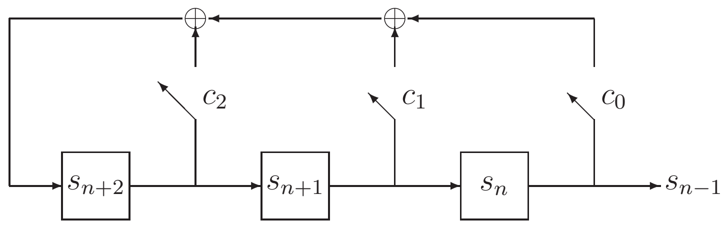
Sample Problems for Quiz 5

Name:________________ Student ID:________________ Score:____________



1. For the pseudo-noise sequence generator above, we have $s_{n+3} = c_2 s_{n+2} \oplus c_1 s_{n+1} \oplus c_0 s_n$, where each s_n and c_i are in $\{0, 1\}$.

   (a) (30%) With $(s_0, s_1, s_2) = (0, 0, 1)$, find the period of the output sequences for $(c_0, c_1, c_2) = (1, 0, 0)$.

   (b) (30%) Is the sequence in (a) a maximum-length shift register sequence? Justify your answer.

   (c) (40%) Does the periodic sequence in (a) satisfy the balance property? Does the periodic sequence in (a) satisfy the run property? Justify your answer by explicitly checking the two properties with numbers.

   Hint:

   1. Balance property $\equiv$ the (absolute value of the) difference between the number of one's and the number of zeros is at most one.

   2. Run property $\equiv N_{\ell+1} = \left\lceil \frac{N_\ell}{2} \right\rceil$ or $\left\lfloor \frac{N_\ell}{2} \right\rfloor$, where N_ℓ is the number of runs of length ℓ.

Solution.

(a) $(c_0, c_1, c_2) = (1, 0, 0)$ implies $s_{n+3} = s_n$. Hence, the period is 3.

Note: There are eight possible initial values for (s_0, s_1, s_2). The output sequence simply repeats the initial values for the particular choice of (c_0, c_1, c_2). Thus, if $(s_0, s_1, s_2) = (0, 0, 0)$ or $(1, 1, 1)$ initially, the period will be reduced to 1. As a result, except for the maximum-length shift-register sequence, the output periodic pattern will be a function of the initial values.

(b) No, because its period is not equal to $2^3 - 1 = 7$.

(c) The periodic sequence in (a) is 001. It satisfies the balance property because the number of ones is only one less than the number of zeros. It satisfies the run property because $N_1 = 1$ and $N_2 = 1$, and $N_2 = \left\lceil \frac{N_1}{2} \right\rceil$.