Corrections

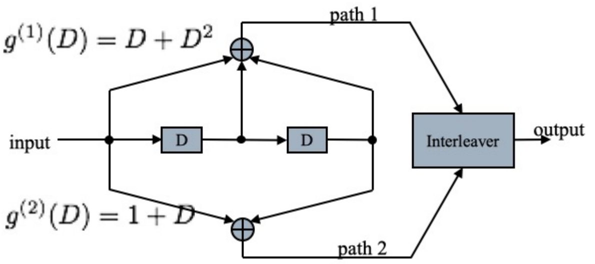
- Slide IDC7-19:

$$\Rightarrow d_{\min} = \min_{\boldsymbol{c}_i, \boldsymbol{c}_j \in \mathcal{C}, \boldsymbol{c}_i \neq \boldsymbol{c}_j} d_{\mathrm{H}}(\boldsymbol{c}_i, \boldsymbol{c}_j) = \min_{\boldsymbol{c} \in \mathcal{C}} w_{\mathrm{H}}(\boldsymbol{c})$$

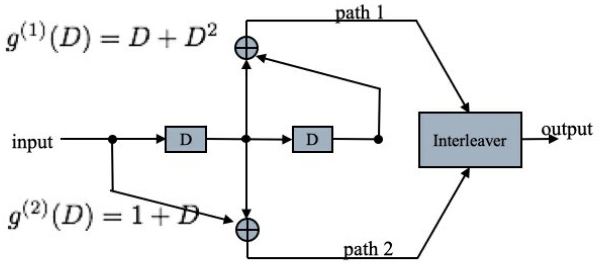
  should be replaced by

$$\Rightarrow d_{\min} = \min_{\boldsymbol{c}_i, \boldsymbol{c}_j \in \mathcal{C}, \boldsymbol{c}_i \neq \boldsymbol{c}_j} d_{\mathrm{H}}(\boldsymbol{c}_i, \boldsymbol{c}_j) = \min_{\boldsymbol{c} \in \mathcal{C}, \boldsymbol{c} \neq \boldsymbol{0}} w_{\mathrm{H}}(\boldsymbol{c})$$

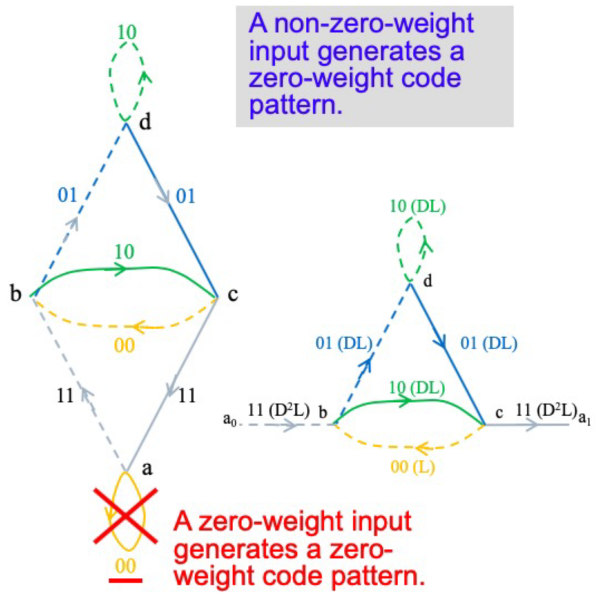
- Slide IDC7-55: "Only the first two" should be replaced by "All three".
- Slide IDC7-56: "It is a code of minimum distance $2^m - 1$" should be "It is a code of minimum distance 2^{m-1}."
- Slide IDC7-84:



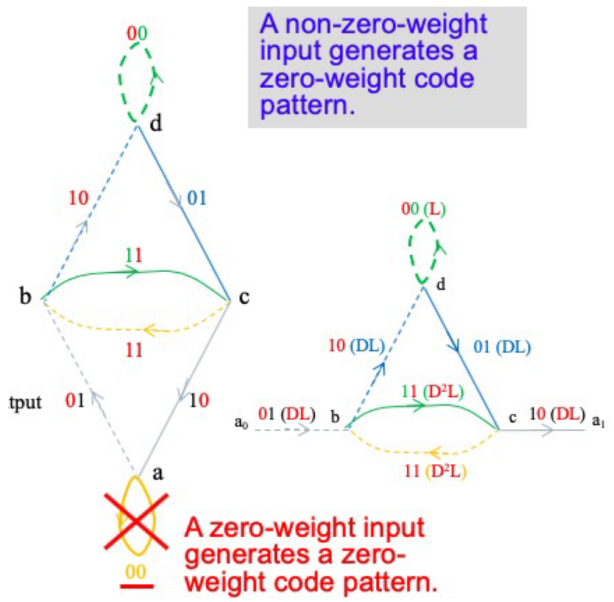
  should be



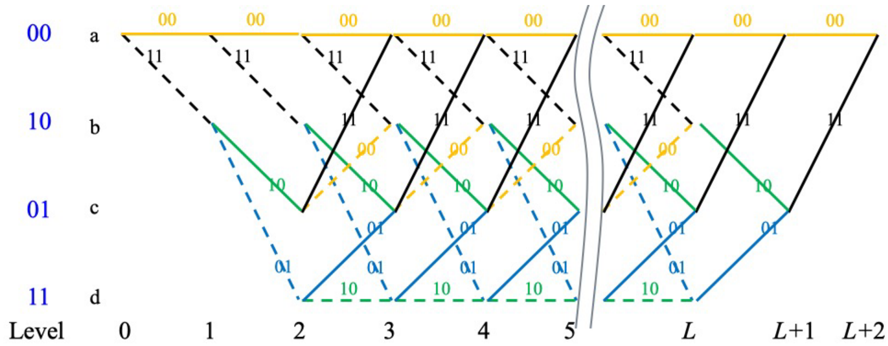
- Slide IDC7-84:



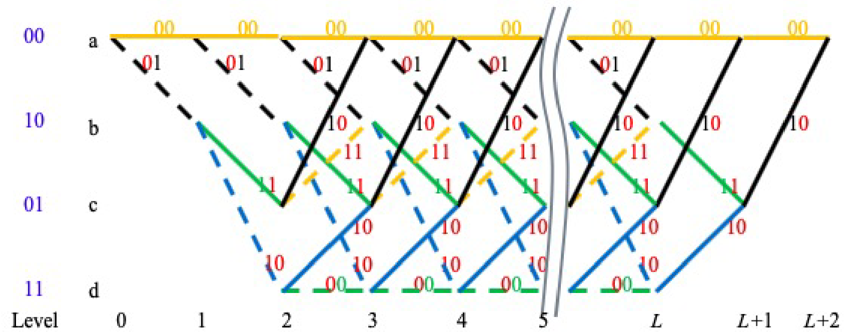
should be



- Slide IDC7-84:



should be



- Slide IDC7-87: “Slide 6-32” should be “Slide IDC 1-30”.

Sample Problems for Quiz 11

1. Continue from Sample Problem 4 for Quiz 10, where we list all code polynomials of a polynomial code of length $n = 5$ with generator polynomial $g(X) = X^3 + X + 1$:

$c(X)$	$(a_0 + a_1 X)(1 + X + X^3)$
$0 + 0 \cdot X + 0 \cdot X^2 + 0 \cdot X^3 + 0 \cdot X^4$	$(0 + 0 \cdot X)(1 + X + X^3)$
$0 + 1 \cdot X + 1 \cdot X^2 + 0 \cdot X^3 + 1 \cdot X^4$	$(0 + 1 \cdot X)(1 + X + X^3)$
$1 + 1 \cdot X + 0 \cdot X^2 + 1 \cdot X^3 + 0 \cdot X^4$	$(1 + 0 \cdot X)(1 + X + X^3)$
$1 + 0 \cdot X + 1 \cdot X^2 + 1 \cdot X^3 + 1 \cdot X^4$	$(1 + 1 \cdot X)(1 + X + X^3)$

(a) Does $g(X) = X^3 + X + 1$ divide $X^5 + 1$? Justify your answer.

Hint: $X^5 + 1 = (X + 1)(X^4 + X^3 + X^2 + X + 1)$

(b) Is the polynomial code of length 5 with generator polynomial $g(X) = X^3 + X + 1$ a cyclic code? Justify your answer.

Hint: Slide IDC 7-48 states that a polynomial code is cyclic iff its generator polynomial divides $X^n + 1$.

(c) Prove that a polynomial code is cyclic iff its generator polynomial divides $X^n + 1$.

Hint: The claim trivially holds when taking $g(X) = 1$. Thus, it suffices to prove the claim for $g(X) = X^{n-k} + g_{n-k-1}X^{n-k-1} + \cdots + g_1 X + 1$ with $n - k \geq 1$.

Hint: You can use the fact that a cyclic codeword must contain at least two 1's, i.e., $(100\ldots000)$ cannot be a codeword of a cyclic code (except trivially $g(X) = 1$).

Solution.

(a) Since

$$X^4 + X^3 + X^2 + X + 1 = (X + 1)(X^3 + X + 1) + X,$$

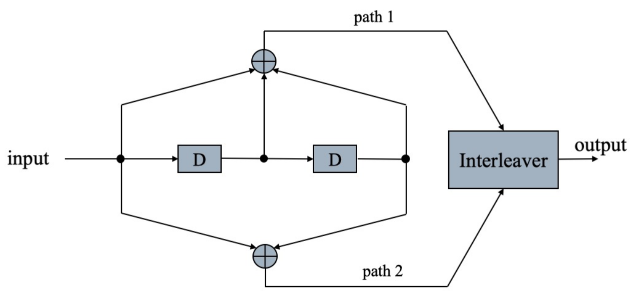
$g(X)$ does not divide $X^5 + 1$.

(b) The answer is negative because $X^3 + X + 1$ does not divide $X^5 + 1$.

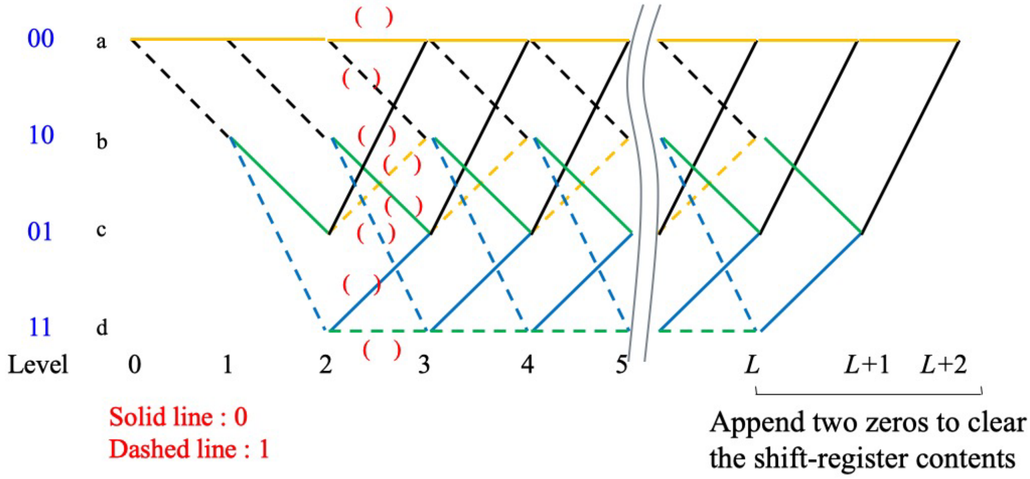
Note: $0 + 1 \cdot X + 1 \cdot X^2 + 0 \cdot X^3 + 1 \cdot X^4$ is a code polynomial but not all five circular shifts of it are code polynomials, e.g., $1 + X^2 + X^4$ is not a code polynomial.

(c) See Slides IDC 7-47∼7-48.

2. (a) For the convolutional code give below,



fill in the code bits inside the red-color parentheses on the code trellis below.



(b) Draw the state diagram and signal graph of this convolutional code, and determine the free distance of this convolutional code.

(c) If the received vector is equal to 11 00 01 . . ., give the four survivor paths up to level 3.

Solution.

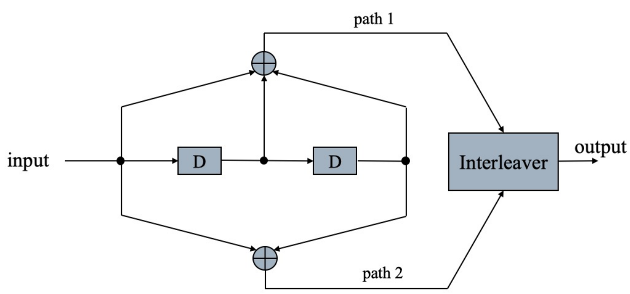
(a) Let me give you an example of how these parentheses can be filled. At state c, where the two shift registers contain 01, when input is 0, then path 1 gives $0 \oplus 0 \oplus 1 = 1$ and path 2 gives $0 \oplus 1 = 1$; thus, the output code bits are 11 (the next state is 00). When input is 1, then path 1 gives $1 \oplus 0 \oplus 1 = 0$ and path 2 gives $1 \oplus 1 = 0$; thus, the output code bits are 00 (and the next state is 10).

For the solution, see Slide IDC 7-69.

(b) See Slides IDC 7-80 $\sim$ 7-82.

(c) See Slide IDC 7-77.

3. (a) For the convolutional code given below,



find the equivalent block code corresponding to information sequences of length $L = 2$, i.e., 00, 01, 10 and 11. What is the minimum pairwise Hamming distance $d_{\min}$ of this $[8, 2, d_{\min}]$ block code?

Hint: Do not forget the two zero tail bits.

(b) What is the effective code rate of the convolutional code in (a)?

Solution.

(a) Performing $(a_0 + a_1 D)(1 + D + D^2) = a_0 + (a_0 + a_1)D + (a_0 + a_1)D^2 + a_1 D^3$ and $(a_0 + a_1 D)(1 + D^2) = a_0 + a_1 D + a_0 D^2 + a_1 D^3$, we obtain the codewords are

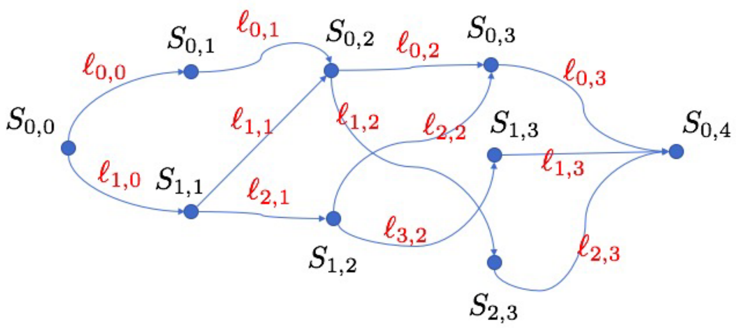
$$(a_0 \ a_0 \ (a_0 + a_1) \ a_1 \ (a_0 + a_1) \ a_0 \ a_1 \ a_1)$$

info sequence of length $L = 2$ $a_0 a_1$	codeword of length $2(L + m) = 8$ $c_0 c_1 c_2 c_3 c_4 c_5 c_6 c_7$
00	00000000
01	00111011
10	11101100
11	11010111

Since the minimum Hamming weight of non-zero codewords is 5, $d_{\min} = 5$.

(b) $2/8 = 1/4$.

4. (Viterbi algorithm) For the graph illustrated in the following figure:



a person wishes to find the most "similar" path from $S_{0,0}$ to $S_{0,4}$ to a received vector $[r_0, r_1, r_2, r_3]$ according to the criterion of minimizing the *Euclidean distance square* between $[r_0, r_1, r_2, r_3]$ and the branch labels, where the *Euclidean distance square* between $[r_0, r_1, r_2, r_3]$ and path $S_{0,0} \to S_{0,1} \to S_{0,2} \to S_{0,3} \to S_{0,4}$ is $(r_0 - \ell_{0,0})^2 + (r_1 - \ell_{0,1})^2 + (r_2 - \ell_{0,2})^2 + (r_3 - \ell_{0,3})^2$.

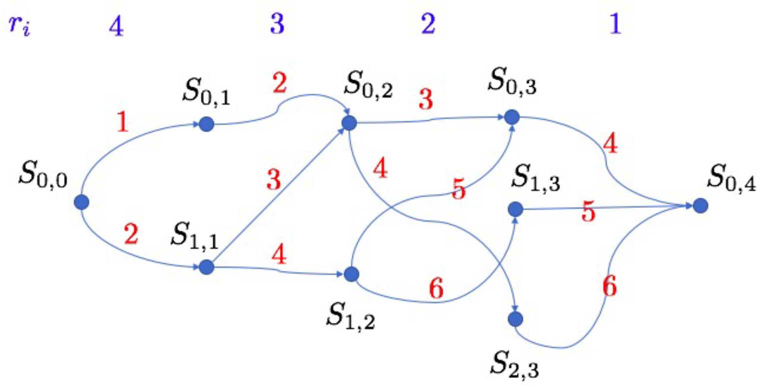
(a) Let $\ell_{i,j} = i + j + 1$ and $r_i = 4 - i$. Draw the two survivor paths ending at node $S_{0,2}$ and $S_{1,2}$, respectively, according to the Viterbi algorithm.

(b) Continue from (a), Draw the three survivor paths ending at node $S_{0,3}$, $S_{1,3}$ and $S_{2,3}$, respectively, according to the Viterbi algorithm.

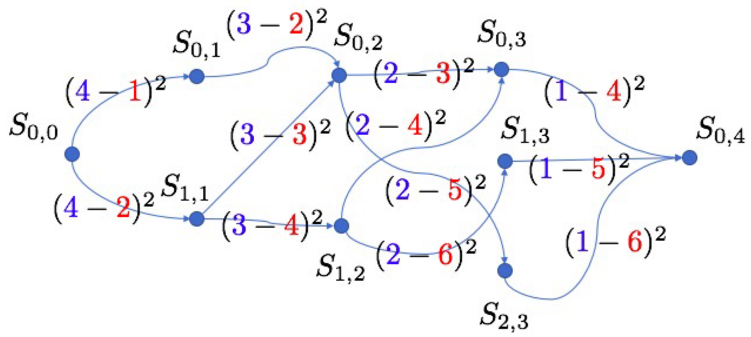
(c) Show the most similar path, i.e., the survivor path ending at node $S_{0,4}$.

Solution.

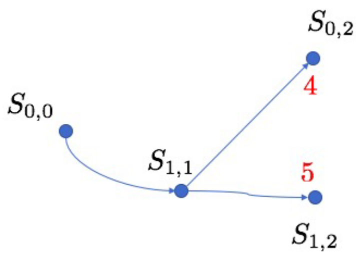
(a) The problem finds the most "similar" path from $S_{0,0}$ to $S_{0,4}$ to received vector $[r_0, r_1, r_2, r_3]$, which can be summarized as the following figure.



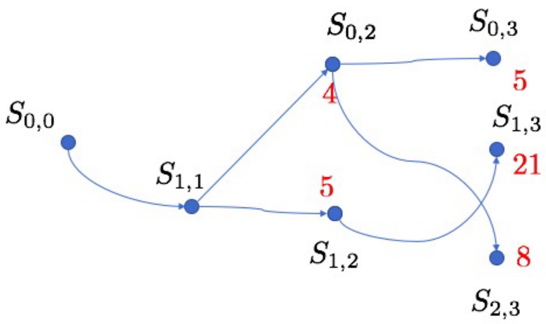
This is equivalent to finding the path with the smallest accumulative distance square shown below.



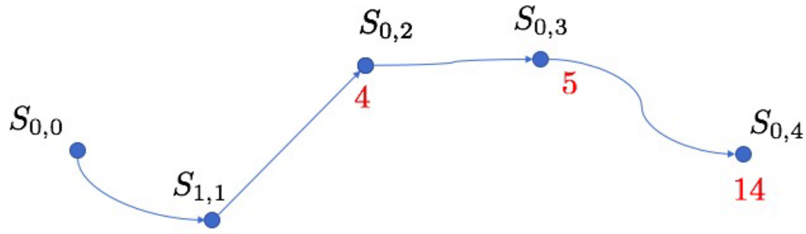
The survivor paths up to level 2 (with their accumulative distance squares marked in red) are:



(b)



(c)


$S_{0,0}$
$S_{1,1}$
$S_{0,2}$
4
$S_{0,3}$
5
$S_{0,4}$
14