Problems for Quiz 7

Name:________________ Student ID:________________ Score:____________

1. (40%) Below are delay spreads for several indoor environments.

Indoor space	typical delay spread
New York stock exchange	120 ns
Meeting room (5m×5m) with metal walls	55 ns
Single room with stone walls	35 ns
Indoor sports arena	120 ns
Factory	125 ns
Office buildings	30ns – 130 ns

Which one (i.e., single choice) of the below signal durations T **cannot** result in a (near) frequency non-selective communication for all the above indoor spaces?

(a) $T = 3.2$ msec; (b) $T = 20$ μsec; (c) $T = 500$ ns; (d) $T = 1$ msec.

Hint: In order to result in a near frequency-flat or frequency non-selective channel, it often requires that the signal duration is (at least) ten times larger than the delay spread.

Solution. (c)

2. (60%) For a time-flat frequency-flat fading channel, the input signal suffers a single multiplicative factor α. The signal-to-noise ratio of the system becomes $\gamma = \alpha^2 \frac{E_b}{N_0}$. Suppose α can be perfectly estimated at the receiver. Subject to

$$\Pr\left[\alpha^2 = \frac{1}{2}\right] = \Pr\left[\alpha^2 = \frac{3}{2}\right] = \frac{1}{2},$$

express the average bit error rate (BER) of the binary BPSK transmission as a function of the average SNR γ_0, where $\gamma_0 = E[\gamma] = E[\alpha^2]\frac{E_b}{N_0} = \frac{E_b}{N_0}$. The answer should be in the form of

$$\text{BER} = (\quad\ ) \cdot \Phi\big(-\sqrt{(\quad\ )\gamma_0}\big) + (\quad\ ) \cdot \Phi\big(-\sqrt{(\quad\ )\gamma_0}\big).$$

Hint: When the pdf of γ is $f_\gamma(\gamma)$, the average BER in a fading environment is given by

$$\text{BER} = \int_0^\infty \Phi(-\sqrt{2\gamma}) f_\gamma(\gamma) \mathrm{d}\gamma, \tag{1}$$

where for binary BPSK transmission, the BER without fading is equal to $\Phi\left(-\sqrt{2\gamma}\right)$. You may think of what the equivalent discrete counterpart of (1) is.

Solution. Noting that

$$\Pr\left[\gamma = \frac{1}{2}\gamma_0\right] = \Pr\left[\gamma = \frac{3}{2}\gamma_0\right] = \frac{1}{2},$$

we have

$$\begin{aligned} \text{BER} &= \frac{1}{2}\Phi\left(-\sqrt{2 \cdot \frac{1}{2}\gamma_0}\right) + \frac{1}{2}\Phi\left(-\sqrt{2 \cdot \frac{3}{2}\gamma_0}\right) \\ &= \frac{1}{2}\Phi\left(-\sqrt{\gamma_0}\right) + \frac{1}{2}\Phi\left(-\sqrt{3\gamma_0}\right). \end{aligned}$$