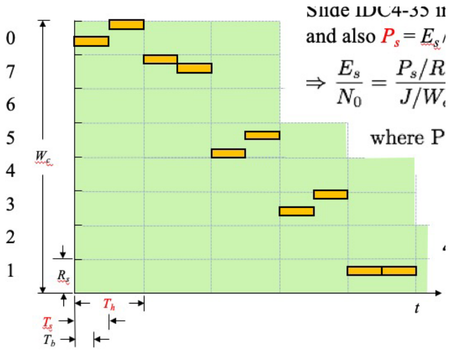
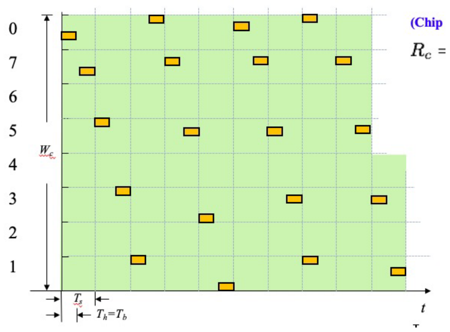
Corrections to slides.

- Slide IDC4-42: Please exchange T_s and T_h. Also, please remove $T_c =$. The corrected slide should look like:



- Slide IDC4-43: Please remove $T_c =$. The corrected slide should look like:



- The problem statement of Sample Problem 1(b) for Quiz 5 should be corrected as follows.

$$\hat{\tau} \;=\; \arg \max_{\sout{\theta\in[0,\pi)}\ \tau} \sum_{k=0}^{L_0-1} \boldsymbol{x}_k^T(\tau)\, \boldsymbol{s}_k(\alpha_k, \theta, \tau|\tau_0)?$$

- The solution to Sample Problem 2(d) for Quiz 5 should be corrected as follows.

j	$C(j)$	
0	3	
1	−1	
2	~~3~~	−1
3	~~3~~	−5
4	~~−5~~	3
5	~~−1~~	3
6	−1	

- Slide IDC 5-46: Please replace “looks” by ”look” in the following sentence: does the cover area of a base station looks like a hexagon.

Sample Problems for Quiz 6

1. (a) For $m = 3$, there are only two maximum-length shift-register sequences (as can be obtained from Sample Problem 2 for Quiz 5). Their cross-correlations are exactly three-valued, i.e., -1, $-t(3) = -5$ and $t(3) - \not{1}\, 2 = 3$, where

$$t(m) = \begin{cases} 2^{(m+1)/2} + 1, & m \text{ odd} \\ 2^{(m+2)/2} + 1, & m \text{ even} \end{cases}$$

Use the two maximum-length shift-register sequences for $m = 3$ to generate all $2^m+1 = 9$ Gold sequences.

(b) Determine the autocorrelations of these $2^m + 1 = 9$ Gold sequences.

(c) Do Gold sequences guarantee satisfying the balance property and run property?

(d) Determine the smallest m such that $\frac{t(m)}{2^m-1} < \frac{1}{8}$.

(e) Select the number that is closest to the (ideal) processing gain (in dB) if the smallest m in (d) is applied to generate a maximum-length shift register sequence?

i) 27 dB *ii*) 18 dB *ii*) 33 dB *ii*) 12 dB

Solution.

(a) In addition to the two maximum-length shift-register sequences $a_0a_1a_2a_3a_4a_5a_6 =$ 0010111 and $b_0b_1b_2b_3b_4b_5b_6 =$ 0011101, the other seven Gold sequences are

$a_0a_1a_2a_3a_4a_5a_6$	circular right shift of $b_0b_1b_2b_3b_4b_5b_6$	Gold sequences (Xor the previous two columns)
0010111	0011101	0001010
0010111	1001110	1011001
0010111	0100111	0110000
0010111	1010011	1000100
0010111	1101001	1111110
0010111	1110100	1100011
0010111	0111010	0101101

(b) The autocorrelation function of all sequences is equal to 7 when the displacement is zero. The autocorrelation function of the two maximum-length shift-register sequences equals -1 for any non-zero displacement. Below, we list the autocorrelation function of the remaining seven Gold sequences based on

$$A(j) \triangleq \sum_{i=0}^{T-1} (-1)^{a_i} (-1)^{a_{(i+j) \bmod T}} \quad \text{for } 0 \le j \le T-1.$$

Gold sequences	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
0001010	-1	3	-1	-1	3	-1
1011001	-1	-5	3	3	-5	-1
0110000	3	-1	-1	-1	-1	3
1000100	1−	-1	3	3	-1	-1
1111110	3	3	3	3	3	3
1100011	3	-1	5	5	-1	3
0101101	-5	3	-1	-1	3	-5

Note: Gold has proved that the autocorrelation function of Gold sequences is also three-valued, i.e., -1, $-t(m)$ and $t(m) -$ ~~3~~2. Hence, if $t(m)$ is adequately small in comparison to $2^m - 1$, Gold sequences can provide acceptably good autocorrelation and crosscorrelation properties.

(c) The answer is apparently no.

(d) We require

$$t(m) = \left\{ \begin{array}{ll} 2^{(m+1)/2} + 1, & m \text{ odd} \\ 2^{(m+2)/2} + 1, & m \text{ even} \end{array} \right\} < 2^{-3}(2^m - 1) = 2^{m-3} - \frac{1}{8}.$$

Because $t(m)$ is an integer, the requirement is equivalent to

$$t(m) = \begin{cases} 2^{(m+1)/2} + 1, & m \text{ odd} \\ 2^{(m+2)/2} + 1, & m \text{ even} \end{cases} < 2^{m-3} \Leftrightarrow \begin{cases} 1 < 2^{(m+1)/2}(2^{(m-7)/2} - 1), & m \text{ odd} \\ 1 < 2^{(m+2)/2}(2^{(m-8)/2} - 1), & m \text{ even} \end{cases}$$

Hence, the smallest m that satisfies the requirement is $m = 9$.

(e) The processing gain is equal to $2^9 - 1 = 511$, approximately 2^9. Since 2 is 3dB, the closest number (in dB) should be $3 \cdot 9 = 27$ dB.

2. From Slide IDC 5-36, we obtain a general noise figure formula for a balanced load system:

$$F - 1 = F_1 - 1 + \frac{1}{G_1}(F_2 - 1) + \frac{1}{G_1 G_2}(F_3 - 1) + \cdots.$$

Suppose three devices respectively have power gains $G_a = 1$, $G_b = 10$ and $G_c = 100$ and NFs $F_a = 1$, $F_b = 11$ and $F_c = 101$. Find the cascaded sequence of these three devices that minimizes F.

Solution.

cascaded sequence	resulting noise figure
abc	$0 + \frac{1}{1}10 + \frac{1}{1 \cdot 10}100 = 20$
acb	$0 + \frac{1}{1}100 + \frac{1}{1 \cdot 100}10 = 100.1$
bac	$10 + \frac{1}{10}0 + \frac{1}{10 \cdot 1}100 = 20$
bca	$10 + \frac{1}{10}100 + \frac{1}{10 \cdot 100}0 = 20$
cab	$100 + \frac{1}{100}0 + \frac{1}{100 \cdot 1}10 = 100.1$
cba	$100 + \frac{1}{100}10 + \frac{1}{100 \cdot 10}0 = 100.1$

Thus, either abc or bac or bca achieves the minimum NF.

Note: This indicates that it is less favorable to place a large NF device c in front, even if it has a large power gain. Also, which one should be placed first, depending on both power gain G and noise figure F.

3. The Friis free space equation gives that

$$\text{Path Loss} = -10\log_{10}(G_t G_r) + 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2.$$

(a) If the distance increases 10 times, how many dB of Path Loss increases?

(b) Given that the antenna gains of the satellite and ground-based antennas are 44 dB and 48 dB, respectively, determine the isotropic free space loss at 4 GHz from an earth station to a geosynchornous satellite with distance $35,000$ km. Re-do the problem when the carrier frequency is changed to 12 GHz.

Note: Light speed $= 3 \times 10^8$ m/sec

Solution.

(a) We need to cmpare

$$\text{Path Loss} = -10\log_{10}(G_t G_r) + 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2$$

with

$$\begin{aligned}\text{Path Loss} &= -10\log_{10}(G_t G_r) + 10\log_{10}\left(\frac{4\pi 10 d}{\lambda}\right)^2 \\ &= -10\log_{10}(G_t G_r) + 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2 + 20\text{dB}\end{aligned}$$

Thus, the path loss increases by 20 dB when the distance has a ten-times increase.

(b) For $f_c = 4$ GHz,

$$\begin{aligned}\text{Path Loss} &= -10\log_{10}(G_t G_r) + 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2 \\ &= -44\text{ dB} - 48\text{ dB} + 10\log_{10}\left(\frac{4\pi \cdot 35\times 10^6 \cdot 4\times 10^9}{3\times 10^8}\right)^2 \\ &= -44\text{ dB} - 48\text{ dB} + 195.36\text{ dB} \\ &= 103.36\text{ dB.}\end{aligned}$$

For $f_c = 12$ GHz,

$$\begin{aligned}\text{Path Loss} &= -44\text{ dB} - 48\text{ dB} + 10\log_{10}\left(\frac{4\pi \cdot 35\times 10^6 \cdot 12\times 10^9}{3\times 10^8}\right)^2 \\ &= -44\text{ dB} - 48\text{ dB} + \sout{195.36}\, 204.91\text{ dB} \\ &= 112.91\text{ dB.}\end{aligned}$$

Note: Thus we have additional 9.55 dB path loss due to the frequency increase.

4. (a) If we wish to find a zero-one sequence to follow the exact run property (in the sense that the number of length-ℓ runs doubles the number of length-2ℓ runs for every ℓ), what will the formula of the length of the sequence be?

(b) The length of the shortest sequence satisfying the requirement in (a) is 4 (two runs of length 1 and one run of length 2). Can we find a sequence of such length that also satisfies the balance property? If Yes, give such sequence; else, explain why such sequence does not exist.

Solution.

(a) Let n_ℓ be the number of length-ℓ runs. Then the length of the PN-sequence should satisfy

$$\begin{aligned} n &= n_1 \times 1 + \frac{n_1}{2} \times 2 + \frac{n_1}{2^2} \times 3 + \cdots + \frac{n_1}{2^k} \times (k+1) \\ &= \sum_{i=0}^{k} 2^i \times (k+1-i) \\ &= 2^{k+2} - k - 3, \end{aligned}$$

where $n_1 = 2^k$. As a result, for $k = 1, 2, 3, 4, \ldots$, we have $n = 4, 11, 26, 57, \ldots$.

Note: The Barker sequence 00111011010 used by Wifi is of length 11. Thus, it can perfectly match the run property.

(b) The one run of length 2 must be either "00" or "11". Thus, the corresponding two runs of length 1 must be two "1"' or two "0". Accordingly, the answer can be either 1001 or 0110.