Problems for Quiz 4

Name:________________ Student ID:________________ Score:____________

1. A binary communication system can be characterized by $\mathbf{x} = \mathbf{s} + \mathbf{n}$, where $\mathbf{s}$ is either $\mathbf{s}_1$ or $\mathbf{s}_2$ with equal probability, and $\mathbf{n}$ has the pdf $\frac{1}{(2\pi\sigma^2)^{N/2}} \exp\{-\|\mathbf{n}\|^2/(2\sigma^2)\}$. It can be derived that the optimal decision rule is

$$\langle \mathbf{x}, \mathbf{s}_1 - \mathbf{s}_2 \rangle \underset{\mathbf{s}_1 \text{ is transmitted}}{\overset{\mathbf{s}_2 \text{ is trasmitted}}{\lessgtr}} \frac{\|\mathbf{s}_1\|^2 - \|\mathbf{s}_2\|^2}{2}$$

and the resulting optimal error rate is $\Phi\big(-\frac{\|\mathbf{s}_1 - \mathbf{s}_2\|}{2\sigma}\big)$, where $\Phi(\cdot)$ is the standard normal cdf.

(a) (35%) Let $\mathbf{s}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. What is the $\mathbf{s}_2$ that minimizes the error rate, subject to $\|\mathbf{s}_2\| = \|\mathbf{s}_1\|$?

Solution. $\mathbf{s}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(b) (35%) Let $\mathbf{s}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. What is the $\mathbf{s}_2$ that minimizes the error rate, subject to $\|\mathbf{s}_2\| = \|\mathbf{s}_1\|$ and $\langle \mathbf{s}_1, \mathbf{s}_2 \rangle = 0$?

Solution. Either $\mathbf{s}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ or $\mathbf{s}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$.

Providing one of the two is sufficient to get the full credit of this sub-problem.

(c) (30%) Let $\mathbf{s}_1 - \mathbf{s}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Can the minimum error rate be changed by adjusting $\mathbf{s}_1$ and $\mathbf{s}_2$? Justify your answer.

Solution. No, because the minimum error probability is simply a function of $\|\mathbf{s}_1 - \mathbf{s}_2\|$, which is fixed.