Problems for Quiz 10

Name:________________ Student ID:________________ Score:____________

1. (50%) Prove that among all continuous random variables of mean μ and variance σ^2, Gaussian random variable has the largest differential entropy.

   Hint: Let X be the Gaussian random variable of mean μ and variance σ^2. Let Y be an arbitrary random variable of the same mean and variance as X. Then,

$$\begin{aligned} h(X) &= \int_{\Re} f_X(x)\log_2 \frac{1}{f_X(x)} dx \\ &= \int_{\Re} f_X(x)\left[\frac{1}{2}\log_2(2\pi\sigma^2) + \log_2(e)\cdot\frac{(x-\mu)^2}{2\sigma^2}\right] dx \\ &= \int_{\Re} f_Y(x)\left[\frac{1}{2}\log_2(2\pi\sigma^2) + \log_2(e)\cdot\frac{(x-\mu)^2}{2\sigma^2}\right] dx \\ &= \int_{\Re} f_Y(x)\log_2 \frac{1}{f_X(x)} dx \qquad (1) \end{aligned}$$

   Complete the proof based on (1).

   Solution. See Slide IDC 6-54. Other than the fundamental inequality, you can also use Jensen's inequality, or log-sum inequality (or the non-negativity of divergence) to prove the result.

2. (50%) Let the generator polynomial of a polynomial code of length $n = 5$ be $g(X) = X^3 + X + 1$. List all the code polynomials of this polynomial code via

$$c(X) = a(X)g(X).$$

   What is the minimum pairwise Hamming distance of this code?

   Hint:

$$d_{\min} = \min_{\boldsymbol{c}_i,\boldsymbol{c}_j\in\mathcal{C},\boldsymbol{c}_i\neq\boldsymbol{c}_j} d_{\mathrm{H}}(\boldsymbol{c}_i,\boldsymbol{c}_j) = \min_{\boldsymbol{c}\in\mathcal{C},\boldsymbol{c}\neq\boldsymbol{0}} w_{\mathrm{H}}(\boldsymbol{c})$$

   Solution. $c(X) = (a_0 + a_1X)(1 + X + X^3)$ implies

$c(X)$	$(a_0 + a_1X)(1 + X + X^3)$
$0+0\cdot X+0\cdot X^2+0\cdot X^3+0\cdot X^4$	$(0+0\cdot X)(1+X+X^3)$
$0+1\cdot X+1\cdot X^2+0\cdot X^3+1\cdot X^4$	$(0+1\cdot X)(1+X+X^3)$
$1+1\cdot X+0\cdot X^2+1\cdot X^3+0\cdot X^4$	$(1+0\cdot X)(1+X+X^3)$
$1+0\cdot X+1\cdot X^2+1\cdot X^3+1\cdot X^4$	$(1+1\cdot X)(1+X+X^3)$

   The minimum pairwise Hamming distance is the smallest Hamming weight of non-zero codewords, which is 3.