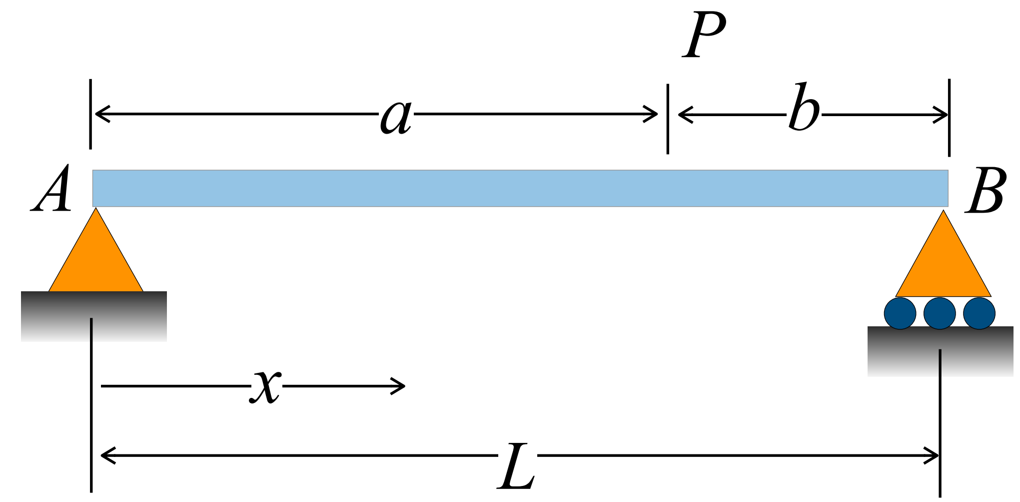
  忽略微小項 $\Rightarrow M_1 = M_0$

例：

Given : as shown in fig. 4-17(a)

Find : draw

(1) shear-force diagram (V-D).
(2) bending-moment diagram (M-D).



剪力變化量是
集中載重的大小

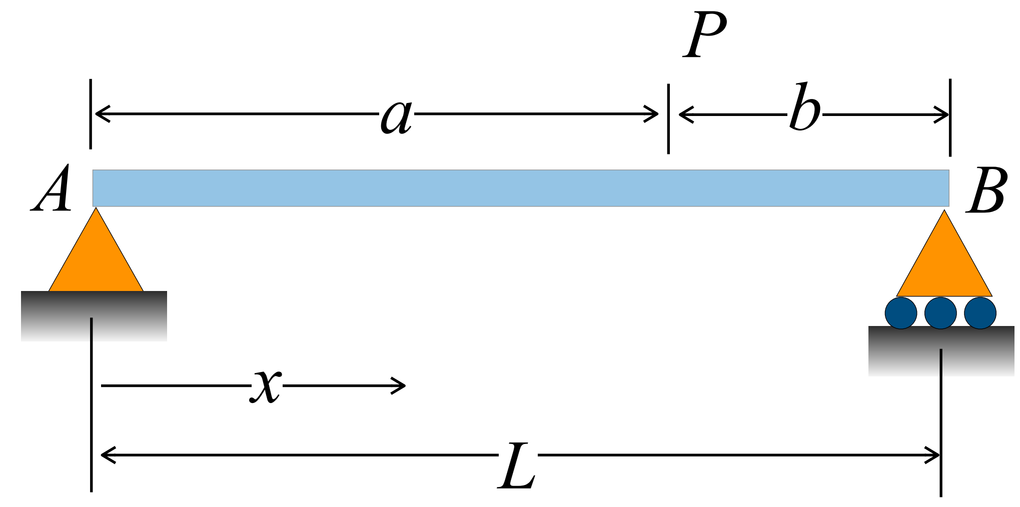
剪力變化量是
分佈負載面積

剪力圖的斜率是相對應
點上分佈負載的值

彎矩圖的斜率是相對應
點上的剪力值

彎矩變化量是
剪力圖的面積

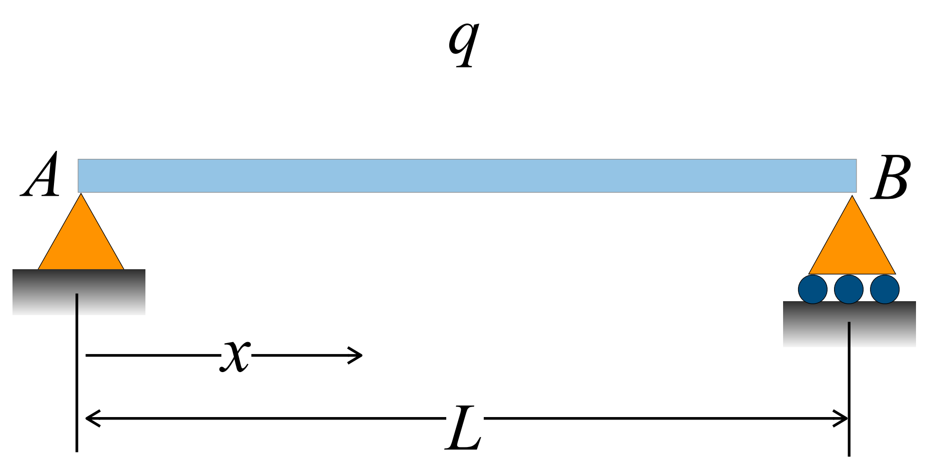
彎矩的變化量的**值**
是外施彎矩的大小



Given : as shown in fig. 4-18(a)

Find : draw

(1) shear-force diagram (V-D).
(2) bending-moment diagram (M-D).



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彎矩變化量是
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彎矩的變化量的**值**
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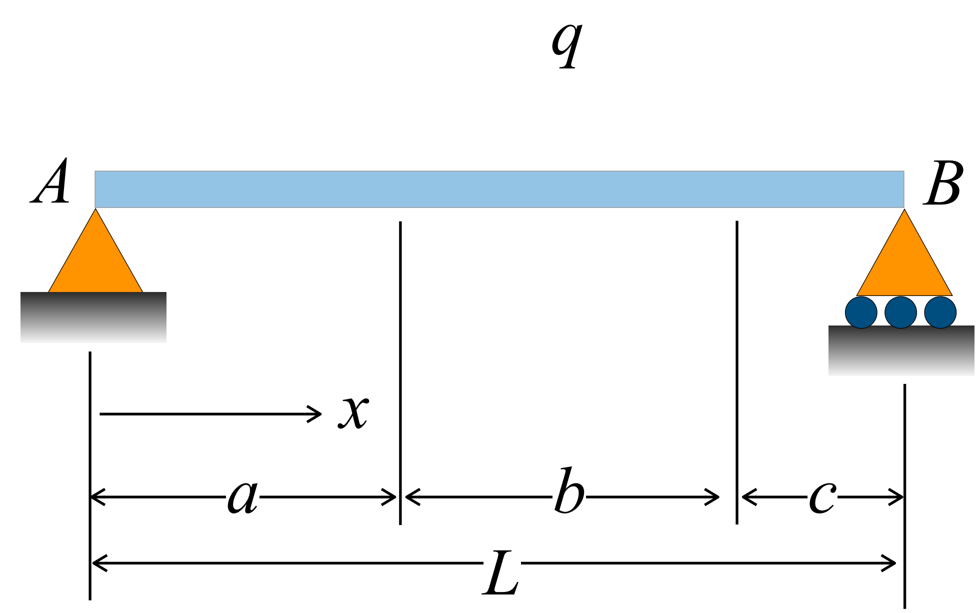
Given : as shown in fig. 4-20(a)

Find : draw

(1) shear-force diagram(V-D).
(2) bending-moment diagram(M-D).



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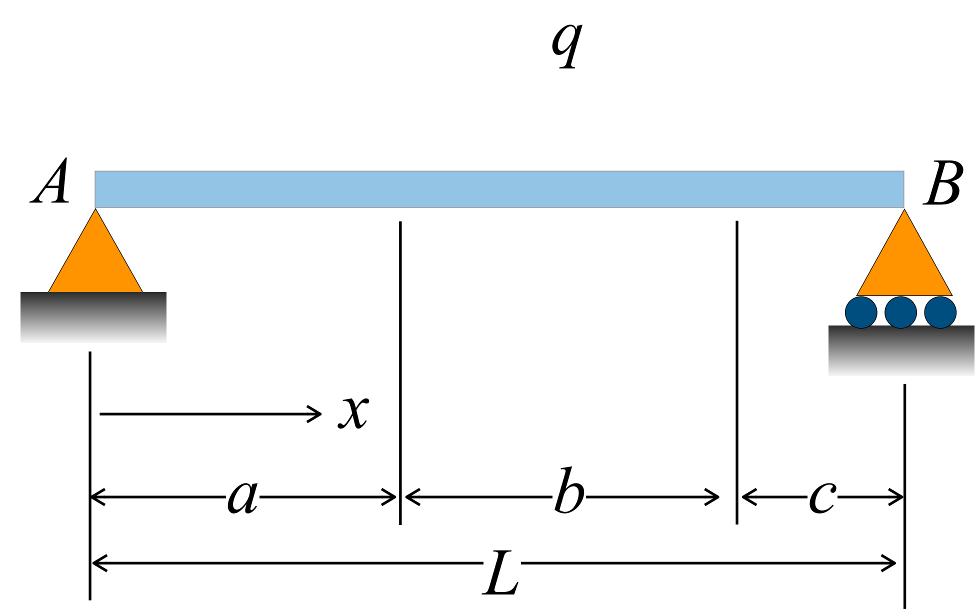
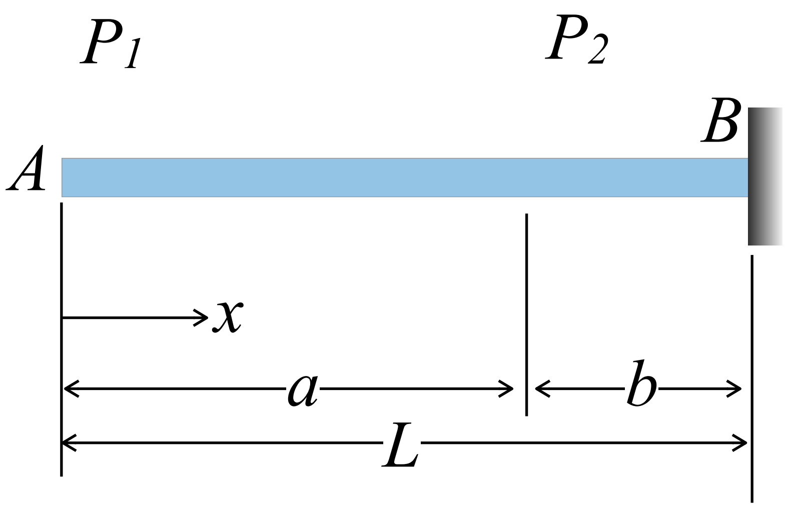


Given：as shown in figure (a).

Find : draw

(1) shear-force diagram(V-D).
(2) bending-moment diagram(M-D).

Sol ：



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剪力圖的面積

彎矩的變化量的**值**
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Given：as shown in figure (a).

Find : draw

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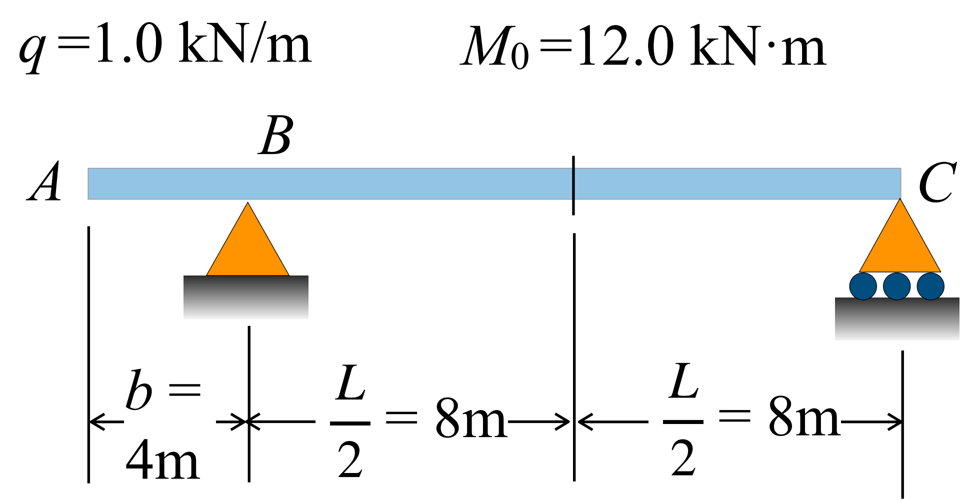
(2) bending-moment diagram(M-D).

Sol ：



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