Problems for Quiz 6

Name:________________ Student ID:________________ Score:____________

1. A general noise figure formula for a balanced load system is given below:

$$(F-1) = (F_1 - 1) + \frac{1}{G_1}(F_2 - 1) + \frac{1}{G_1 G_2}(F_3 - 1) + \cdots.$$

   (a) (30%) Suppose three devices respectively have power gains $G_a = 1$, $G_b = 10$ and $G_c = 100$ and NFs $F_a = F_b = F_c$. Find the cascaded sequence of the three devices that **minimizes** the overall noise figure F. You shall provide either derivation or argument about why your cascaded sequence minimizes the overall noise figure.

   (b) (30%) Re-do (a) by finding the cascaded sequence of the three devices that **maximizes** the overall noise figure F.

   (c) (40%) What are the resulting noise figures in (a) and (b) if $F_a = F_b = F_c = 2$?

Solution.

(a) Since

$$(F-1) = (F_a - 1) + \frac{1}{G_1}(F_a - 1) + \frac{1}{G_1 G_2}(F_a - 1) = (F_a - 1)\left(1 + \frac{1}{G_1} + \frac{1}{G_1 G_2}\right), \quad (1)$$

it is clear that G_1 should be made as large as possible, and G_2 should be the second largest. Thus, the cascaded sequence of the three devices, which minimizes the overall noise figure, is cba.

(b) Again, from the formula in (1), we shall make G_1 as small as possible, and G_2 should be the second smallest. Thus, the cascaded sequence of the three devices, which maximizes the noise figure, is abc.

(c) For (a), we have $(F_1 - 1) = (F_a - 1) \times 1.011 = 1.011$ since $G_1 = 100$ and $G_2 = 10$. For (b), we have $(F_1 - 1) = (F_a - 1) \times 2.1 = 2.1$ since $G_1 = 1$ and $G_2 = 10$.

Note: If $F_a = F_b = F_c = 1$ (i.e., ideal devices are designed and built), then the cascaded order has nothing to do with the performance. However, for non-ideally constructed devices, we do favor putting a high-gain device in front. As you can see, by cascading devices in an improper order, the noise figure can be doubled.