Problems for Quiz 8

Name:_____ Student ID:_____ Score:_____

1. (50%) With $L$-diversity, a receiver receives

$$\begin{aligned} r_1 &= \pm\alpha_1\sqrt{\mathcal{E}} + z_1 \\ r_2 &= \pm\alpha_2\sqrt{\mathcal{E}} + z_2 \\ &\vdots \\ r_L &= \pm\alpha_L\sqrt{\mathcal{E}} + z_L \end{aligned}$$

where $\{z_k\}_{k=1}^L$ are zero-mean i.i.d. with variance $\sigma^2$ and $\{\alpha_k\}_{k=1}^L$ are assumed to be perfectly estimated. Which ones (i.e., multiple choices) of the below combiners (that combine $r_1$, $r_2$, ..., $r_L$ into one value for threshold-type decision) is classified as a linear combiner?

(a) $r = \sum_{k=1}^L w_k\, r_k$

(b) $r = \sum_{k=1}^L w_{L-k}\, r_k$

(c) $r = \sum_{k=1}^L w_k\, \log(r_k)$

(d) $r = \prod_{k=1}^L r_k$

**Solution.** (a) and (b) are both linear combiners.

2. (50%) Compute the entropy $H(X)$ (in bits) of random variable $X$, where

$$\Pr[X=0] = \frac{1}{8}, \quad \Pr[X=1] = \frac{1}{8}, \quad \Pr[X=2] = \frac{1}{4}, \quad \text{and} \quad \Pr[X=3] = \frac{1}{2}.$$

**Solution.**

$$\begin{aligned} H(X) &= \frac{1}{8}\log_2\frac{1}{(\frac{1}{8})} + \frac{1}{8}\log_2\frac{1}{(\frac{1}{8})} + \frac{1}{4}\log_2\frac{1}{(\frac{1}{4})} + \frac{1}{2}\log_2\frac{1}{(\frac{1}{2})} \\ &= \frac{1}{8}\times 3 + \frac{1}{8}\times 3 + \frac{1}{4}\times 2 + \frac{1}{2}\times 1 \\ &= \frac{7}{4} \text{ bits} \end{aligned}$$