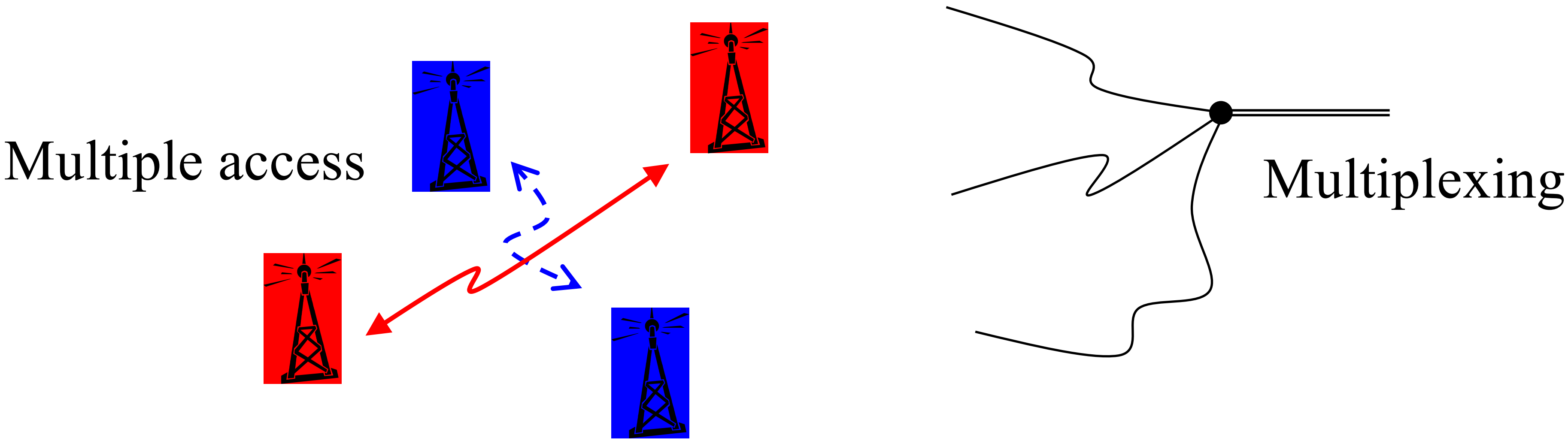
Part 5 Multiuser Communications

Introduction

- Multiuser communications refer to the simultaneous use of a communication channel by a number of users.

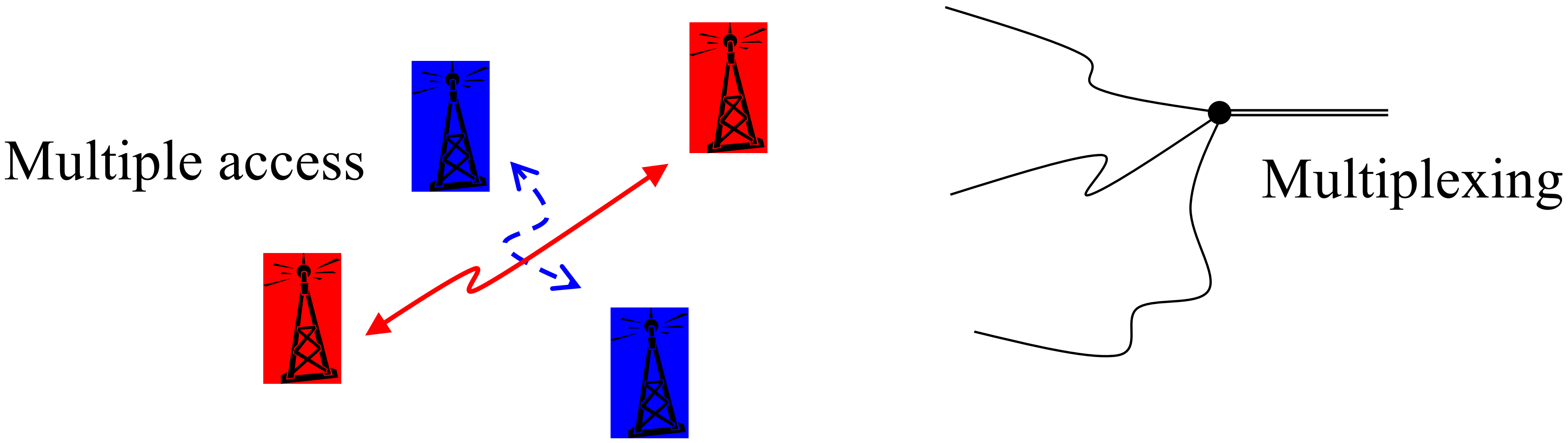
Multiple Access Techniques

- Difference between multiple access and multiplexing
  - Sources of multiple access may be geographically dispersed, while sources of multiplexing are confined within a local site (or point).



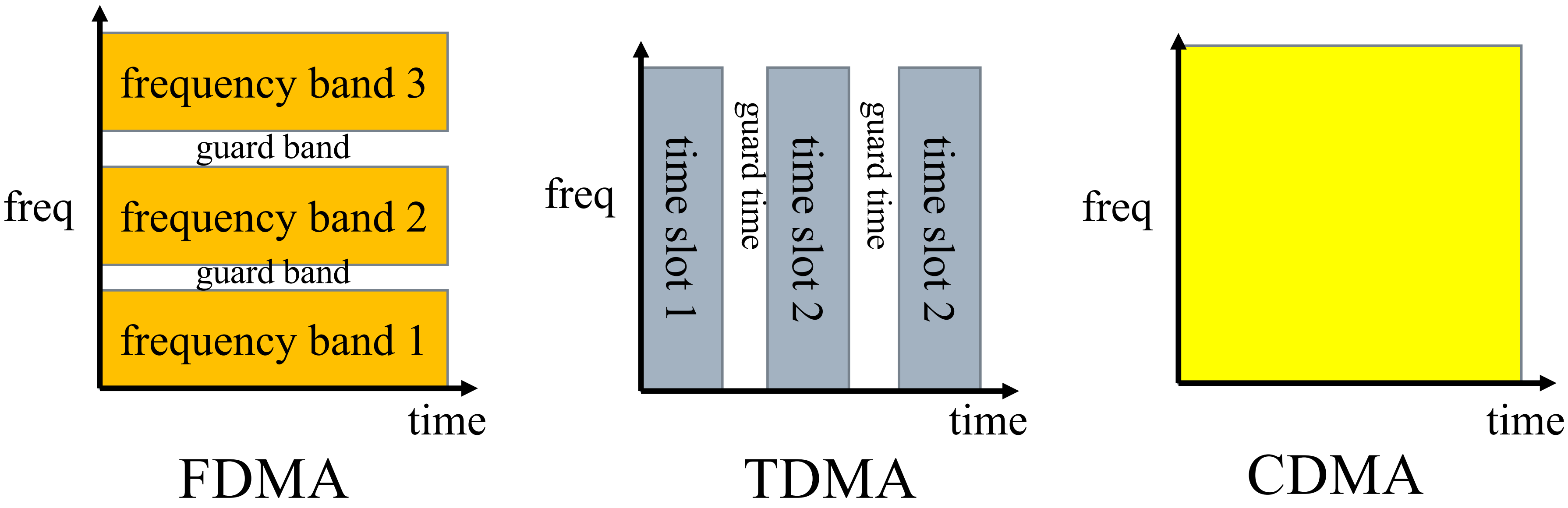
Multiple Access Techniques

- Difference between multiple access and multiplexing
  - Sources of multiple access are often homogeneous in requirements and characteristics, while sources of multiplexing are not.



Multiple Access Techniques

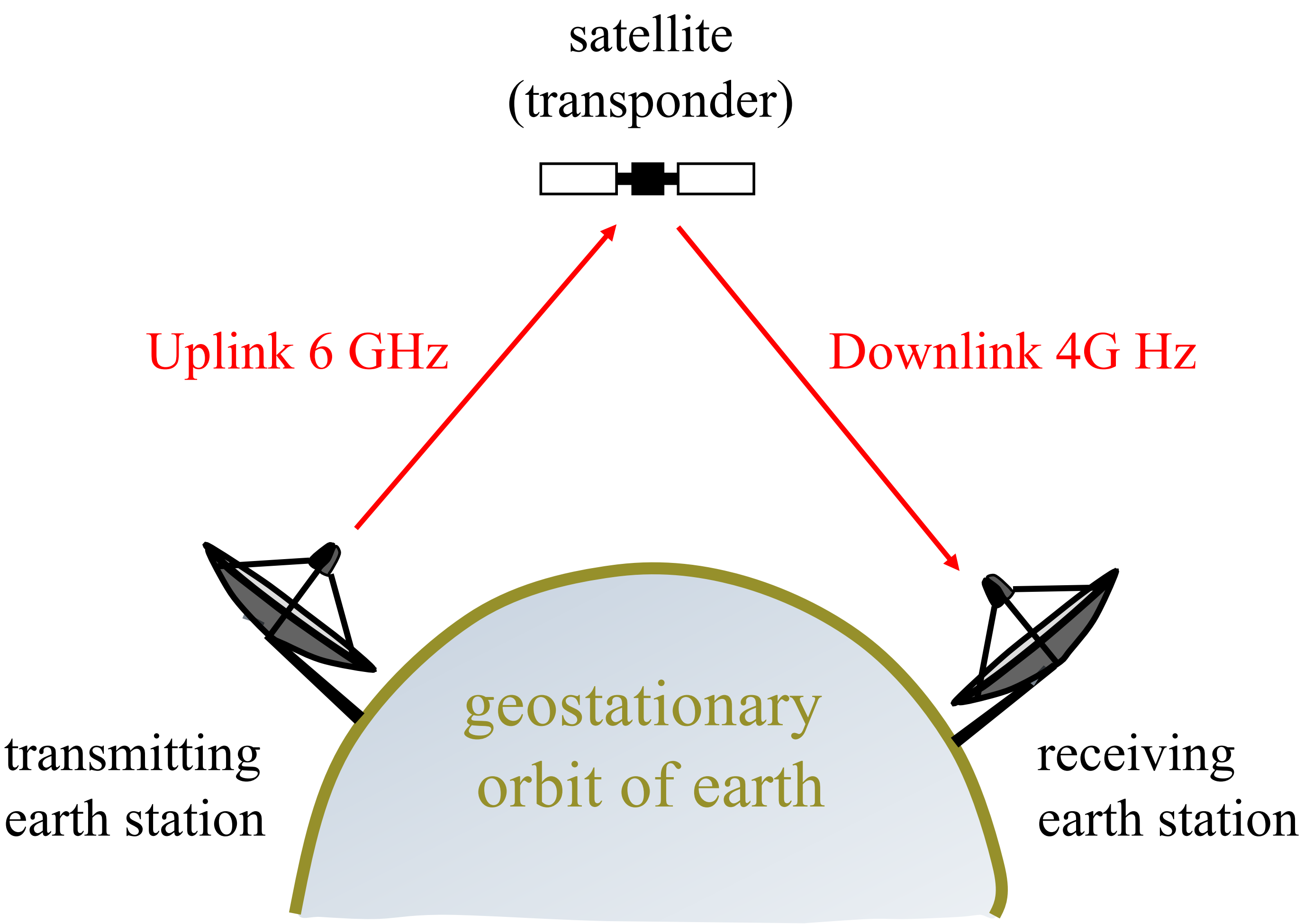
- Three basic types of multiple access



Multiple Access Techniques

- A fourth type of multiple access
  - Space-division multiple access (SDMA)
    - Spatial separation of individual users

Satellite Communications



Satellite Communications

- Advantage of 6/4 GHz band
  - Relatively inexpensive microwave equipment
  - Low attenuation due to weather change, such as rainfall
  - Insignificant sky background noise (from 1~10 GHz)
- However, these bands conflict with those used in terrestrial microwave systems.
  - Hence, 6/4 GHz band has been replaced by 14/12 GHz band (Ku-band)

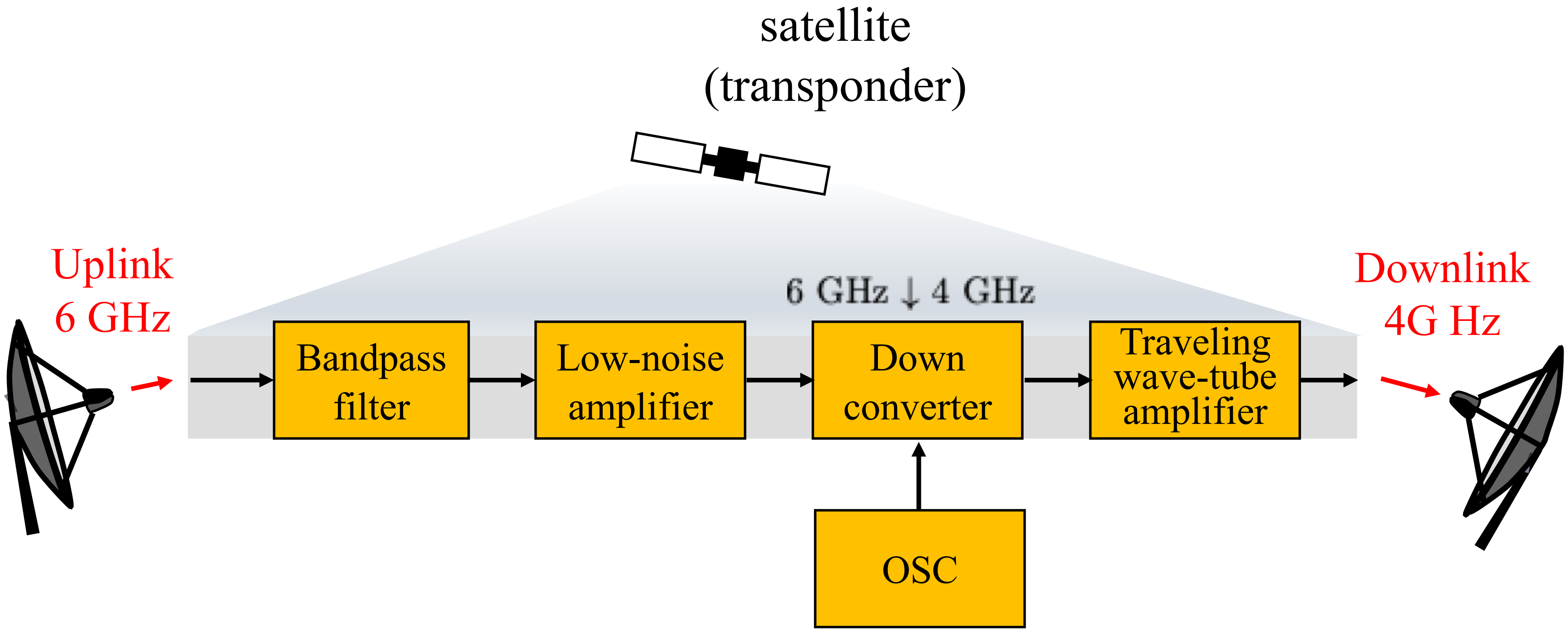
ITU	Designation	Frequency	Wavelength
ELF	extremely low frequency	3Hz to 30Hz	100'000km to 10'000 km
SLF	superlow frequency	30Hz to 300Hz	10'000km to 1'000km
ULF	ultralow frequency	300Hz to 3000Hz	1'000km to 100km
VLF	very low frequency	3kHz to 30kHz	100km to 10km
LF	low frequency	30kHz to 300kHz	10km to 1km
MF	medium frequency	300kHz to 3000kHz	1km to 100m
HF	high frequency	3MHz to 30MHz	100m to 10m
VHF	very high frequency	30MHz to 300MHz	10m to 1m
UHF	ultrahigh frequency	300MHz to 3000MHz	1m to 10cm
SHF	superhigh frequency	3GHz to 30GHz	10cm to 1cm
EHF	extremely high frequency	30GHz to 300GHz	1cm to 1mm

IEEE Radar Band Designations

Frequency	Wavelength	IEEE Radar Band designation
1 - 2 GHz	30 - 15 cm	L Band
2 - 4 GHz	15 - 7.5 cm	S Band
4 - 8 GHz	7.5 - 3.75 cm	C Band
8 - 12 GHz	3.75 - 2.50 cm	X Band
12 - 18 GHz	2.5 - 1.67 cm	Ku Band
18 - 27 GHz	1.67 - 1.11 cm	K Band
27 - 40 GHz	11.1 - 7.5 mm	Ka Band
40 - 75 GHz		V Band
75 - 110 GHz		W Band
110 - 300 GHz		mm Band
300 - 3000 GHz		u mm Band

Satellite Communications

- Block diagram of transponder

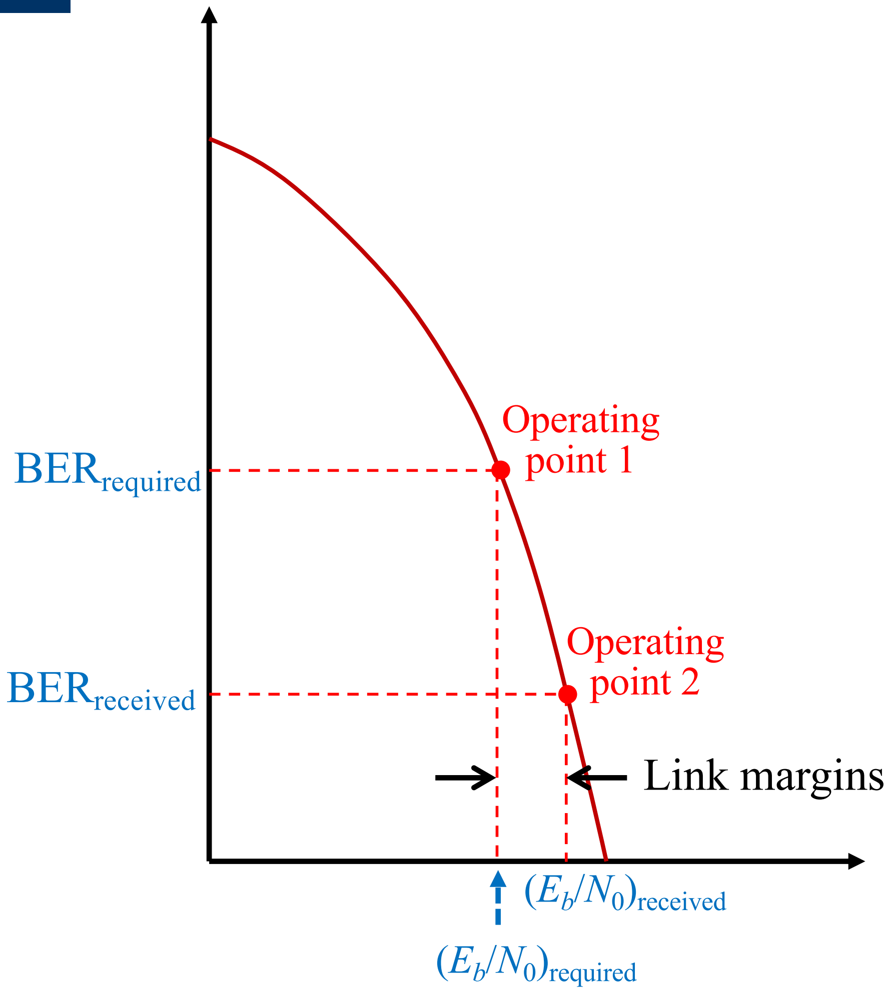


Satellite Communications

- Observations on satellite communication
  - Long propagation delay: As large as 270ms one-way
    - Echo canceller may be necessary for speech signals due to impedance mismatch at the receiver end that induces "bounce-back" echo signal.
  - Well modeled by AWGN
  - Nonlinearity in transponder will cause serious interference between users.
    - The amplifier thus is purposely operated at the linear region whenever possible, and thus operated below capacity.

Radio Link Analysis

- *Link budget* or *link power budget*
  - Definition: Accounting of all the gains and losses incurred in a communication link.
- With the analysis, we can ensure the system is operating at the desired E_b/N_0 region.



Radio Link Analysis

- Apparently,

$$\left(\frac{E_b}{N_0}\right)_{\text{received}} > \left(\frac{E_b}{N_0}\right)_{\text{required for given } P_e}$$

- Hence, we usually set a *link margin M* defined as

$$M \text{ (dB)} = \left(\frac{E_b}{N_0}\right)_{\text{received}} \text{(dB)} - \left(\frac{E_b}{N_0}\right)_{\text{required}} \text{(dB)}$$

- The received E_b/N_0 requires a model for the calculation of received power.

Radio Link Analysis

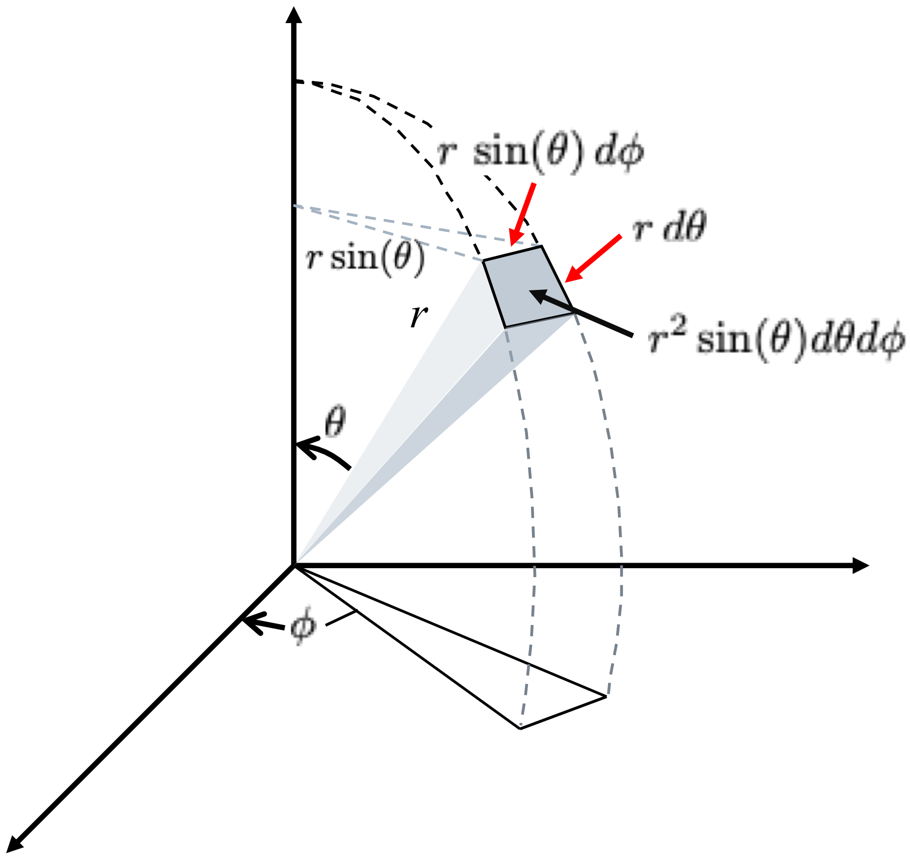
- The link analysis requires:
  - Calculation of the average received power
    - *Friis free-space equation*
  - Calculation of the average noise power
    - *Noise figure*

Radio Link Analysis

$$\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} |\sin(\theta)|d\theta d\phi = 4\pi$$

- *Received power density $\rho(r)$*
  - Rate of energy flow per unit area (e.g., watts per square meter)
    - E.g., for omnidirectional antenna,

$$\rho(r) = \frac{P_t}{4\pi r^2} \text{ (watt/m}^2\text{)}$$



Radio Link Analysis

$$\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}|\sin(\theta)|d\theta d\phi = 4\pi$$

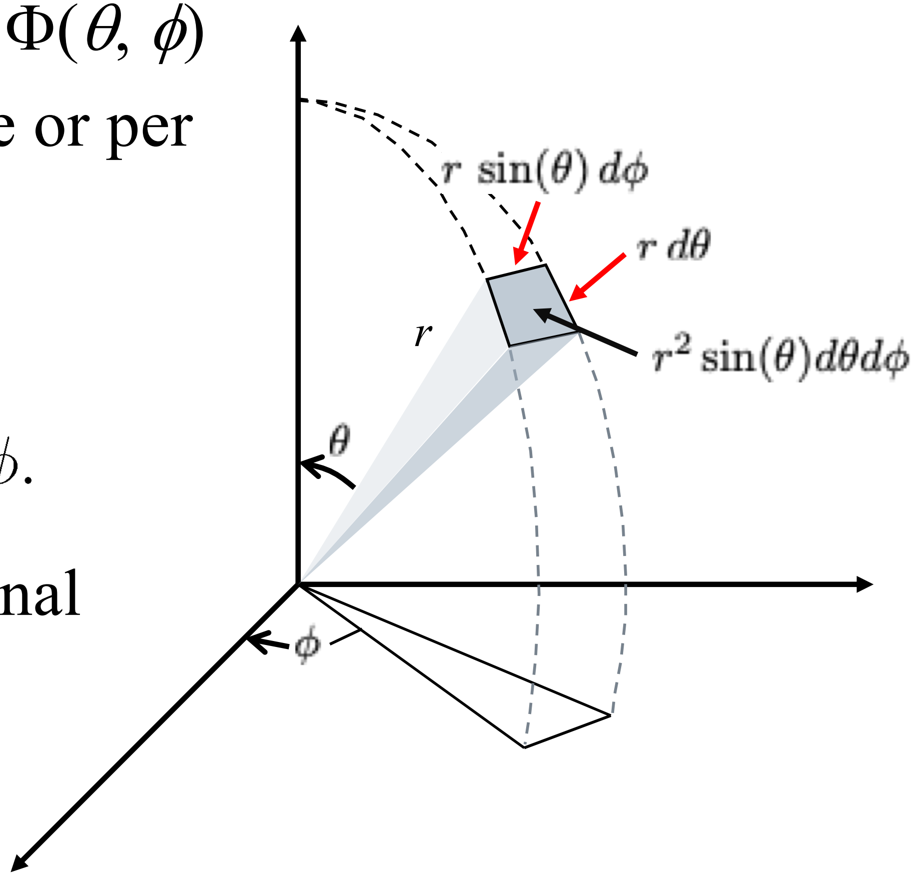
- *Received radiation intensity* $\Phi(\theta, \phi)$
  - Watts per unit solid angle or per steradian

$$P_t = \int \Phi(\theta, \phi) d\Omega$$

where $d\Omega = \sin(\theta) d\theta d\phi$.

  - E.g., for omnidirectional antenna,

$$\Phi(\theta, \phi) = r^2 \rho(r)$$



Radio Link Analysis

- *Average power radiated per unit solid angle (watts per steradian)*

$$P_{\text{av}} = \frac{\displaystyle\int \Phi(\theta, \phi) d\Omega}{\displaystyle\int d\Omega} = \frac{1}{4\pi} \int \Phi(\theta, \phi) d\Omega \text{ watts/steradian}$$

- *Directive gain of an antenna (normalized radiation intensity)*

$$g(\theta, \phi) = \frac{\Phi(\theta, \phi)}{P_{av}}$$

Radio Link Analysis

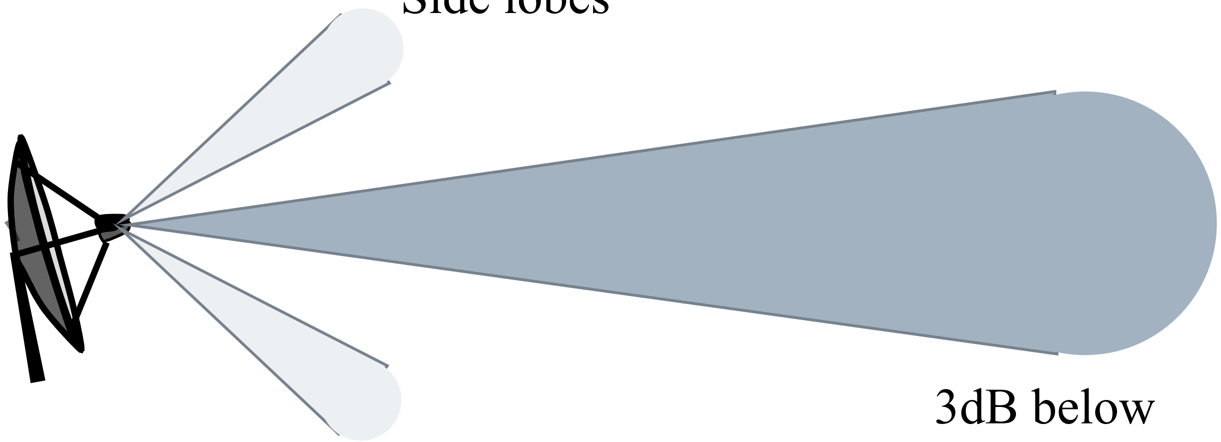
- *Directivity of an antenna* $D = \max_{\theta,\phi} g(\theta, \phi)$

- *Power gain of an antenna* $G = \eta_{\text{radiation}} D$
  - *Efficiency of an antenna: The ratio of the maximum radiation intensity from the antenna to the radiation intensity from a lossless isotropic source under the constraint that the same input power is applied to both antennas.*

$\eta_{\text{radiation}} = 1$ if the antenna is 100 percent efficient.
$\eta_{\text{radiation}} < 1$ usually.

Radio Link Analysis

- Notion of power gain
  - Concentrating the power density in a restricted region smaller than 4π steradians

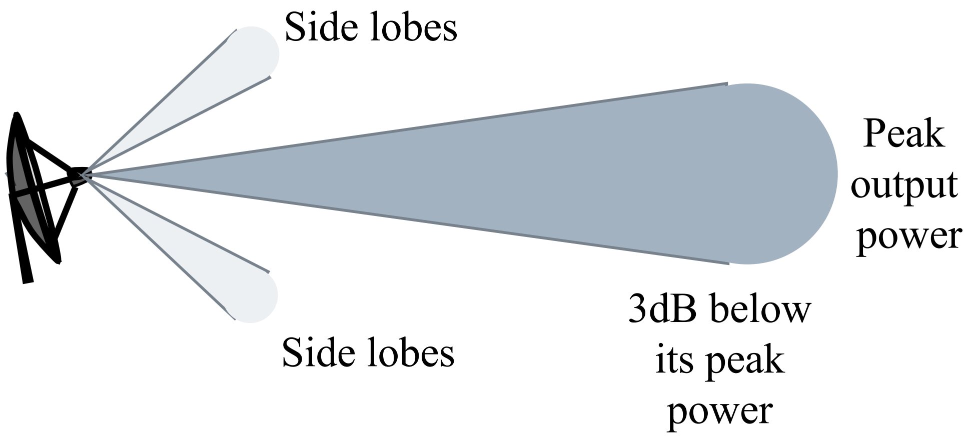


Radio Link Analysis

- Effective radiated power referencing to an isotropic source (EIRP = Equivalent Isotropically Radiated Power) defined only for transmitting antenna

$$\text{EIRP} = G_t \times P_t, \text{ where } P_t \text{ is the transmitted power}$$

- Antenna beamwidth
  - Measure of the antenna's solid angle such that the peak field power is reduced to 3 dB



Radio Link Analysis

- Effective aperture (is proportional to λ^2)
  - The effective aperture of an antenna is sometimes called its capture area. It is the frontal area from which a receiving antenna extracts energy from passing electromagnetic waves.

$$A_r = \lambda^2 \frac{G_r}{4\pi} \text{ (meter}^2\text{)}$$

where λ is the wavelength of the carrier.

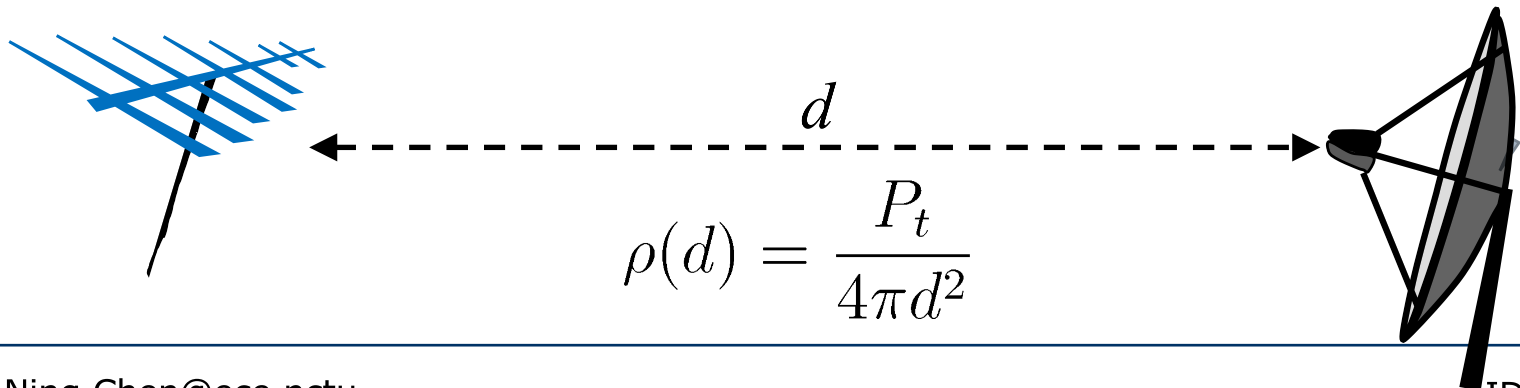
$$P_r = \text{received power density} \times A_r$$

Radio Link Analysis

- Friis free-space equation

$$
\boxed{P_r} = \left(\frac{P_t}{4\pi d^2} G_t\right) A_r = \left(\frac{P_t}{4\pi d^2} G_t\right)\left(\frac{\lambda^2}{4\pi} G_r\right)
$$

$$
\boxed{= P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2}
$$



$$
\rho(d) = \frac{P_t}{4\pi d^2}
$$

Radio Link Analysis

- Path loss (The smaller, the better)

$$PL = 10\log_{10}\left(\frac{P_t}{P_r}\right) = -10\log_{10}(G_t G_r) + \underbrace{10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2}_{\text{free-space loss}}$$

- Observation
  - The larger the distance, the higher in preference to lower carrier frequency.

Radio Link Analysis

- Noise figure (NF)
  - NF is a measure of degradation of the signal-to-noise ratio (SNR) through a device.
  - Under the same signal power (sinusoidal input), *spot NF* is a measure of the noise power increased by the device under test with respect to a specific frequency *f*.

$$F = F_{\text{spot}}(f) = \frac{1}{G(f)} \frac{S_{NO}(f)}{S_{NS}(f)}$$

$$\text{where } \begin{cases} S_{NO}(f) & \text{spectral density of noise power at device output} \\ S_{NS}(f) & \text{spectral density of noise power at device input(source)} \\ G(f) & \text{Power gain of the device} \end{cases}$$

Radio Link Analysis

- With the above definition on $G(f)$,

$$P_O(f) = G(f) P_S(f)$$

- Suppose we concern about the average noise figure. Then,

$$F_0 = \frac{\int_{-\infty}^{\infty} S_{NO}(f)\,df}{\int_{-\infty}^{\infty} G(f) S_{NS}(f)\,df} \quad \underbrace{\left(\approx \frac{S_{NO}(f_c)\Delta f}{G(f_c) S_{NS}(f_c)\Delta f}\right)}_{\text{Certered at } f_c \text{ with width } \Delta f}$$

Radio Link Analysis

- Equivalent noise temperature T_e
  - Noise voltage level across a resistor R due to thermal noise at degree Kelvin T can be approximated by

$$V_{RMS}^2 = \overline{V^2} = 4kTR\Delta f,$$

where $k = 1.38 \times 10^{-23}$ (Joule/Kelvin) Boltzmann's constant, and Δf is the circuit bandwidth in Hz.

$$\text{Power}_{RMS} = \frac{\overline{V^2}}{R} = 4kT\Delta f$$

Radio Link Analysis

- Hence, the noise power is proportional to the temperature.

$$N_{input,RMS} = 4kT\Delta f$$

$$\text{Assume} \begin{cases} N_{output,RMS} = G \cdot N_{input,RMS} + G \cdot N_{additional,RMS} \\ N_{additinal,RMS} = 4kT_e\Delta f \end{cases}$$

where G is the power gain of the device under test.

$$\Rightarrow F = \frac{S_{NO}(f)}{G(f)S_{NS}(f)} = \frac{4GkT\Delta f + 4GkT_e\Delta f}{4GkT\Delta f} = \frac{T + T_e}{T}$$

$$\Rightarrow T_e = T(F-1) \text{ and } \boxed{N_{additional,RMS} = N_{input,RMS}(F-1)}$$

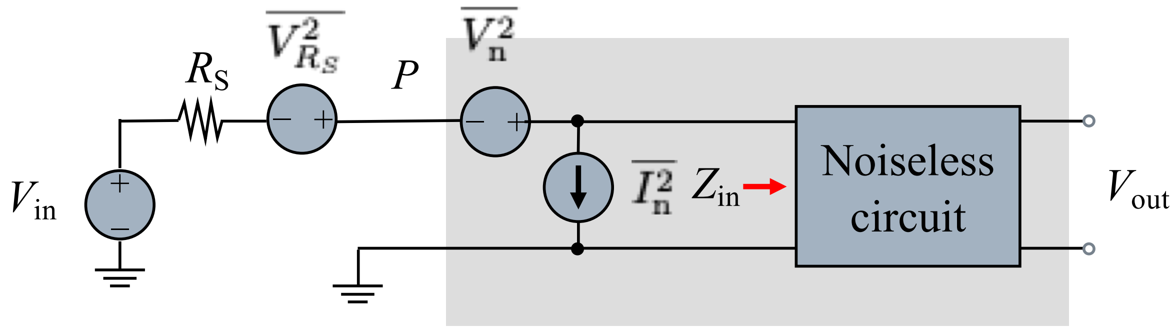
Radio Link Analysis

- An alternative and perhaps more formal definition of (spot) noise figure

$$
\begin{aligned}
F &\approx \frac{S_{NO}(f_c)}{G(f_c)S_{NS}(f_c)} \\
&= \frac{P_S(f_c)S_{NO}(f_c)}{P_S(f_c)G(f_c)S_{NS}(f_c)} \\
&= \frac{P_S(f_c)S_{NO}(f_c)}{P_O(f_c)S_{NS}(f_c)} \\
&= \frac{P_S(f_c)/S_{NS}(f_c)}{P_O(f_c)/S_{NO}(f_c)}
\end{aligned}
$$

The ratio between input SNR and output SNR

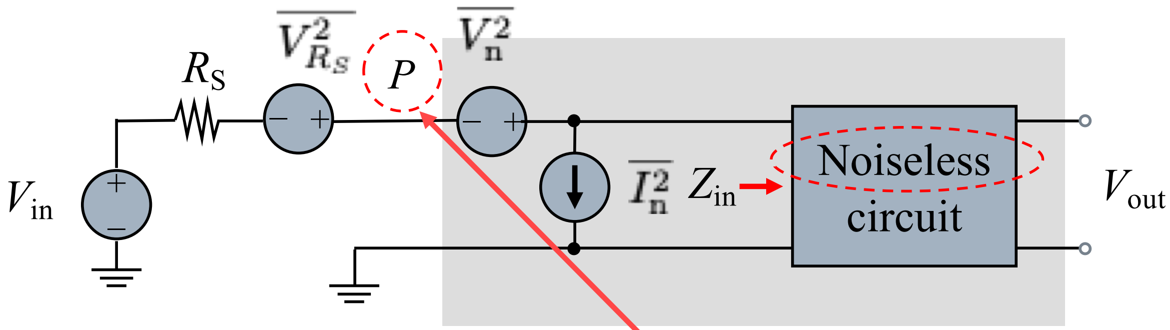
- Example (Fig. 2-29: Calculation of noise figure, in Behzad Ravavi, *RF Microelectronics*, Prentice Hall, 1998)



$\overline{V^2_{R_S}} = 4kTR_S$ is the thermal noise voltage level from resistor R_S.

$\overline{V^2_n}$ and $\overline{I^2_n}$ are the equivalent noise voltage and current level respectively corresponding to "Shot noise" and "Flicker noise".

SNR=ratio of voltage square across the same load resistor.



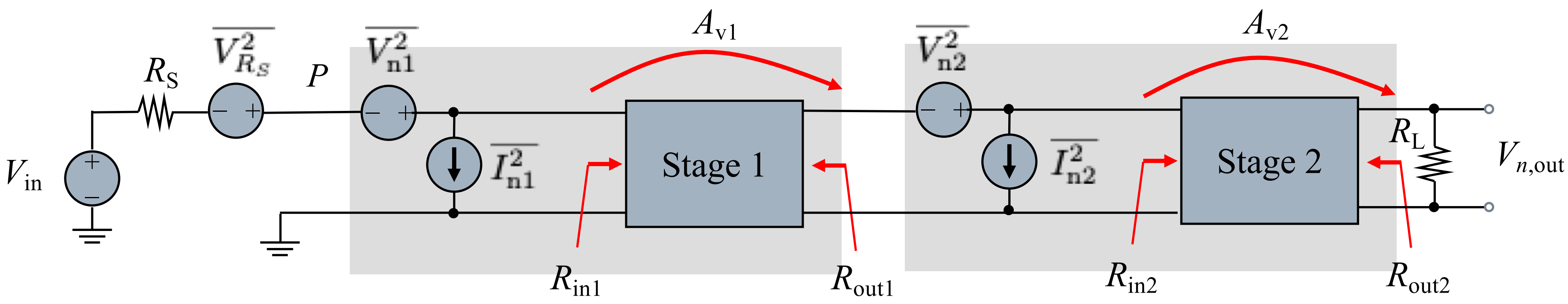
A_v voltage gain between node P and output

$$\Rightarrow \text{SNR}_{in} = \frac{V_{in}^2}{\overline{V_{RS}^2}}$$

$$\text{SNR}_{out} = \frac{A_v^2 V_{in}^2}{A_v^2\left[\overline{V_{RS}^2} + \overline{(V_n + I_n R_S)^2}\right]}$$

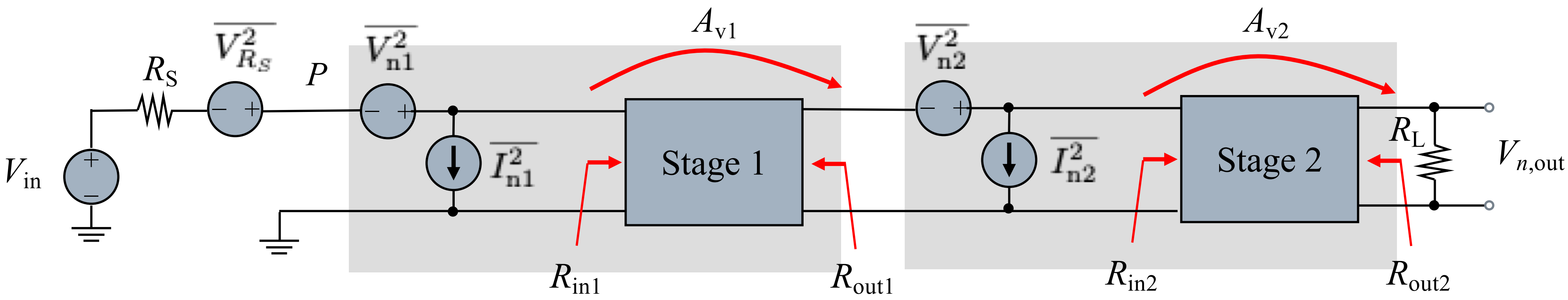
$$F = \frac{\overline{V_{RS}^2} + \overline{(V_n + I_n R_S)^2}}{\overline{V_{RS}^2}} = 1 + \frac{\overline{(V_n + I_n R_S)^2}}{\overline{V_{RS}^2}}$$

☐ Example: Noise figure of cascaded system (Fig. 2-32: Calculation of noise figure, in Behzad Ravavi, *RF Microelectronics*, Prentice Hall, 1998)



$$V^2_{n,in1} = \overline{\left[I_{n1}(R_S \| R_{in1}) + V_{n1}\frac{R_{in1}}{R_{in1}+R_S}\right]^2} + \overline{V^2_{R_S}}\frac{R^2_{in1}}{(R_{in1}+R_S)^2}$$

$$V^2_{n,in2} = \overline{\left[I_{n2}(R_{out1} \| R_{in2}) + V_{n2}\frac{R_{in2}}{R_{in2}+R_{out1}}\right]^2} + A^2_{v1}V^2_{n,in1}\frac{R^2_{in2}}{(R_{in2}+R_{out1})^2}$$



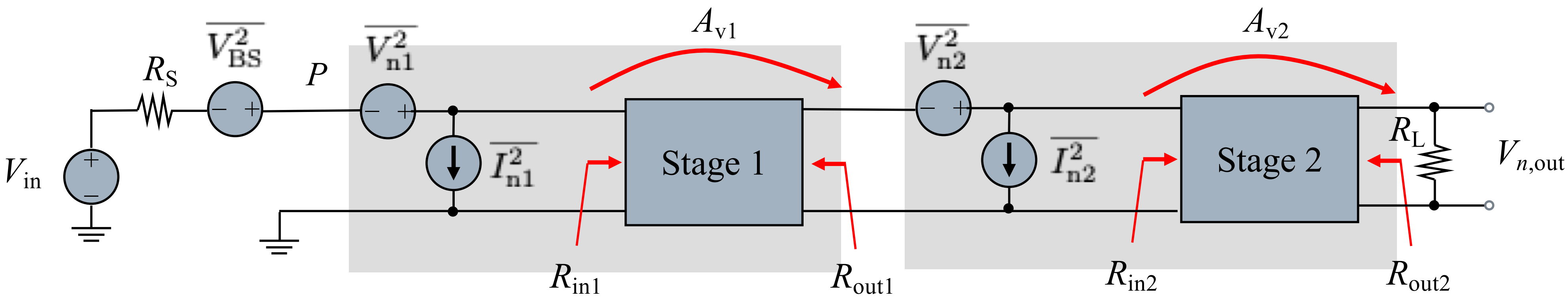
$$V^2_{n,out} = A^2_{v2} V^2_{n,in2} \frac{R_L^2}{(R_{out2} + R_L)^2}$$

$$\mathrm{SNR}_{in} = \frac{V^2_{in}}{\overline{V^2_{R_S}}}$$

$$\mathrm{SNR}_{out} = \frac{A^2_{v,tot} V^2_{in}}{V^2_{n,out}}$$

$$A_{v,tot} = \frac{R_{in1}}{R_S + R_{in1}} A_{v1} \frac{R_{in2}}{R_{out1} + R_{in2}} A_{v2} \frac{R_L}{R_{out2} + R_L}$$

$$F = \frac{\mathrm{SNR}_{out}}{\mathrm{SNR}_{in}} = \frac{V^2_{n,out}}{A^2_{v,tot} \overline{V^2_{R_S}}}$$



(A matched load system)

Assume $R_S = R_{in1} = R_{out1} = R_{in2} = R_{out2} = R_L = R$.

$$\begin{aligned} V_{n,in1}^2 &= \frac{1}{4}\overline{[I_{n1}R + V_{n1}]^2} + \frac{1}{4}\overline{V_{R_S}^2} \\ V_{n,in2}^2 &= \frac{1}{4}\overline{[I_{n2}R + V_{n2}]^2} + \frac{1}{4}A_{v1}^2 V_{n,in1}^2 \\ &= \frac{1}{4}\overline{[I_{n2}R + V_{n2}]^2} + \frac{1}{16}A_{v1}^2\overline{[I_{n1}R + V_{n1}]^2} + \frac{1}{16}A_{v1}^2\overline{V_{R_S}^2} \end{aligned}$$

$$A_{v,tot} = \frac{1}{8}A_{v1}A_{v2}$$

$$
\begin{aligned}
V_{n,out}^2 &= \frac{1}{4} A_{v2}^2 V_{n,in2}^2 \\
&= \frac{1}{16} A_{v2}^2 \overline{[I_{n2}R + V_{n2}]^2} + \frac{1}{64} A_{v2}^2 A_{v1}^2 \overline{[I_{n1}R + V_{n1}]^2} + \frac{1}{64} A_{v2}^2 A_{v1}^2 \overline{V_{R_S}^2} \\
F &= \frac{V_{n,out}^2}{A_{v,tot}^2 \overline{V_{R_S}^2}} \\
&= \frac{\frac{1}{16} A_{v2}^2 \overline{[I_{n2}R + V_{n2}]^2} + \frac{1}{64} A_{v2}^2 A_{v1}^2 \overline{[I_{n1}R + V_{n1}]^2} + \frac{1}{64} A_{v2}^2 A_{v1}^2 \overline{V_{R_S}^2}}{\frac{1}{64} A_{v1}^2 A_{v2}^2 \overline{V_{R_S}^2}} \\
&= \frac{\overline{[I_{n2}R + V_{n2}]^2}}{\frac{1}{4} A_{v1}^2 \overline{V_{R_S}^2}} + \frac{\overline{[I_{n1}R + V_{n1}]^2}}{\overline{V_{R_S}^2}} + 1 \\
\Bigg(&= \frac{\overline{[I_{n2}R + V_{n2}]^2}}{\left(\frac{R_{in1}}{R_S + R_{in1}}\right)^2 A_{v1}^2 \overline{V_{R_S}^2}} + \frac{\overline{[I_{n1}R + V_{n1}]^2}}{\overline{V_{R_S}^2}} + 1 \Bigg) \\
&= \frac{\overline{[I_{n2}R + V_{n2}]^2}}{G_1 \overline{V_{R_S}^2}} + \frac{\overline{[I_{n1}R + V_{n1}]^2}}{\overline{V_{R_S}^2}} + 1
\end{aligned}
$$

$$
\begin{aligned}
F &= \frac{\overline{[I_{n2}R + V_{n2}]^2}}{G_1 \overline{V_{R_S}^2}} + \frac{\overline{[I_{n1}R + V_{n1}]^2}}{\overline{V_{R_S}^2}} + 1 \\
&= \frac{1}{G_1}(F_2 - 1) + F_1
\end{aligned}
$$

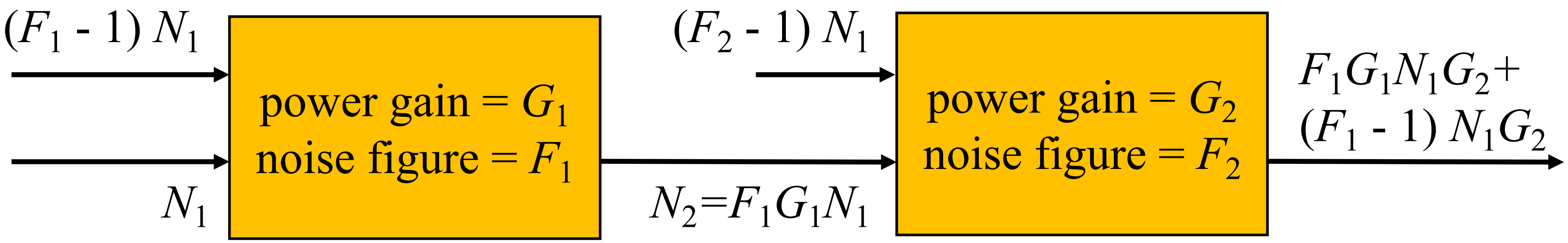
- Note that F_2 is the noise figure of the second stage with respect to a source impedance R_S.
- Also, note that $R_{out1} = R_S$.
- Generally,

$$
F = F_1 + \frac{1}{G_1}(F_2 - 1) + \frac{1}{G_1 G_2}(F_3 - 1) + \frac{1}{G_1 G_2 G_3}(F_4 - 1) + \cdots
$$

Radio Link Analysis

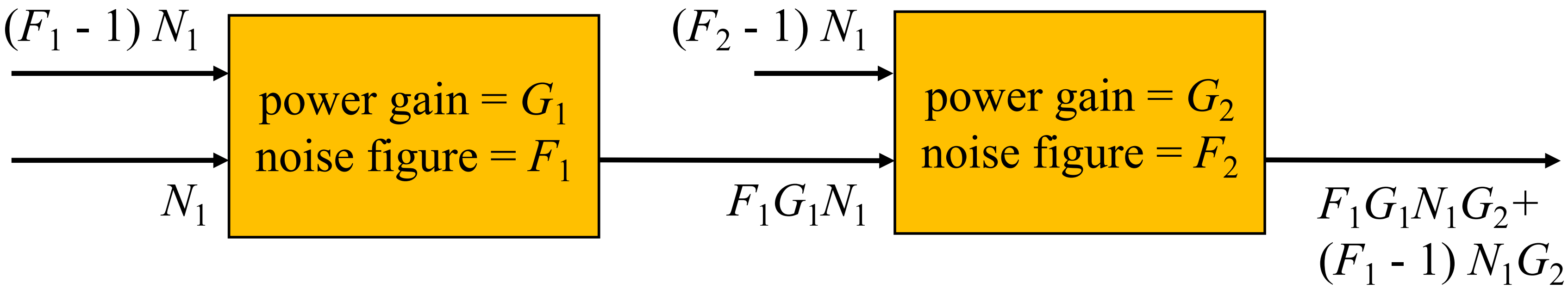
- A simplified analysis of the noise figure of (equivalent) cascade system

F_2 is the (single-stage) noise figure with respect to input N_1.



By definition, $F_1 = \frac{N_2}{G_1 N_1} \Rightarrow N_2 = F_1 G_1 N_1 = G_1[N_1 + (F_1 - 1)N_1]$.

Thus, $N_3 = G_2[F_1 G_1 N_1 + (F_2 - 1)N_1]$.



$$\begin{aligned} F_{tot} &= \frac{N_3}{G_1 G_2 N_1} \\ &= \frac{F_1 G_1 G_2 N_1 + (F_2 - 1) G_2 N_1}{G_1 G_2 N_1} \\ &= F_1 + \frac{1}{G_1}(F_2 - 1). \end{aligned}$$

$$F = F_1 + \frac{1}{G_1}(F_2 - 1) + \frac{1}{G_1 G_2}(F_3 - 1) + \frac{1}{G_1 G_2 G_3}(F_4 - 1) + \cdots$$

$$F = F_1 + \frac{1}{G_1}(F_2 - 1) + \frac{1}{G_1 G_2}(F_3 - 1) + \frac{1}{G_1 G_2 G_3}(F_4 - 1) + \cdots$$

- Observations
  - If the first stage has high power gain, the overall noise figure will be dominated by the first stage.
  - In other words, a high-gain stage can suppress the noise figure of the following stages.
  - Based on the above formula, we can easily obtain the famous *Friis formula* for equivalent noise temperatures.

$$T_{e,system} = T_{e,1} + \frac{1}{G_1}T_{e,2} + \frac{1}{G_1 G_2}T_{e,3} + \frac{1}{G_1 G_2 G_3}T_{e,4} + \cdots$$

(Recall that $T_e = T(F-1)$.)

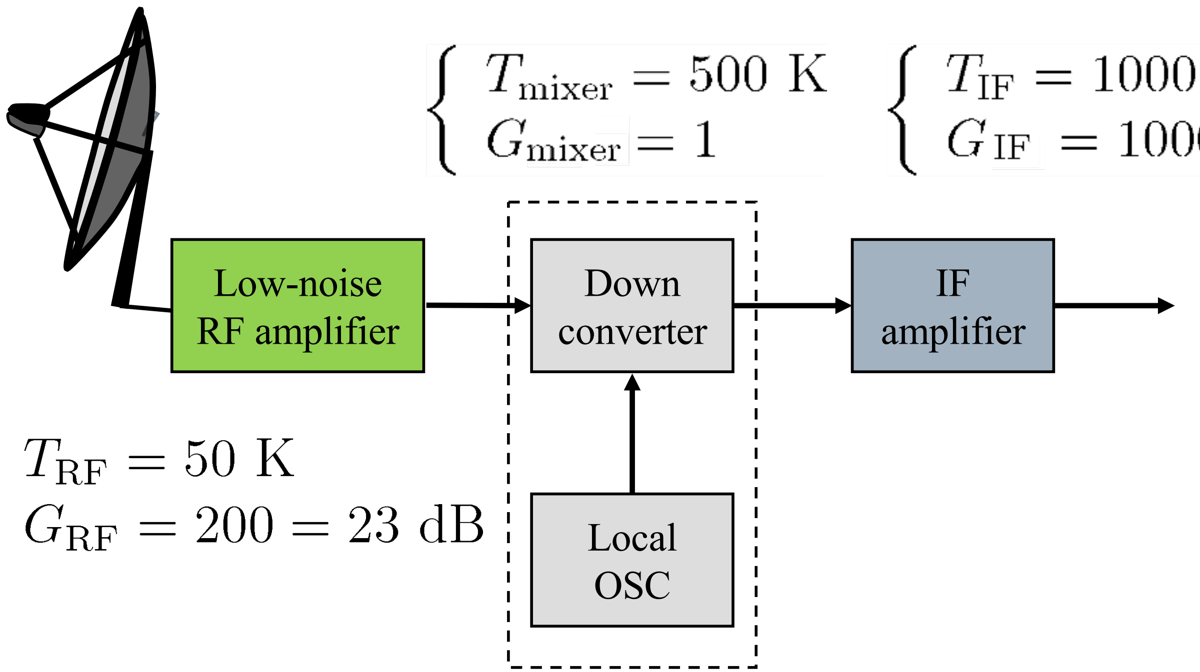
Radio Link Analysis

- Example 1: Below are all equivalent noise temperatures.

$$\begin{cases} T_{\text{antenna}} = 50 \text{ K} \\ G_{\text{antenna}} = 1 \end{cases}$$

$$\begin{cases} T_{\text{mixer}} = 500 \text{ K} \\ G_{\text{mixer}} = 1 \end{cases}$$

$$\begin{cases} T_{\text{IF}} = 1000 \text{ K} \\ G_{\text{IF}} = 1000 = 30 \text{ dB} \end{cases}$$



$$\begin{cases} T_{\text{RF}} = 50 \text{ K} \\ G_{\text{RF}} = 200 = 23 \text{ dB} \end{cases}$$

$$
\begin{aligned}
T_{e,system} &= T_{\text{antenna}} + \frac{T_{\text{RF}}}{G_{\text{antenna}}} + \frac{T_{\text{mixer}}}{G_{\text{antenna}}G_{\text{RF}}} + \frac{T_{\text{IF}}}{G_{\text{antenna}}G_{\text{RF}}G_{\text{mixer}}} \\
&= 50 + \frac{50}{1} + \frac{500}{1 \times 200} + \frac{1000}{1 \times 200 \times 1} \\
&= 107.5 \text{ K}
\end{aligned}
$$

Radio Link Analysis

- Example 2: Friis free-space equation

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 = \text{EIRP} \cdot G_r \left(\frac{\lambda}{4\pi d}\right)^2$$

- For a downlink channel of digital satellite communication system,

$$\left(\frac{P_r}{N_0}\right)_{\text{downlink}} = \text{EIRP}_{\text{satellite}} \left(\frac{G_r}{T_e}\right)_{\text{earth/terminal}} \left(\frac{1}{k}\right) \left(\frac{\lambda}{4\pi d}\right)^2$$

where $N_0 = kT_e$.

$$\text{EIRP}_{\text{satellite}} = 46.5 \text{ dBW or } 10^{46.5/10} = 44668.4 \text{ Watt}$$

$$(G_r)_{\text{2m−dish antenna}} = 45 \text{ dB}$$

$$T_e = 107.5 \text{ K as Example 1}$$

$$(G_r/T_e)_{\text{earth terminal}} = 45 - 10\log_{10}(107.5) = 24.7 \text{ dB/K}$$

$$10\log_{10}(k) = 10\log_{10}(1.38 \times 10^{-23} \text{ J/K}) = -228.6 \text{ dBW/K-Hz}$$

$$f = 12 \text{ GHz}$$

$$d = 40000 \text{ km}$$

$$\begin{aligned} L_{\text{free-space}} &= 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2 = 20\log_{10}\left(\frac{4\pi}{3\times 10^{-4} \text{ km-GHz}} f \text{ GHz} \cdot d \text{ km}\right) \\ &= [92.4 + 20\log_{10}(12) + 20\log_{10}(40000)] = 206 \text{ dB} \end{aligned}$$

$$\left(\frac{P_r}{N_0}\right)_{\text{downlink}} = 46.5 + 24.7 - (-228.6) - 206 = 93.8 \text{ dB-Hz}$$

(dBW)+(dB/K)−(dBW/K-Hz)−dB=dB-Hz

$$\left(\frac{P_r}{N_0}\right)_{\text{downlink}} = \left(\frac{E_s/T_s}{N_0}\right)_{\text{received}} = 93.8 \text{ dB-Hz}$$

$$M \text{ (dB)} = \left(\frac{E_b}{N_0}\right)_{\text{received}} \text{(dB)} - \left(\frac{E_b}{N_0}\right)_{\text{required}} \text{(dB)}$$

n symbols carry k information bits

$\Rightarrow E_s \cdot n = E_b \cdot k$

$\Rightarrow E_s = E_b \tilde{R}$, where $\tilde{R} = k/n$ (bits/symbol period)

$$\begin{aligned}\left(\frac{E_s/T_s}{N_0}\right)_{\text{received}} &= \left(\frac{E_b}{N_0}\right)_{\text{received}} + 10\log_{10}\left(\frac{\tilde{R}}{T_s}\right) \\ &= \left(\frac{E_b}{N_0}\right)_{\text{required}} + M \text{ dB} + 10\log_{10}\left(\frac{\tilde{R}}{T_s}\right)\end{aligned}$$

The link margin M is usually 4 dB for C-band and 6 dB for Ku-band. From Slide IDC1-62,

$$\begin{aligned} P_{e,\text{8PSK}} &\approx 2\Phi\left(-\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{8}\right)\right) = 2\Phi\left(-\sqrt{\frac{6E_b}{N_0}}\sin\left(\frac{\pi}{8}\right)\right) \\ &= \text{erfc}\left(\sqrt{\frac{3E_b}{N_0}}\sin\left(\frac{\pi}{8}\right)\right) \le 10^{-5} \end{aligned}$$

$$\Phi(-x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$\Rightarrow \left(\frac{E_b}{N_0}\right)_{\text{required}} \ge 13.4424 \text{ dB}$$

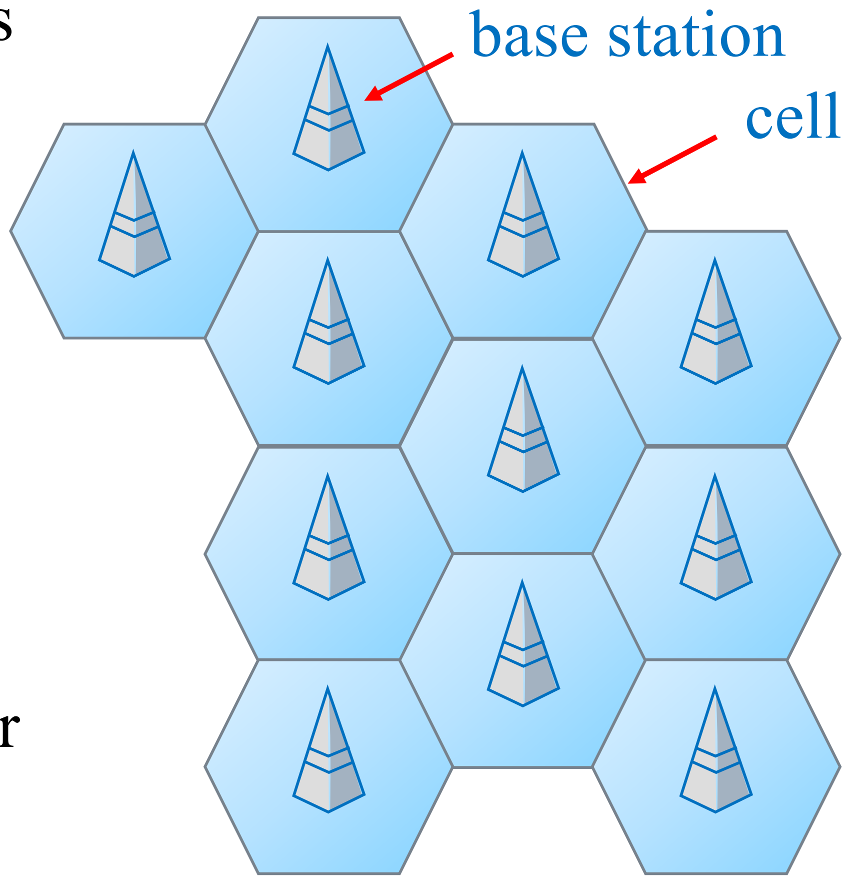
$$\begin{aligned} \Rightarrow 10\log_{10}\left(\frac{\tilde{R}}{T_s}\right) &= \left(\frac{P_r}{N_0}\right)_{\text{received}} - \left(\frac{E_b}{N_0}\right)_{\text{required}} - M \text{ dB} \\ &= 93.8 - 13.4424 - 6 = 74.3576 \end{aligned}$$

dB-Hz$-$(dB/bits)$-$dB=bits/second

$$\Rightarrow \frac{\tilde{R}}{T_s} = 27.2747 \text{ Mbits/second}$$

Wireless Communications

- This section actually concerns a special type of wireless communications, namely, *mobile (cellular) radio*.
- Usual model of the cellular radio system
  - Base station centered in a hexagonal cell
  - Base station = interface between mobile subscriber and (mobile) switching center

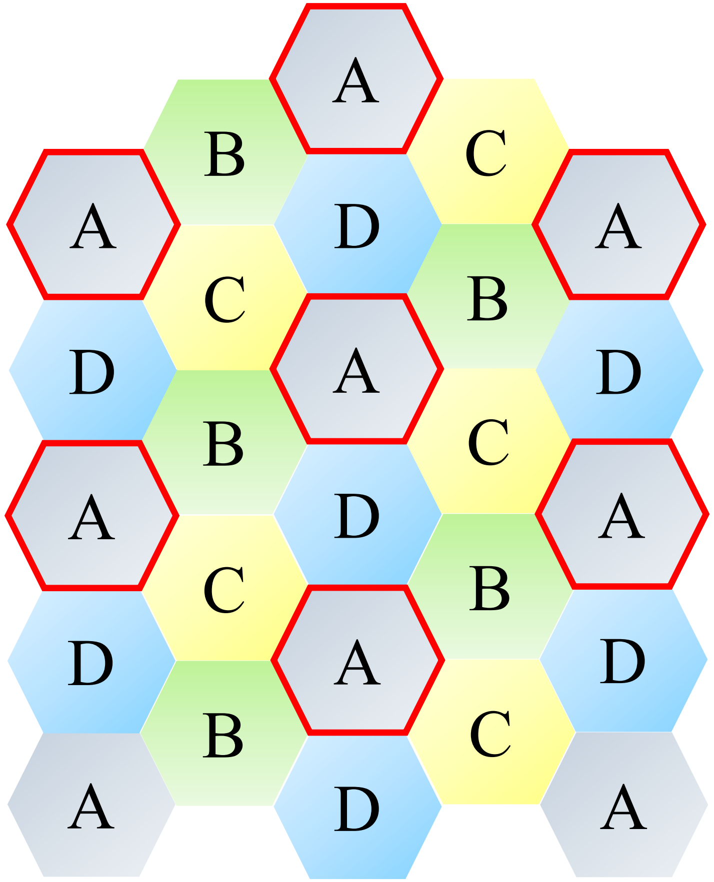


Wireless Communications

- Handover or handoff = Switching process from one cell to another
- Two important techniques for mobile cellular radio
  - Frequency reuse
    - Co-channel interference is acceptable.
  - Cell splitting
    - With the help of, e.g., directional antenna

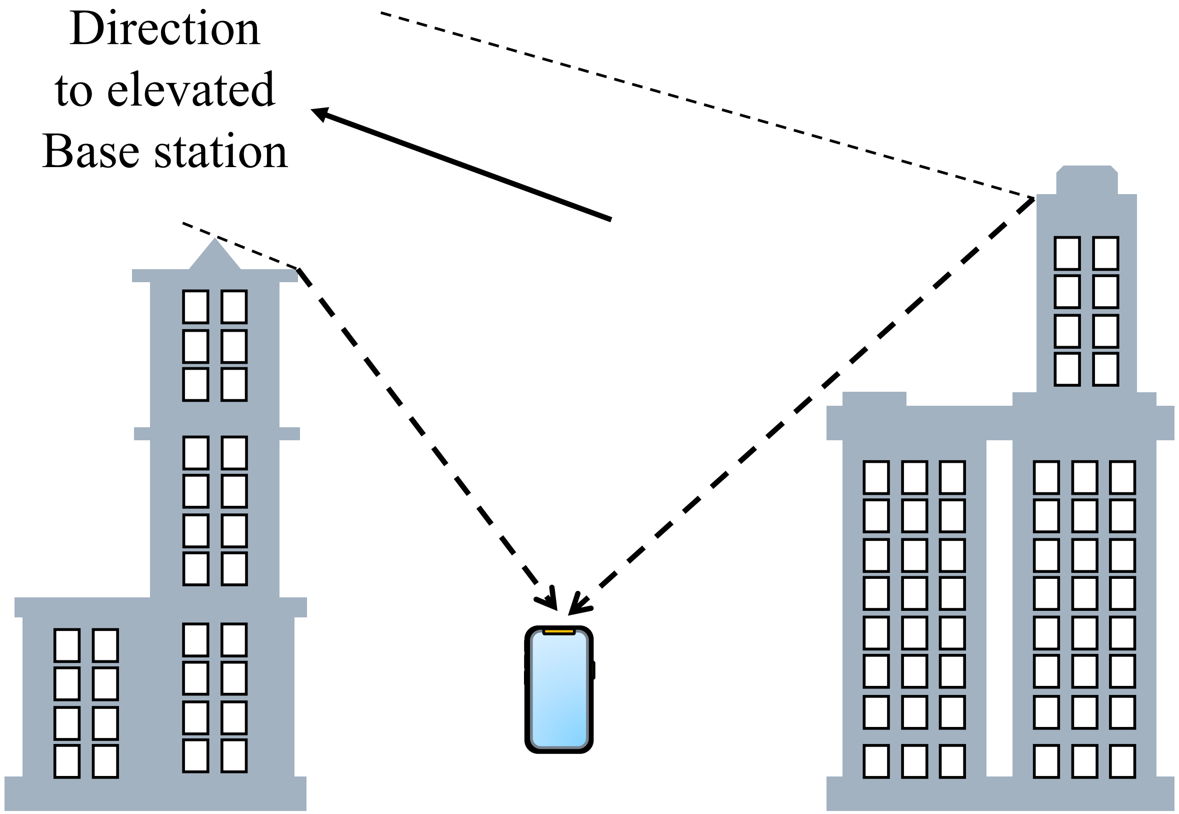
Wireless Communications

- One way to determine the co-channel cells
  - Philosophy behind : Equal distances among base stations at co-channel cells
  - Thinking:
    - More specifically, does the cover area of a base station look like a hexagon?
    - How to efficiently assign co-channel cells based on the true "cell topology"?



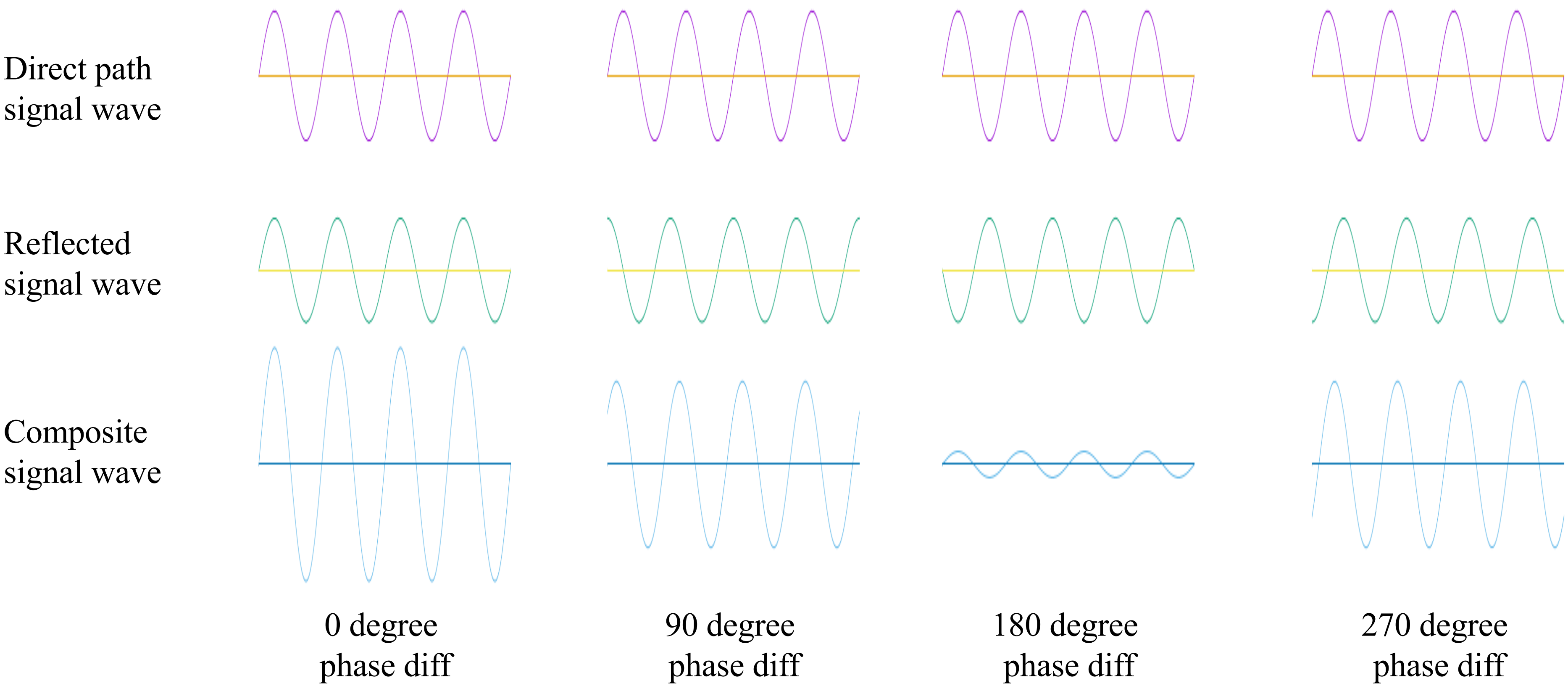
Wireless Communications

- Multipath phenomenon



Wireless Communications

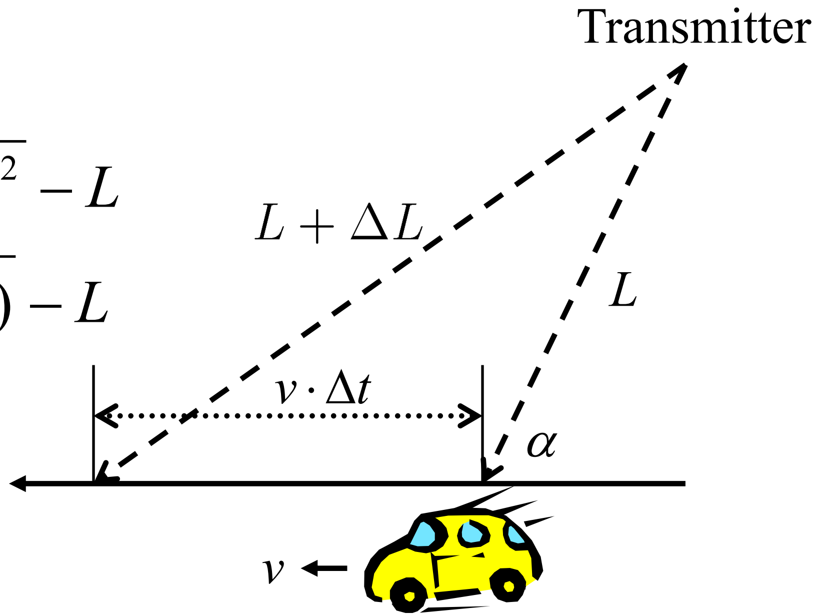
- Signal fading due to multipath phenomenon



Wireless Communications

– *Difference in path length*

$$\Delta L = \sqrt{(L\sin(\alpha))^2 + (L\cos(\alpha) + v\cdot\Delta t)^2} - L$$
$$= \sqrt{L^2 + v^2(\Delta t)^2 + 2L\cdot v\cdot\Delta t\cdot\cos(\alpha)} - L$$

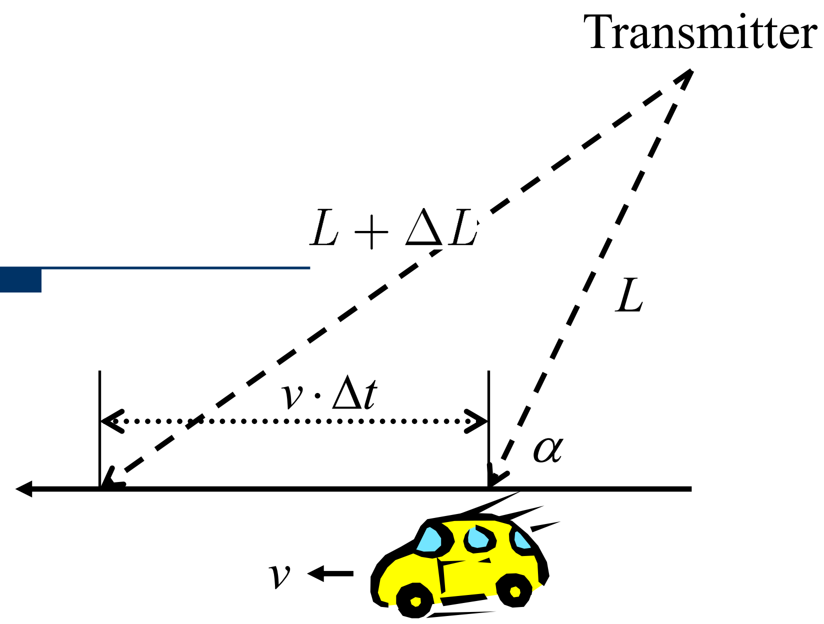


– *Phase change (during time Δt)*

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda} \quad \text{(radian)} \quad \text{and} \quad \Delta f = -\frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = -\frac{1}{\lambda}\frac{\Delta L}{\Delta t} \text{ (Hz)}$$

Fourier$\{g(t-\Delta t)\} = G(f)e^{-j2\pi f\cdot\Delta t}$
Thus, conceptually, $e^{j\Delta\phi} = e^{-j2\pi\Delta f\cdot\Delta t}$

Wireless Communications



– *Doppler shift*

$$d(f) = -\frac{1}{\lambda}\lim_{\Delta t\to 0}\frac{\Delta L}{\Delta t}$$

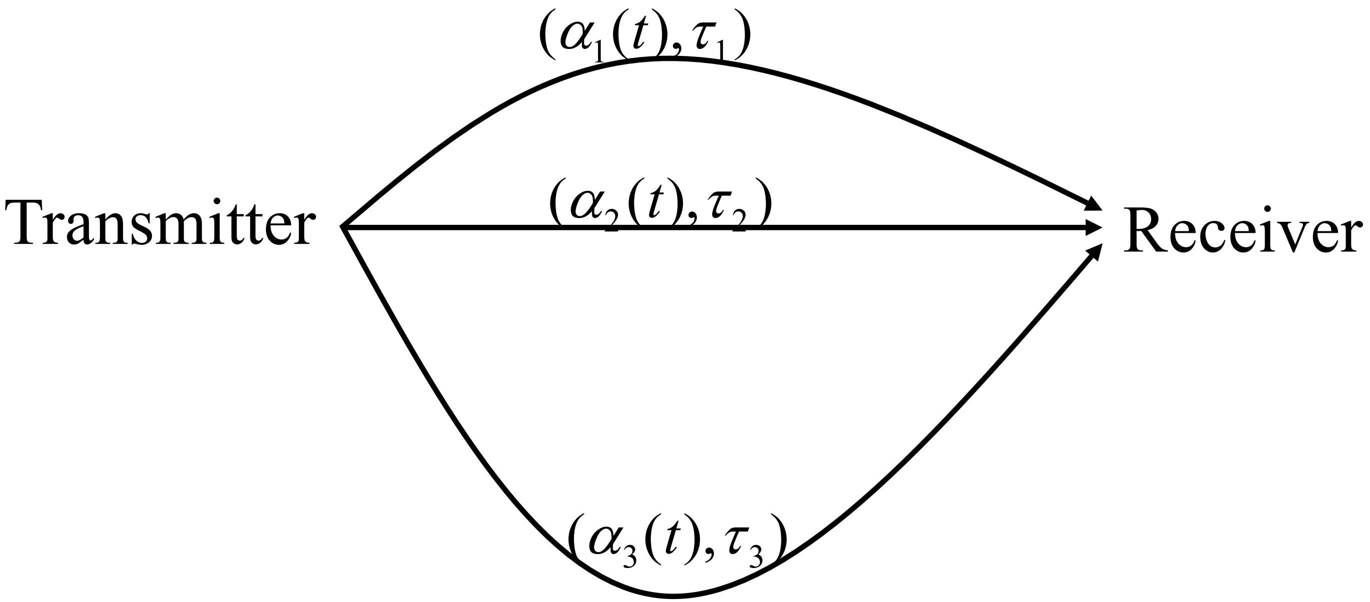
$$= -\frac{1}{\lambda}\lim_{\Delta t\to 0}\frac{\sqrt{L^2 + v^2(\Delta t)^2 + 2L\cdot v\cdot \Delta t\cdot \cos(\alpha)} - L}{\Delta t}$$

$$= -\frac{1}{\lambda} v \cdot \cos(\alpha)$$

$$f_{Doppler} = f_c + d(f) = f_c - \frac{v}{\lambda}\cos(\alpha)$$

Statistical Characterization of Multipath Channels

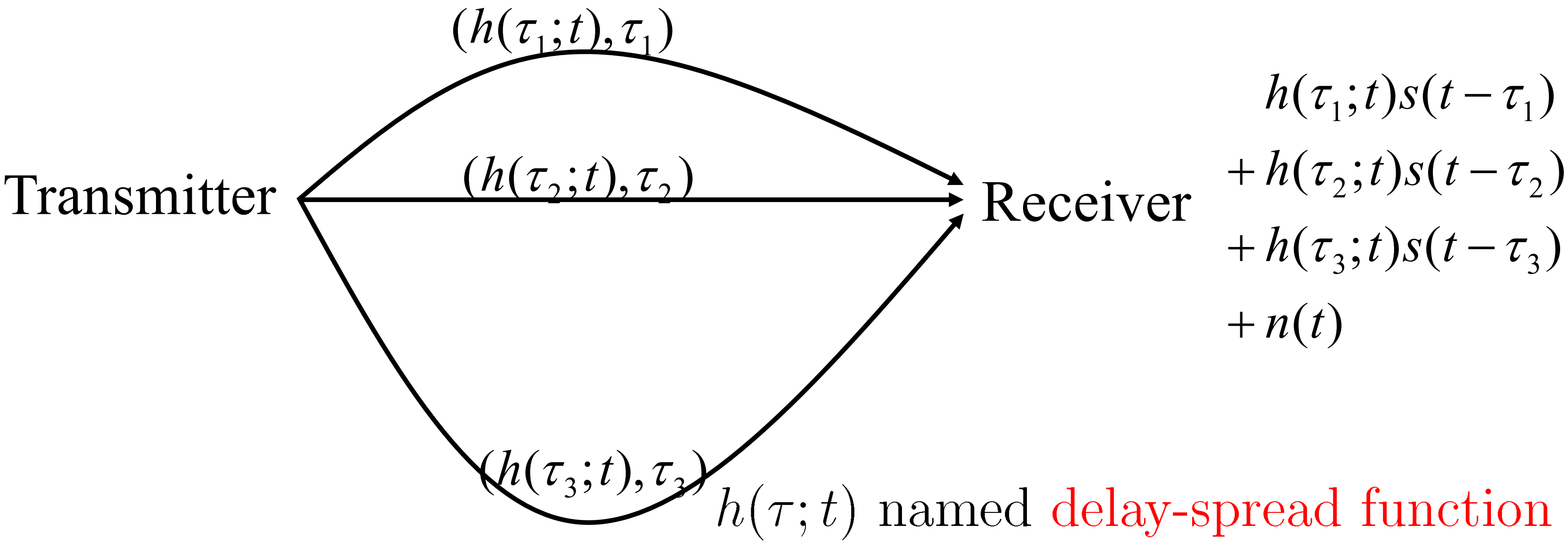
- A formal definition of multipath fading channels



$$
\begin{aligned}
&\alpha_1(t)s(t-\tau_1) \\
&+\alpha_2(t)s(t-\tau_2) \\
&+\alpha_3(t)s(t-\tau_3) \\
&+n(t)
\end{aligned}
$$

Statistical Characterization of Multipath Channels

- A formal definition of multipath fading channels



$h(\tau;t)$ named delay-spread function

$$\text{Receiver output} = \int_{-\infty}^{\infty} h(\tau;t)s(t-\tau)d\tau + n(t)$$

Statistical Characterization of Multipath Channels

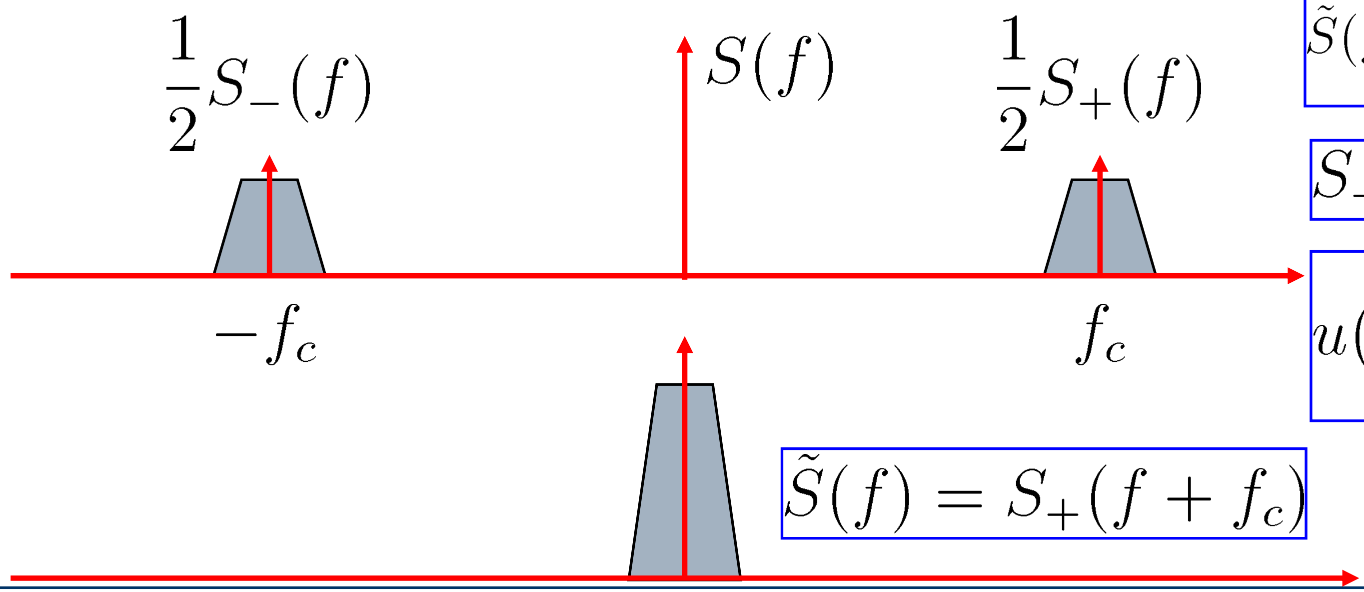
- Canonical representation of low-pass complex envelope

$$s(t) = \mathrm{Re}\left\{\tilde{s}(t)\exp(j2\pi f_c t)\right\}$$
$$\Rightarrow S(f) = \frac{1}{2}[\tilde{S}(f-f_c) + \tilde{S}^*(-f-f_c)]$$

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-j2\pi f t}dt$$
$$\tilde{S}(f) = \int_{-\infty}^{\infty} \tilde{s}(t)e^{-j2\pi f t}dt$$

$$S_+(f) = 2u(f)S(f)$$

$$u(f) = \begin{cases} 1, & f > 0 \\ 0, & f < 0 \end{cases}$$



$$\tilde{S}(f) = S_+(f+f_c)$$

Given

$$\begin{cases} h(\tau) = \mathrm{Re}\left\{\tilde{h}(\tau)\exp(j2\pi f_c\tau)\right\} \\ s_o(t) = \mathrm{Re}\left\{\tilde{s}_o(t)\exp(j2\pi f_c t)\right\} \\ s_o(t) = \int_{-\infty}^{\infty} h(\tau)s(t-\tau)d\tau \\ S_o(f) = S(f)H(f) \end{cases}$$

convolution

$$H(f) = \int_{-\infty}^{\infty} h(\tau)e^{-j2\pi f\tau}d\tau$$

$$S_o(f) = \int_{-\infty}^{\infty} s_o(t)e^{-j2\pi ft}dt$$

$$\tilde{H}(f) = \int_{-\infty}^{\infty} \tilde{h}(\tau)e^{-j2\pi f\tau}d\tau$$

$$\tilde{S}_o(f) = \int_{-\infty}^{\infty} \tilde{s}_o(t)e^{-j2\pi ft}dt$$

Then

$$\begin{aligned} \tilde{S}(f)\tilde{H}(f) &= S_+(f+f_c)H_+(f+f_c) \\ &= (2u(f+f_c)S(f+f_c))(2u(f+f_c)H(f+f_c)) \\ &= 4u(f+f_c)S(f+f_c)H(f+f_c) \\ &= 4u(f+f_c)S_o(f+f_c) \\ &= 2[2u(f+f_c)S_o(f+f_c)] \\ &= 2\tilde{S}_o(f) \quad \Rightarrow \tilde{s}_o(t) = \frac{1}{2}\int_{-\infty}^{\infty}\tilde{h}(\tau)\tilde{s}(t-\tau)d\tau \end{aligned}$$

For time-varying channel,

$$\begin{cases} h(\tau;t) = \mathrm{Re}\left\{\tilde{h}(\tau;t)\exp(j2\pi f_c \tau)\right\} \\ s_o(t) = \mathrm{Re}\left\{\tilde{s}_o(t)\exp(j2\pi f_c t)\right\} \end{cases}$$

$$H(f;t) = \int_{-\infty}^{\infty} h(\tau;t)e^{-j2\pi f\tau}d\tau$$

$$S_o(f) = \int_{-\infty}^{\infty} s_o(t)e^{-j2\pi ft}dt$$

$$\tilde{H}(f;t) = \int_{-\infty}^{\infty} \tilde{h}(\tau;t)e^{-j2\pi f\tau}d\tau$$

$$\tilde{S}_o(f) = \int_{-\infty}^{\infty} \tilde{s}_o(t)e^{-j2\pi ft}dt$$

Eq. (8.38) in text is mistakenly put *t* in here, which is wrong.

$$\begin{aligned} s_o(t) &= \int_{-\infty}^{\infty} h(\tau;t)s(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} H(f;t)e^{j2\pi f\tau}df\right)s(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} s(t-\tau)e^{j2\pi f\tau}d\tau\right)H(f;t)df \end{aligned}$$

$$
\begin{cases}
s_o(t) &= \displaystyle\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} s(u)e^{j2\pi f(t-u)}du\right)H(f;t)df, \text{ where } u = t-\tau \\
&= \displaystyle\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} s(u)e^{-j2\pi f u}du\right)H(f;t)e^{j2\pi f t}df \\
&= \displaystyle\int_{-\infty}^{\infty} S(f)H(f;t)e^{j2\pi f t}df
\end{cases}
$$

It is however not correct to infer $S_o(f) = S(f)H(f;t)$! (See Slide IDC5-58.)

$$
\begin{cases}
S(f) = \frac{1}{2}\left[\tilde{S}(f-f_c) + \tilde{S}^*(-f-f_c)\right] \\
H(f;t) = \frac{1}{2}\left[\tilde{H}(f-f_c;t) + \tilde{H}^*(-f-f_c;t)\right]
\end{cases}
$$

(See Slide IDC5-58.)

$$
\begin{aligned}
\Rightarrow\ & S(f)H(f;t) \\
&= \frac{1}{4}\left[\tilde{S}(f-f_c) + \tilde{S}^*(-f-f_c)\right]\left[\tilde{H}(f-f_c;t) + \tilde{H}^*(-f-f_c;t)\right] \\
&= \frac{1}{4}\left[\tilde{S}(f-f_c)\tilde{H}(f-f_c;t) + \tilde{S}^*(-f-f_c)\tilde{H}^*(-f-f_c;t)\right]
\end{aligned}
$$

$$
\begin{aligned}
s_o(t) \\
&= \int_{-\infty}^{\infty} \frac{1}{4}\left[\tilde{S}(f-f_c)\tilde{H}(f-f_c;t)+\tilde{S}^*(-f-f_c)\tilde{H}^*(-f-f_c;t)\right]e^{j2\pi ft}df \\
&= \frac{1}{4}\int_{-\infty}^{\infty}\tilde{S}(f-f_c)\tilde{H}(f-f_c;t)e^{j2\pi ft}df+\frac{1}{4}\int_{-\infty}^{\infty}\tilde{S}^*(-f-f_c)\tilde{H}^*(-f-f_c;t)e^{j2\pi ft}df \\
&= \frac{1}{4}e^{j2\pi f_c t}\int_{-\infty}^{\infty}\tilde{S}(f')\tilde{H}(f';t)e^{j2\pi f't}df'+\frac{1}{4}\left(e^{j2\pi f_c t}\int_{-\infty}^{\infty}\tilde{S}(f'')\tilde{H}(f'';t)e^{j2\pi f''t}df''\right)^* \\
&= \frac{1}{4}e^{j2\pi f_c t}\int_{-\infty}^{\infty}\tilde{h}(\tau;t)\tilde{s}(t-\tau)d\tau+\frac{1}{4}\left(e^{j2\pi f_c t}\int_{-\infty}^{\infty}\tilde{h}(\tau;t)\tilde{s}(t-\tau)d\tau\right)^* \\
&= \mathrm{Re}\left\{e^{j2\pi f_c t}\underbrace{\frac{1}{2}\int_{-\infty}^{\infty}\tilde{h}(\tau;t)\tilde{s}(t-\tau)d\tau}_{\tilde{s}_o(t)}\right\}
\end{aligned}
$$

Hence, we can equivalently operate on "lowpass domain" as

$$
\tilde{s}_o(t)=\frac{1}{2}\int_{-\infty}^{\infty}\tilde{h}(\tau;t)\tilde{s}(t-\tau)d\tau
$$

with no information loss on the "bandpass domain".

Note that in a time-varying environment,

$$\begin{aligned}
H(f;t) &= \int_{-\infty}^{\infty} h(\tau;t)e^{-j2\pi f\tau}d\tau \\
&= \int_{-\infty}^{\infty} \mathrm{Re}\left\{\tilde{h}(\tau;t)\, e^{j2\pi f_c\tau}\right\} e^{-j2\pi f\tau}d\tau \\
&= \frac{1}{2}\int_{-\infty}^{\infty}\left(\tilde{h}(\tau;t)e^{j2\pi f_c\tau} + \tilde{h}^*(\tau;t)e^{-j2\pi f_c\tau}\right) e^{-j2\pi f\tau}d\tau \\
&= \frac{1}{2}\int_{-\infty}^{\infty}\tilde{h}(\tau;t)e^{-j2\pi(f-f_c)\tau}d\tau + \frac{1}{2}\left(\int_{-\infty}^{\infty}\tilde{h}(\tau;t)e^{-j2\pi(-f-f_c)\tau}d\tau\right)^* \\
&= \frac{1}{2}\left[\tilde{H}(f-f_c;t) + \tilde{H}^*(-f-f_c;t)\right]
\end{aligned}$$

$$\begin{aligned}
S_o(f) &= \int_{-\infty}^{\infty} s_o(t)e^{-j2\pi ft}dt = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} S(f')H(f';t)e^{j2\pi f't}df'\right)e^{-j2\pi ft}dt \\
&= \int_{-\infty}^{\infty}\left(\underbrace{\int_{-\infty}^{\infty} H(f';t)e^{-j2\pi(f-f')t}dt}_{\neq H(f';t)\delta(f-f')}\right)S(f')df'
\end{aligned}$$

Statistical Characterization of Multipath Channels

- Usual **assumptions** on statistical characterization of the channel

- $\tilde{h}(\tau;t)$ stationary, zero-mean, complex-valued Gaussian process in t

- **Uncorrelated scattering**: $\tilde{h}(\tau_1;t)$ and $\tilde{h}(\tau_2;t)$ are uncorrelated for different τ_1 and τ_2

$$\begin{aligned} R_{\tilde{h}}(\tau_1,\tau_2;\Delta t) &= E\left[\tilde{h}^*(\tau_1;t)\tilde{h}(\tau_2;t+\Delta t)\right] \\ &= E\left[\tilde{h}^*(\tau_1;t)\tilde{h}(\tau_1;t+\Delta t)\right]\delta(\tau_1-\tau_2) \\ &= r_{\tilde{h}}(\tau_1;\Delta t)\delta(\tau_1-\tau_2) \end{aligned}$$

where $r_{\tilde{h}}(\tau;\Delta t)$ is called *multipath autocorrelation profile*.

$$
\begin{aligned}
R_{\tilde{H}}(f_1, f_2; \Delta t) &= E\left[\tilde{H}^*(f_1; t)\tilde{H}(f_2; t+\Delta t)\right] \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{\tilde{h}}(\tau_1, \tau_2; \Delta t) e^{j2\pi(f_1\tau_1 - f_2\tau_2)} d\tau_1 d\tau_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r_{\tilde{h}}(\tau_1; \Delta t)\delta(\tau_1 - \tau_2) e^{j2\pi(f_1\tau_1 - f_2\tau_2)} d\tau_1 d\tau_2 \\
&= \int_{-\infty}^{\infty} r_{\tilde{h}}(\tau_1; \Delta t) e^{j2\pi(f_1 - f_2)\tau_1} d\tau_1 \qquad \boxed{\Delta f = f_2 - f_1} \\
&= \int_{-\infty}^{\infty} r_{\tilde{h}}(\tau_1; \Delta t) e^{-j2\pi(\Delta f)\tau_1} d\tau_1 = r_{\tilde{H}}(\Delta f; \Delta t)
\end{aligned}
$$

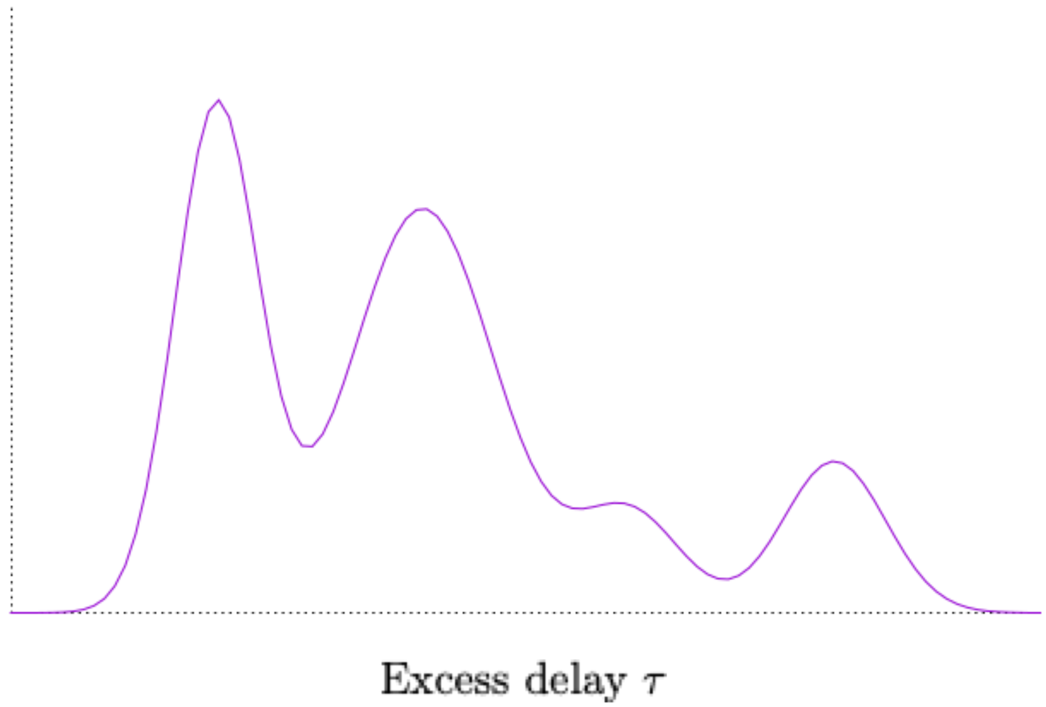
For zero-mean, stationary, uncorrelated scattering channels, the autocorrelation function of the channel transfer function only depends on time difference and frequency difference.

It is thus named spaced-frequency spaced-time correlation function.

Statistical Characterization of Multipath Channels

- A typical multipath intensity profile

$$P_{\tilde{h}}(\tau) = r_{\tilde{h}}(\tau; \Delta t = 0)$$



Statistical Characterization of Multipath Channels

- Two major concerns on multipath fading channels
  - Delay spread from τ

    Delay power spectrum or multipath intensity profile
    $r_{\tilde{h}}(\tau; \Delta t = 0) = P_{\tilde{h}}(\tau)$

  - Doppler spread from ν

Scattering function $S(\tau; \nu) = \int_{-\infty}^{\infty} r_{\tilde{h}}(\tau; \Delta t) \exp(-j2\pi\nu(\Delta t)) d(\Delta t)$

Doppler power spectrum $S_{\tilde{H}}(\nu) = \int_{-\infty}^{\infty} S(\tau; \nu) d\tau = \int_{-\infty}^{\infty} r_{\tilde{H}}(0; \Delta t) \exp\left(-j2\pi\nu\, \Delta t\right) d(\Delta t)$

Statistical Characterization of Multipath Channels

- (rms) delay spread

$$\sigma_\tau = \left(\frac{\int_0^\infty (\tau - \tau_{av})^2 P_{\tilde{h}}(\tau) d\tau}{\int_0^\infty P_{\tilde{h}}(\tau) d\tau} \right)^{1/2}, \quad \text{where } \tau_{av} = \frac{\int_0^\infty \tau P_{\tilde{h}}(\tau) d\tau}{\int_0^\infty P_{\tilde{h}}(\tau) d\tau}.$$

- (rms) Doppler spread

$$\sigma_v = \left(\frac{\int_{-\infty}^\infty (\nu - \nu_{av})^2 S_{\tilde{H}}(\nu) d\nu}{\int_{-\infty}^\infty S_{\tilde{H}}(\nu) d\nu} \right)^{1/2}, \quad \text{where } \nu_{av} = \frac{\int_{-\infty}^\infty \nu S_{\tilde{H}}(\nu) d\nu}{\int_{-\infty}^\infty S_{\tilde{H}}(\nu) d\nu}.$$

(Usually, $\nu_{av} = 0$.)

Statistical Characterization of Multipath Channels

- (rms) coherent bandwidth

$$B_c = \frac{1}{\sigma_\tau}$$

- (rms) coherent time

$$\tau_c = \frac{1}{\sigma_v}$$

Statistical Characterization of Multipath Channels

- Classification of channels according to coherent time and coherent bandwidth
  - Let signal bandwidth be B.

    If $B_c \ll B$, frequency-selective channel.

    $\sigma_\tau \ll T$ If $B_c \gg B$, frequency-nonselective or frequency flat channel.

  - Let symbol period be T.

    If $\tau_c \ll T$, time-selective channel.

    $\sigma_v \ll B$ If $\tau_c \gg T$, time-nonselective or time flat channel.

Binary Signaling over Rayleigh Fading Channels

- For a time-flat frequency-flat fading channel, i.e., $\sigma_\tau \ll T \ll \tau_c$, the relation between input and output can be modeled as:

$$\tilde{x}(t) = \alpha \exp(-j\phi) \cdot \tilde{s}(t) + \tilde{w}(t)$$

$$\text{where } \begin{cases} \alpha & \text{Rayleigh distributed} \\ \phi & \text{Some distribution} \\ \tilde{w}(t) & \text{complex AWGN} \end{cases}$$

- Assume the receiver can **perfectly** estimate α and ϕ.

Under the "perfect" assumption, the receiver system can be equivalently transformed to

$$\bar{x}(t) = \tilde{x}(t)\exp(j\phi) = \alpha \cdot \tilde{s}(t) + \tilde{w}(t)\exp(j\phi) = \alpha \cdot \tilde{s}(t) + \bar{w}(t)$$

where $\bar{w}(t)$ and $\tilde{w}(t)$ have exactly the same distributions.

We can then do exactly the same derivation as Slide IDC1-30 by replacing E_b with $\alpha^2 E_b$, and obtain:

$$P(\text{Error}|\alpha,\phi) = \Phi\left(-\sqrt{2\frac{(\alpha^2 E_b)}{N_0}}\right)$$

Coherent Phase-Shift Keying (PSK) – Error Probability

- Error probability of Binary PSK
  - Based on the decision rule $x \underset{\sqrt{E_b}}{\overset{-\sqrt{E_b}}{\lessgtr}} 0$

$$
\begin{aligned}
P(\text{Error}) &= P\left(-\sqrt{E_b}\text{ transmitted}\right) P\left(x > 0 \middle| -\sqrt{E_b}\text{ transmitted}\right) \\
&\quad P\left(+\sqrt{E_b}\text{ transmitted}\right) P\left(x > 0 \middle| +\sqrt{E_b}\text{ transmitted}\right) \\
&= \frac{1}{2}\int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x+\sqrt{E_b})^2/2\sigma^2}dx + \frac{1}{2}\int_{-\infty}^0 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\sqrt{E_b})^2/2\sigma^2}dx \\
&= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x+\sqrt{E_b})^2/2\sigma^2}dx = \Phi\left(\frac{0-\sqrt{E_b}}{\sigma}\right) = \Phi\left(-\sqrt{2\frac{E_b}{N_0}}\right)
\end{aligned}
$$

$$\Phi(-x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$
\begin{aligned}
P(\text{Error}) &= \int_0^{\infty}\int_0^{2\pi} P(\text{Error}|\alpha,\phi) f_{\alpha,\phi}(\alpha,\phi)\,d\phi\,d\alpha \\
&= \int_0^{\infty} \Phi\left(-\sqrt{2\frac{(\alpha^2 E_b)}{N_0}}\right)\left(\int_0^{2\pi} f_{\alpha,\phi}(\alpha,\phi)\,d\phi\right) d\alpha \\
&= \int_0^{\infty} \Phi\left(-\sqrt{2\frac{(\alpha^2 E_b)}{N_0}}\right) f_\alpha(\alpha)\,d\alpha \quad (\text{Let } \gamma = \alpha^2\frac{E_b}{N_0}) \\
&= \int_0^{\infty} \Phi\left(-\sqrt{2\gamma}\right) f_\gamma(\gamma)\,d\gamma
\end{aligned}
$$

(α Rayleigh distribution implies

γ Chi-square distribution with two degree of freedom)

$$
f_\gamma(\gamma) = \frac{1}{\gamma_0} e^{-\gamma/\gamma_0}, \text{ where } \gamma_0 = E[\gamma] = \frac{E_b}{N_0} E[\alpha^2]
$$

$$
\begin{aligned}
P(\text{Error}) &= \int_0^\infty \Phi\left(-\sqrt{2\gamma}\right)\frac{1}{\gamma_0}e^{-\gamma/\gamma_0}d\gamma \\
&\quad \text{(Let } u(\gamma) = \Phi\left(-\sqrt{2\gamma}\right) \text{ and } v(\gamma) = -e^{-\gamma/\gamma_0}. \\
&\quad\ \text{Apply } \int u\cdot dv = u\cdot v| - \int v\cdot du.\text{)} \\
&= \Phi\left(-\sqrt{2\gamma}\right)\left(-e^{-\gamma/\gamma_0}\right)\Big|_0^\infty - \int_0^\infty \left(-e^{-\gamma/\gamma_0}\right)\left(-\frac{1}{\sqrt{2\gamma}}\frac{1}{\sqrt{2\pi}}e^{-\gamma}\right)d\gamma \\
&= \frac{1}{2} - \int_0^\infty \frac{1}{\sqrt{4\pi\gamma}}e^{-\gamma(1+1/\gamma_0)}d\gamma \quad (x = \gamma(1+1/\gamma_0)) \\
&= \frac{1}{2} + \frac{1}{\sqrt{1+1/\gamma_0}}\int_0^\infty \left(-\frac{1}{\sqrt{4\pi x}}e^{-x}\right)dx \quad \text{(This is exactly } u'(x).\text{)} \\
&= \frac{1}{2} + \frac{1}{\sqrt{1+1/\gamma_0}}\Phi\left(-\sqrt{2x}\right)\Big|_0^\infty \\
&= \frac{1}{2} - \frac{1}{2\sqrt{1+1/\gamma_0}}
\end{aligned}
$$

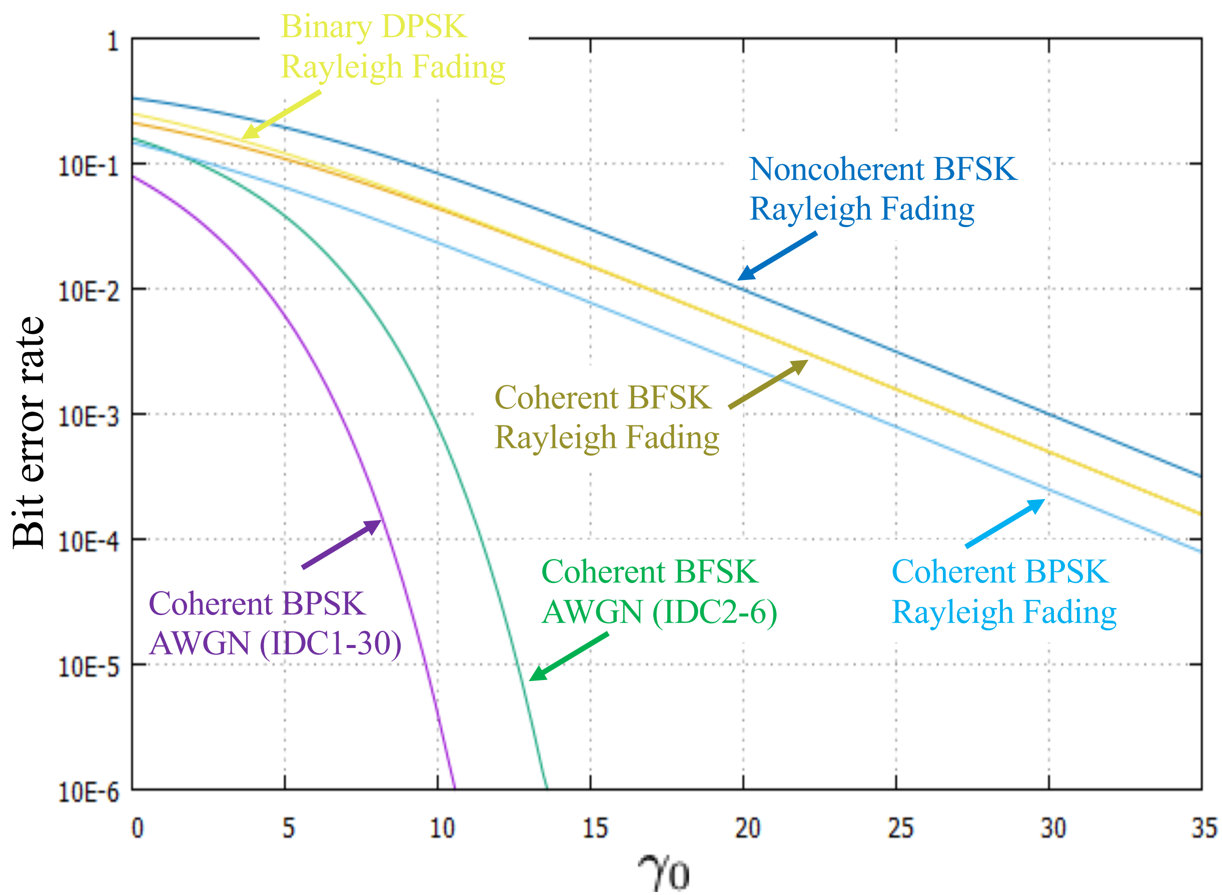
$$P_{\mathrm{BPSK}}(\mathrm{Error}) = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right)$$

We can similarly obtain the error rate for the other transmission schemes as follows.

Type of Signaling	*Exact Formula for the Bit Error Rate*	*Approximate Formula for the Bit Error Rate, Assuming Large* γ_0
Coherent binary PSK	$\frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{1+\gamma_0}}\right)$	$\frac{1}{4\gamma_0}$
Coherent binary FSK	$\frac{1}{2}\left(1 - \sqrt{\frac{\gamma_0}{2+\gamma_0}}\right)$	$\frac{1}{2\gamma_0}$
Binary DPSK	$\frac{1}{2(1+\gamma_0)}$	$\frac{1}{2\gamma_0}$
Noncoherent binary FSK	$\frac{1}{2+\gamma_0}$	$\frac{1}{\gamma_0}$

Binary Signaling over Rayleigh Fading Channels

- The performance degrades significantly in fading channels even with perfect channel estimation.
- How to compensate fading effect without, e.g., greatly increasing the transmitted power ?
  - Diversity technique



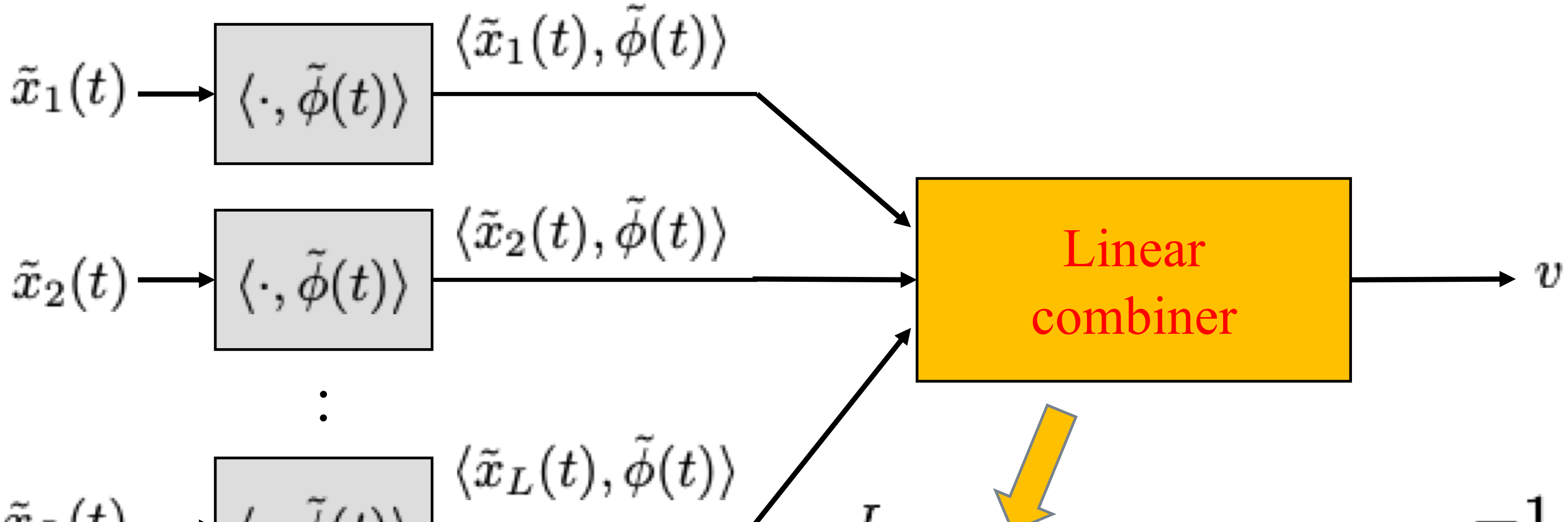
Binary Signaling over Rayleigh Fading Channels

- Categories of diversity technique
  - Frequency diversity
  - Time (signal-repetition) diversity
  - Space diversity

- Basically, they just repetitively transmit the same signal L times, and make decision based on these L replica (assuming that the fadings encountered are uncorrelated)
  - How to combine these L replica is also a research subject ?

Binary Signaling over Rayleigh Fading Channels

- Maximal-ratio combiner (For example, one-dimensional BPSK)



$$v = \sum_{\ell=1}^{L} \alpha_\ell e^{j\phi_\ell} \langle \tilde{x}_\ell(t), \tilde{\phi}(t)\rangle \overset{-1}{\underset{+1}{\lessgtr}} 0$$

$$\tilde{x}_k(t) = \alpha_k e^{-j\phi_k}\tilde{s}(t) + \tilde{w}_k(t)$$

$$
\begin{aligned}
v &= \sum_{\ell=1}^{L} \alpha_\ell e^{j\phi_\ell} \langle \tilde{x}_\ell(t), \tilde{\phi}(t) \rangle \\
&= \left(\sum_{\ell=1}^{L} \alpha_\ell^2 \right) \langle \tilde{s}(t), \tilde{\phi}(t) \rangle + \sum_{\ell=1}^{L} \alpha_\ell e^{j\phi_\ell} \langle \tilde{w}_\ell(t), \tilde{\phi}(t) \rangle \\
&= \pm\sqrt{E_b} \left(\sum_{\ell=1}^{L} \alpha_\ell^2 \right) + \sum_{\ell=1}^{L} \alpha_\ell z_\ell .
\end{aligned}
$$

$$
\Rightarrow \mathrm{Re}\{v\} = \pm\sqrt{E_b} \left(\sum_{\ell=1}^{L} \alpha_\ell^2 \right) + \sum_{\ell=1}^{L} \alpha_\ell \mathrm{Re}\{z_\ell\}.
$$

where $\{\mathrm{Re}\{z_\ell\}\}_{\ell=1}^{L}$ i.i.d. zero-mean normal with variance $N_0/2$.

$$
\begin{aligned}
v_{\text{decision}} &= \pm\sqrt{E_b}\sqrt{\sum_{\ell=1}^{L}\alpha_\ell^2} + \frac{\sum_{\ell=1}^{L}\alpha_\ell \mathrm{Re}\{z_\ell\}}{\sqrt{\sum_{\ell=1}^{L}\alpha_\ell^2}} \\
&= \pm\sqrt{E_b}\alpha + \tilde{w}
\end{aligned}
$$

where $\alpha^2 = \sum_{\ell=1}^{L}\alpha_\ell^2$, and $\tilde{w}$ zero-mean Gaussian with variance $N_0/2$.

Under the "perfect-estimation" assumption, the receiver system can be equivalently transformed to (as similarly did in Slide IDC5-67)

$$v_{\text{decision}} = \pm\sqrt{E_b}\alpha + \tilde{w}$$

We can then do exactly the same derivation as Slide IDC1-30 by replacing E_b with $\alpha^2 E_b$, and obtain:

$$P(\text{Error}|\{\alpha_\ell, \phi_\ell\}_{\ell=1}^{L}) = \Phi\left(-\sqrt{2\frac{(\alpha^2 E_b)}{N_0}}\right)$$

$$P(\text{Error}) = \int_0^\infty \Phi\left(-\sqrt{2\gamma}\right) f_\gamma(\gamma) d\gamma \quad (\text{where } \gamma = \alpha^2 \frac{E_b}{N_0})$$

($\{\alpha_\ell\}_{\ell=1}^L$ i.i.d. Rayleigh distribution implies
γ Chi-square distribution with $2L$ degree of freedom.)

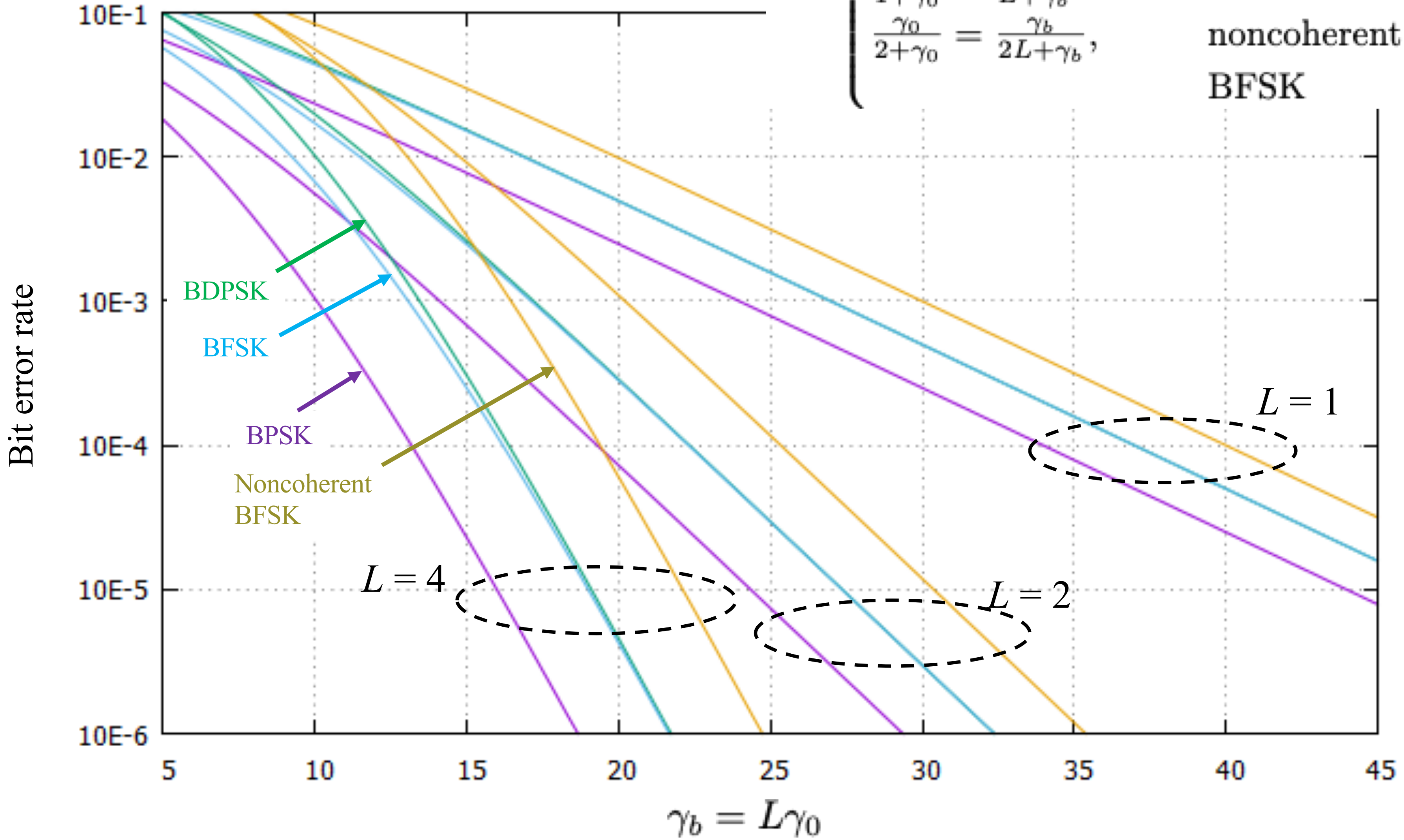
$$f_\gamma(\gamma) = \frac{1}{(L-1)!\gamma_0}\left(\frac{\gamma}{\gamma_0}\right)^{L-1} e^{-\gamma/\gamma_0}, \text{ where } \gamma_0 = E[\gamma_\ell].$$

$$P_{\text{BPSK}}(\text{Error}) = \left[\frac{1}{2}(1-\mu)\right]^L \sum_{\ell=0}^{L-1} \binom{L-1+\ell}{\ell} \left[\frac{1}{2}(1+\mu)\right]^\ell \approx \binom{2L-1}{L}\left(\frac{1}{4\gamma_0}\right)^L,$$

$$\text{where } \mu = \sqrt{\frac{\gamma_0}{1+\gamma_0}}.$$

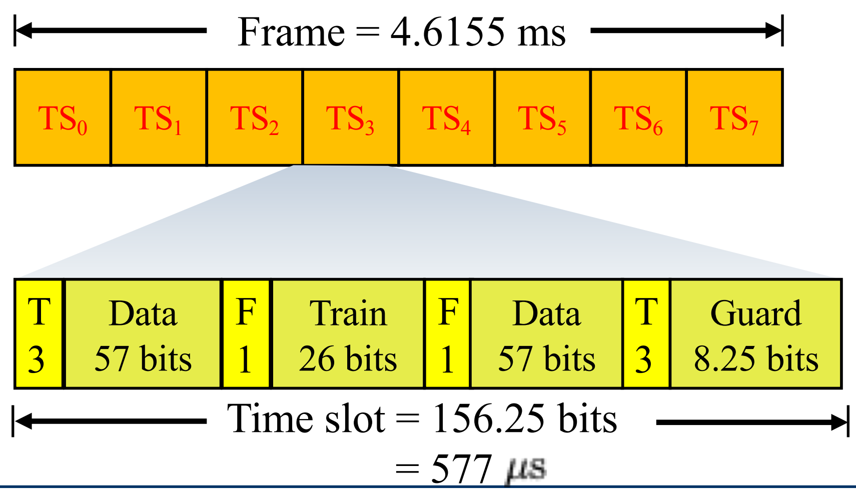
$$P(\text{Error}) = \left(\frac{1-\mu}{2}\right)^L \sum_{\ell=0}^{L-1} \binom{L-1+\ell}{\ell} \left(\frac{1+\mu}{2}\right)^\ell$$

with
$$\mu = \begin{cases} \sqrt{\frac{\gamma_0}{1+\gamma_0}} = \sqrt{\frac{\gamma_b}{L+\gamma_b}}, & \text{BPSK} \\ \sqrt{\frac{\gamma_0}{2+\gamma_0}} = \sqrt{\frac{\gamma_b}{2L+\gamma_b}}, & \text{BFSK} \\ \frac{\gamma_0}{1+\gamma_0} = \frac{\gamma_b}{L+\gamma_b}, & \text{BDPSK} \\ \frac{\gamma_0}{2+\gamma_0} = \frac{\gamma_b}{2L+\gamma_b}, & \text{noncoherent BFSK} \end{cases}$$



Wireless Communication Systems: TDMA/CDMA

- Global System for Mobile Communications (GSM)
  - Modulation type: GMSK
  - Channel bandwidth: 200 KHz
  - Number of duplex channels: 125
  - TDMA/FDD



TS: time slot

T3: three all-zero tail bits
(to reset the convolutional coder)

F1: one flag bit
(to identify whether the data bits are digitized speech or some other information-bearing signal)

Train: training bits for equalizer

Guard: all-zero guard time interval

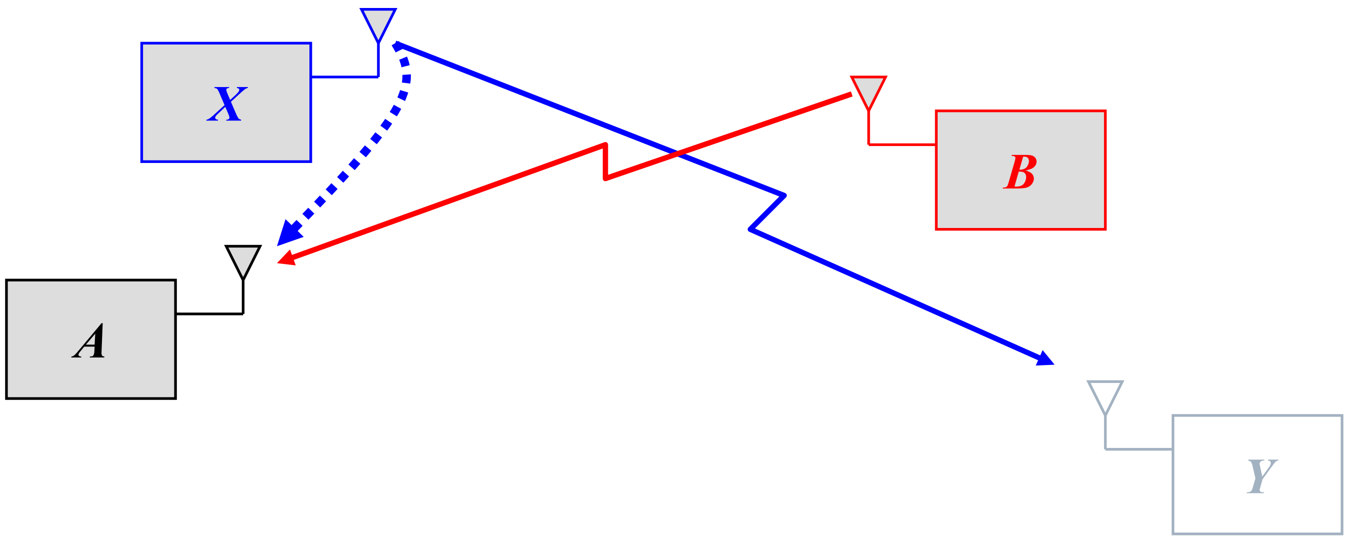
(57+57)/577 = 198.57 Kbps

Wireless Communication Systems: TDMA/CDMA

- IS-95 (Interim Standard)
  - Modulation type: BPSK
  - Channel bandwidth: 1.25 MHz
  - Number of duplex channels: 20
  - CDMA/FDD
  - Access users per channel: 20 to 35 (contrary to 8 for GSM)
  - Frame period 20ms, equal to that of the speech codec
  - Data rate: 9.6 or 14.4 Kbps

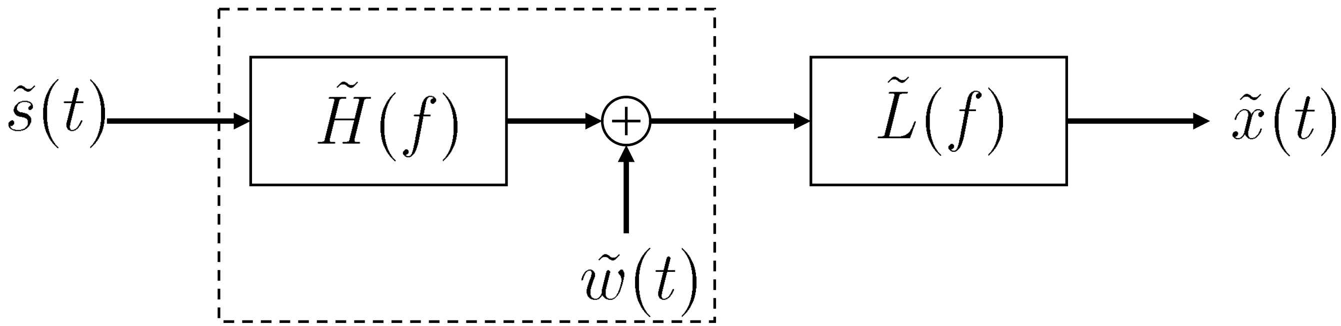
Wireless Communication Systems: TDMA/CDMA

- Technique challenge of CDMA
  - MAI (multiple access interference): Interference from other users
  - Near-far problem – Power control



Wireless Communication Systems: TDMA/CDMA

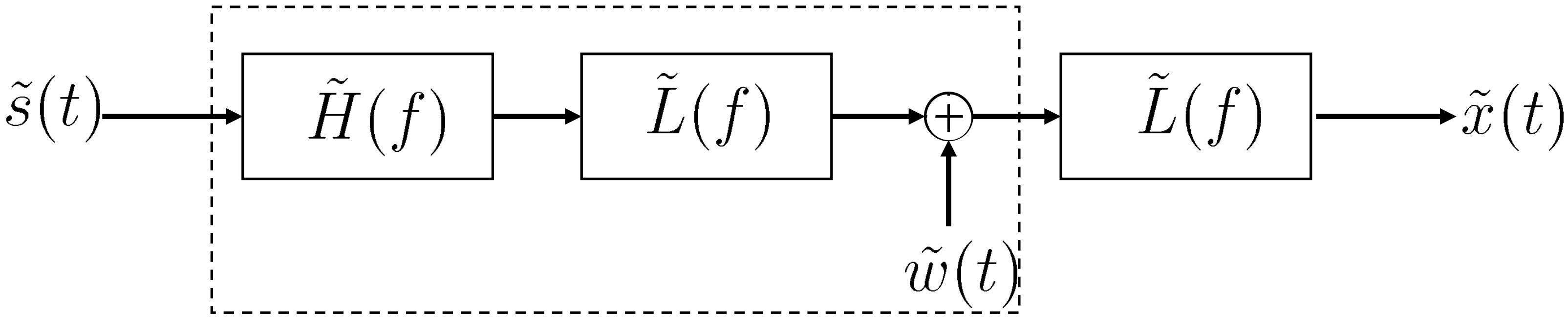
- RAKE receiver



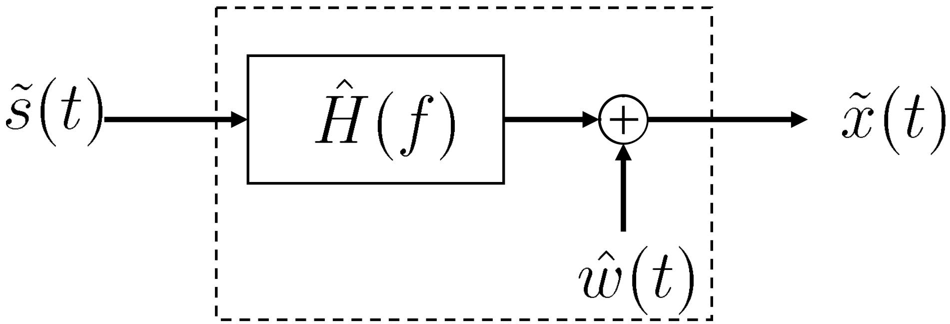
Channel with $\tilde{H}(f)$ random

$$\tilde{s}(t) \text{ bandlimited}$$
$$\Rightarrow |\tilde{S}(f)| = 0 \text{ for } |f| > W/2$$

$$\tilde{L}(f) = \begin{cases} 1, & |f| \le W/2 \\ 0, & \text{otherwise} \end{cases}$$



Equivalent channel with $\hat{H}(f) = \tilde{H}(f)\tilde{L}(f)$ random



Equivalent channel with $\hat{H}(f)$ random and bandlimited, and $\hat{w}(t)$ bandlimited white noise

$\hat{h}(t)$ bandlimited

$$\Rightarrow |\hat{H}(f)| = 0 \text{ for } |f| > W/2$$

$$\Rightarrow \text{Sampling theorem } \hat{h}(t) = \sum_{\ell=-\infty}^{\infty} \hat{h}\left(\frac{\ell}{W}\right) \cdot \text{sinc}\left[W\left(t - \frac{\ell}{W}\right)\right]$$

$$\Rightarrow \hat{H}(f) = \begin{cases} \dfrac{1}{W} \displaystyle\sum_{\ell=-\infty}^{\infty} \hat{h}\left(\frac{\ell}{W}\right) \cdot e^{-j2\pi f\ell/W}, & |f| \le W/2 \\ 0, & \text{otherwise} \end{cases}$$

$$
\begin{aligned}
\tilde{x}(t) &= \int_{-\infty}^{\infty} \tilde{S}(f)\hat{H}(f)e^{j2\pi ft}df + \hat{w}(t) \\
&= \int_{-W/2}^{W/2} \tilde{S}(f)\left(\frac{1}{W}\sum_{\ell=-\infty}^{\infty}\hat{h}\left(\frac{\ell}{W}\right)e^{-j2\pi f\ell/W}\right)e^{j2\pi ft}df + \hat{w}(t) \\
&= \frac{1}{W}\sum_{\ell=-\infty}^{\infty}\hat{h}\left(\frac{\ell}{W}\right)\int_{-W/2}^{W/2}\tilde{S}(f)e^{j2\pi(t-\ell/W)f}df + \hat{w}(t) \\
&= \frac{1}{W}\sum_{\ell=-\infty}^{\infty}\hat{h}\left(\frac{\ell}{W}\right)\tilde{s}\left(t-\frac{\ell}{W}\right) + \hat{w}(t) \\
&= \sum_{\ell=-\infty}^{\infty}\hat{h}_\ell\cdot\tilde{s}\left(t-\frac{\ell}{W}\right) + \hat{w}(t),\ \text{where } \hat{h}_\ell = \frac{1}{W}\hat{h}\left(\frac{\ell}{W}\right)
\end{aligned}
$$

Assume $\Pr[\hat{h}_\ell = 0] = 1$ for $\ell/W \gg$ delay spread, i.e., for $\ell > L$.
By causality, $\Pr[\hat{h}_\ell = 0] = 1$ for $\ell < 0$.

$$
= \sum_{\ell=0}^{L}\hat{h}_\ell\cdot\tilde{s}(t-\ell T) + \hat{w}(t),\ \text{where } T = 1/W.
$$

$$\tilde{x}(t) = \sum_{\ell=0}^{L} \hat{h}_\ell \cdot \tilde{s}_k(t-\ell T) + \hat{w}(t) \text{ for } 1 \le k \le 2$$

Assume that $\{\hat{h}_\ell\}_{\ell=0}^{L}$ can be perfectly estimated, and $\tilde{s}_2(t) = -\tilde{s}_1(t)$ (BPSK). Then, the rake matched filter gives:

$$\left\langle \tilde{x}(t), \sum_{i=0}^{L} \hat{h}_i \tilde{s}_1(t-iT) \right\rangle$$

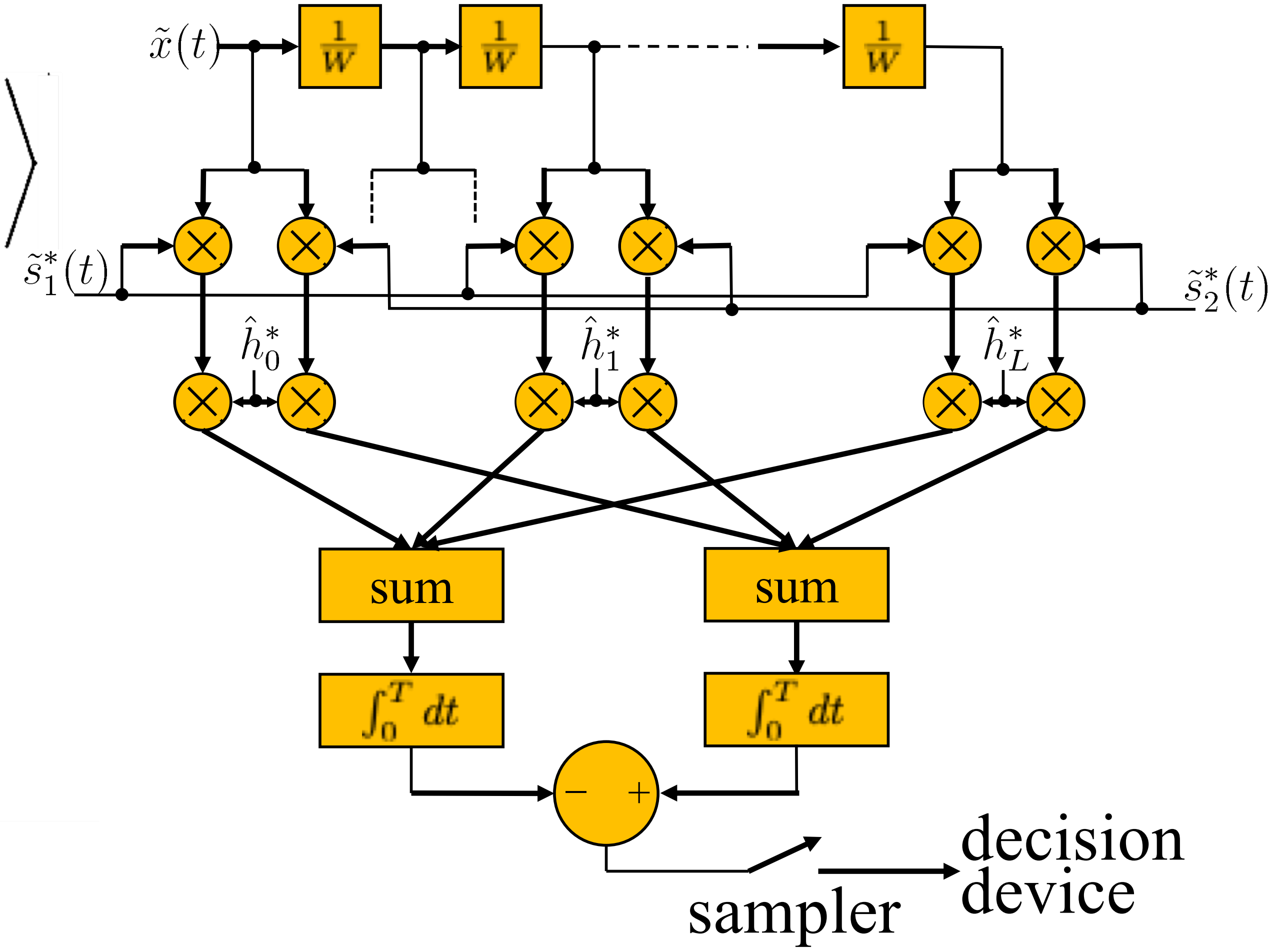
$$= \left\langle \sum_{\ell=0}^{L} \hat{h}_\ell \tilde{s}_k(t-\ell T), \sum_{i=0}^{L} \hat{h}_i \tilde{s}_1(t-iT) \right\rangle + \left\langle \hat{w}(t), \sum_{i=0}^{L} \hat{h}_i \tilde{s}_1(t-iT) \right\rangle$$

$$= \begin{cases} +\sum_{\ell=0}^{L} |\hat{h}_\ell|^2 + \hat{w}, & k=1 \\ -\sum_{\ell=0}^{L} |\hat{h}_\ell|^2 + \hat{w}, & k=2 \end{cases}$$

(equivalent to (L+1)-diversity with maximal ratio combiner)

where $\langle \tilde{s}_1(t-\ell T), \tilde{s}_1(t-iT) \rangle = \begin{cases} 1, & \ell = i \\ 0, & \text{otherwise} \end{cases}$

$$\left\langle \tilde{x}(t), \sum_{i=0}^{L} \hat{h}_i \tilde{s}_1(t - iT) \right\rangle$$
$$= \sum_{i=0}^{L} \hat{h}_i^* \langle \tilde{x}(t), \tilde{s}_1(t - iT) \rangle$$
$$= \sum_{i=0}^{L} \hat{h}_i^* \langle \tilde{x}(t + iT), \tilde{s}_1(t) \rangle$$
$$= \sum_{i=0}^{L} \left\langle \tilde{x}(t + iT), \hat{h}_i \tilde{s}_1(t) \right\rangle$$



The receiver **collects** the signal energy from all the received paths, which is somewhat analogous to the garden **rake** that is used to **gather** together leaves, hays, etc. Consequently, the name “RAKE receiver” has been coined for this receiver structure by Price and Green (1958).

Wireless Communication Systems: TDMA/CDMA

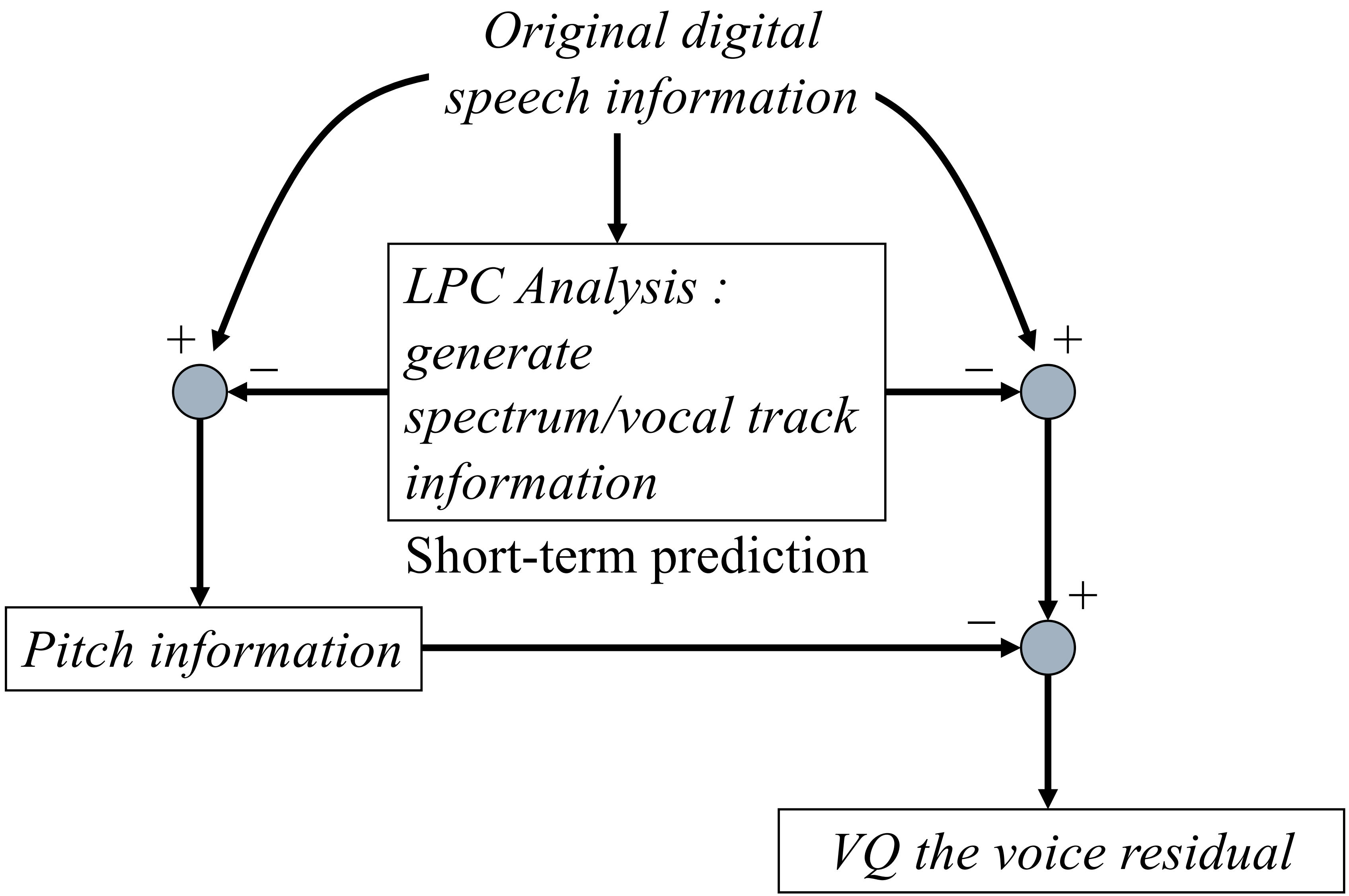
- Final note
  - The rake receiver collects all the significant echoes that are likely to occur in the multipath environment, and behaves as though there was a single propagation path between the transmitter and receiver.

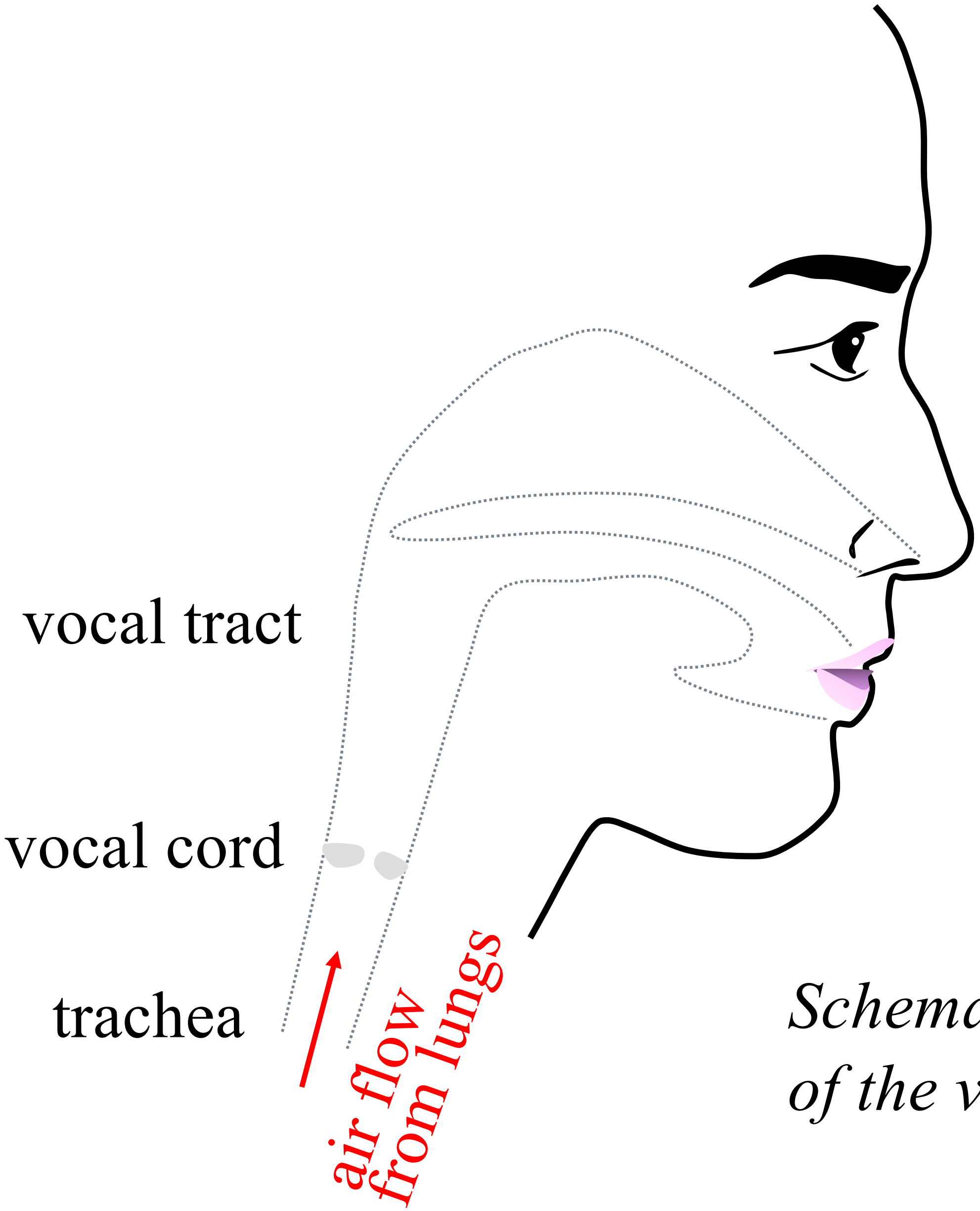
$$\tilde{x}(t) = \sum_{\ell=0}^{L} \hat{h}_\ell \cdot \tilde{s}\,(t - \ell T) + \hat{w}(t), \text{ where } T = 1/W.$$

Source Coding of Speech for Wireless Communication

- Speech coding used in GSM and IS-95
  - Multi-pulse excited linear predictive coding (LPC) – GSM
  - Code-excited LPC – IS-95

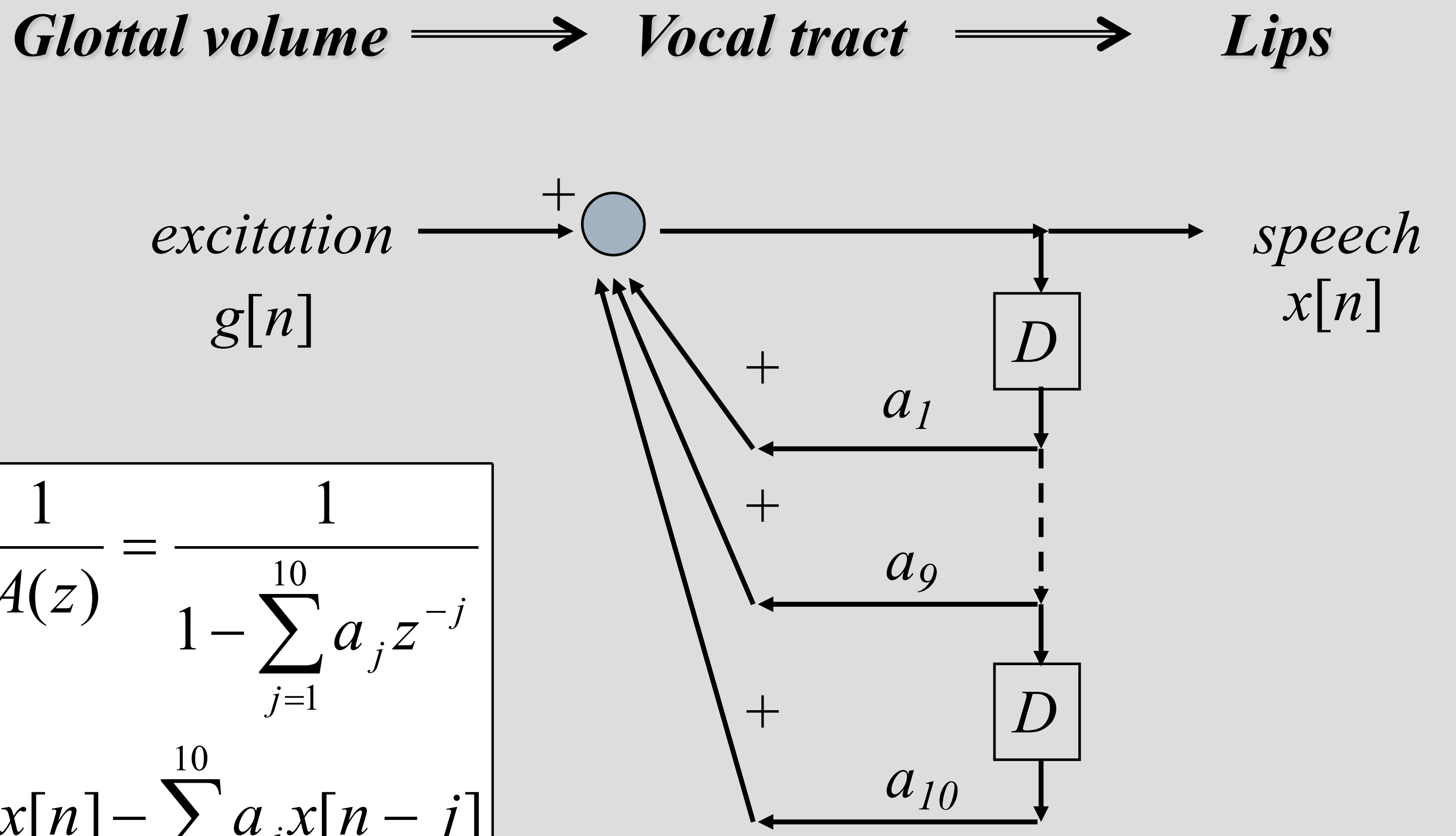
- Principle of analysis by synthesis
  - The encoder (analyzer) includes a copy of the decoder (synthesizer) in its design.


Original digital
speech information
LPC Analysis :
generate
spectrum/vocal track
information
Short-term prediction
+
−
+
−
+
−
Pitch information
VQ the voice residual



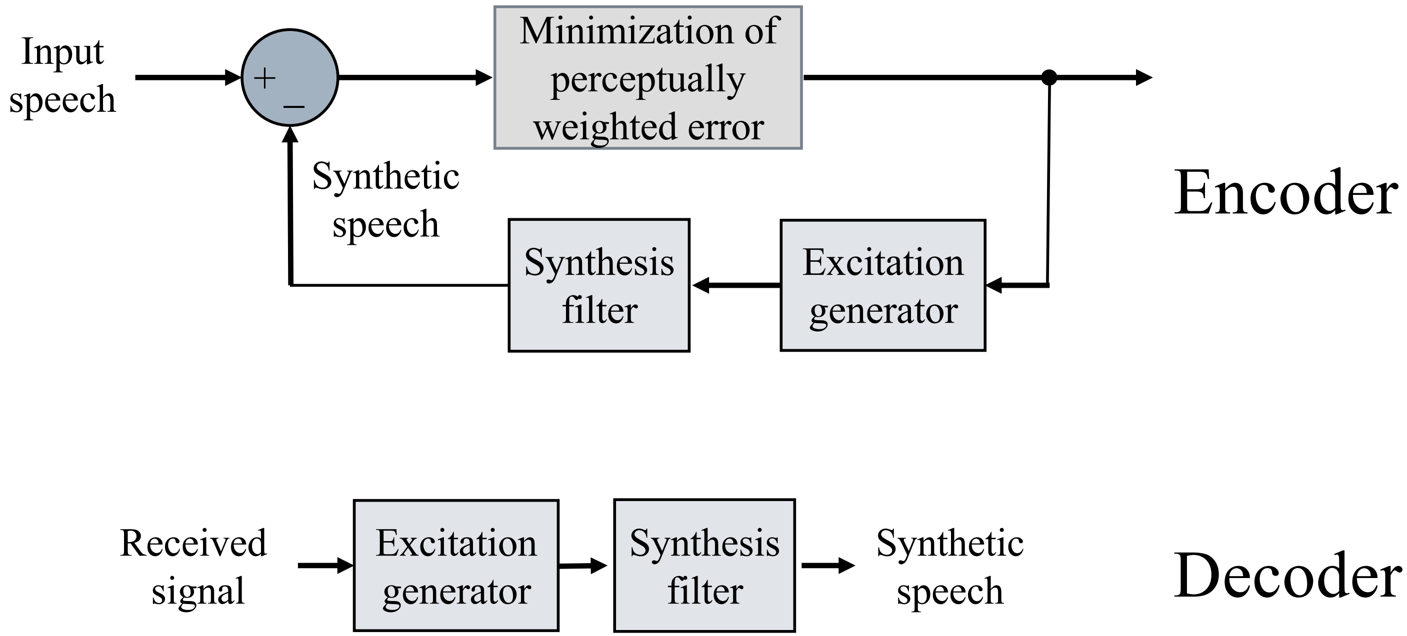
Schematic diagram of the vocal system

Synthesis filter (analysis by synthesis)

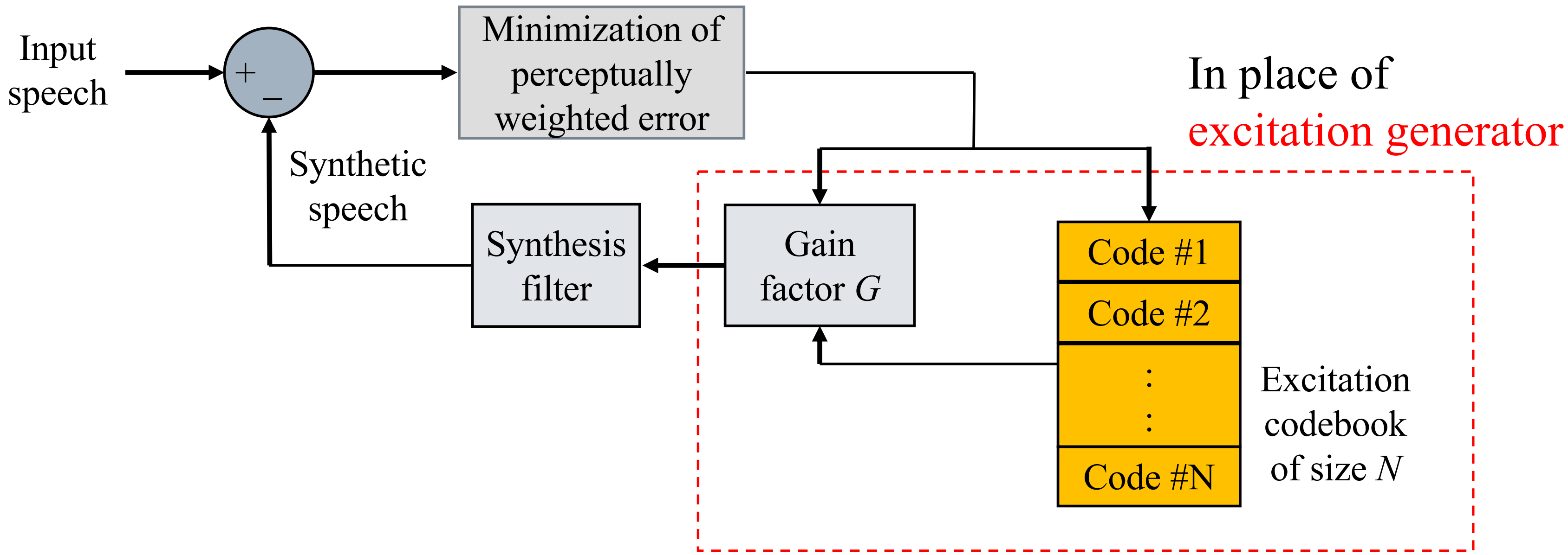


$$\frac{X(z)}{G(z)} = \frac{1}{A(z)} = \frac{1}{1-\sum_{j=1}^{10} a_j z^{-j}}$$

$$g[n] = x[n] - \sum_{j=1}^{10} a_j x[n-j]$$

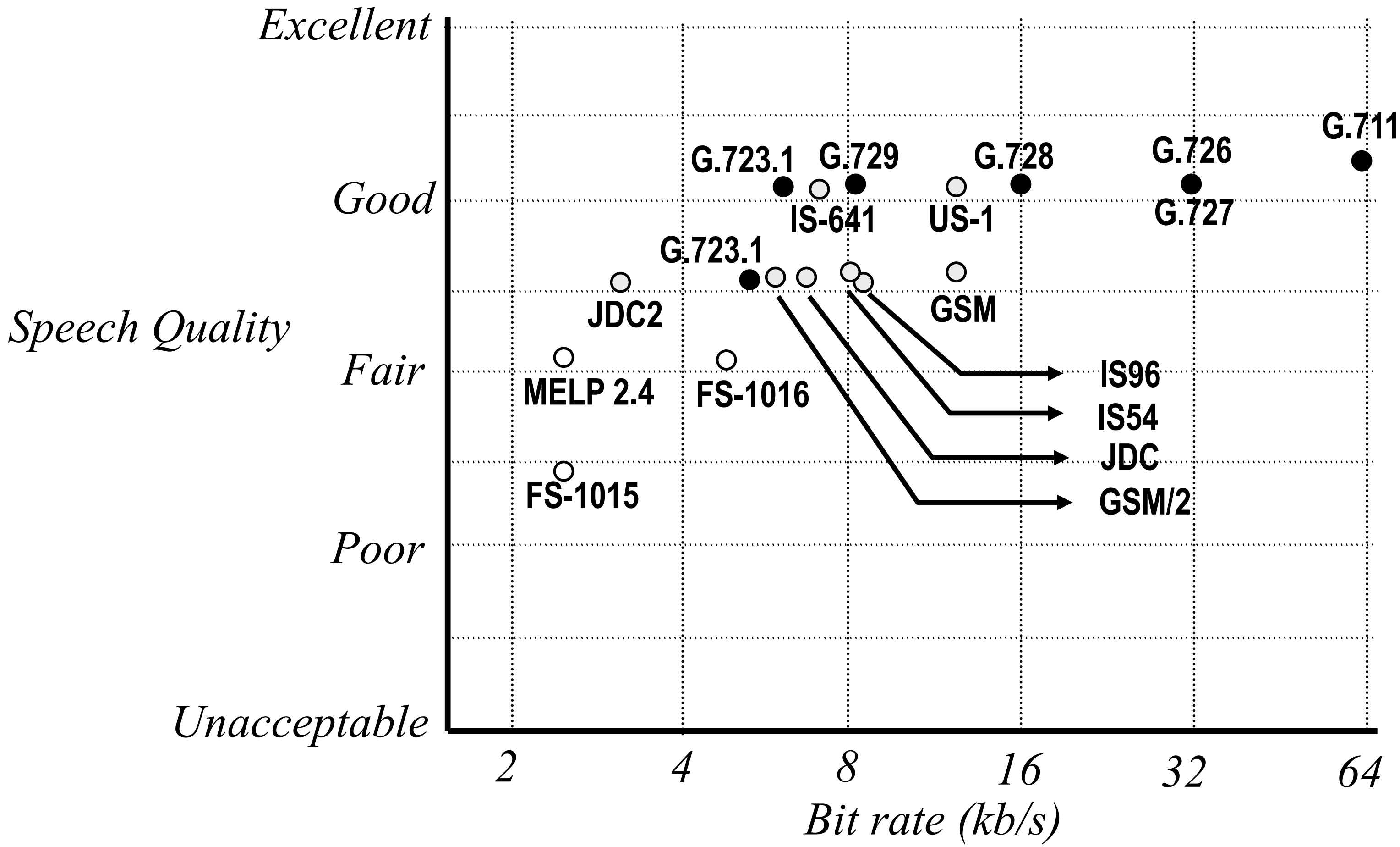


Multi-pulse excited linear predictive codec



Code-excited linear predictive codec (CELP)

(Further enhancement of the speech compression rate below 8 kbps)



Source: IEEE Communications Magazine, September 1997.

ITU Audio Standards

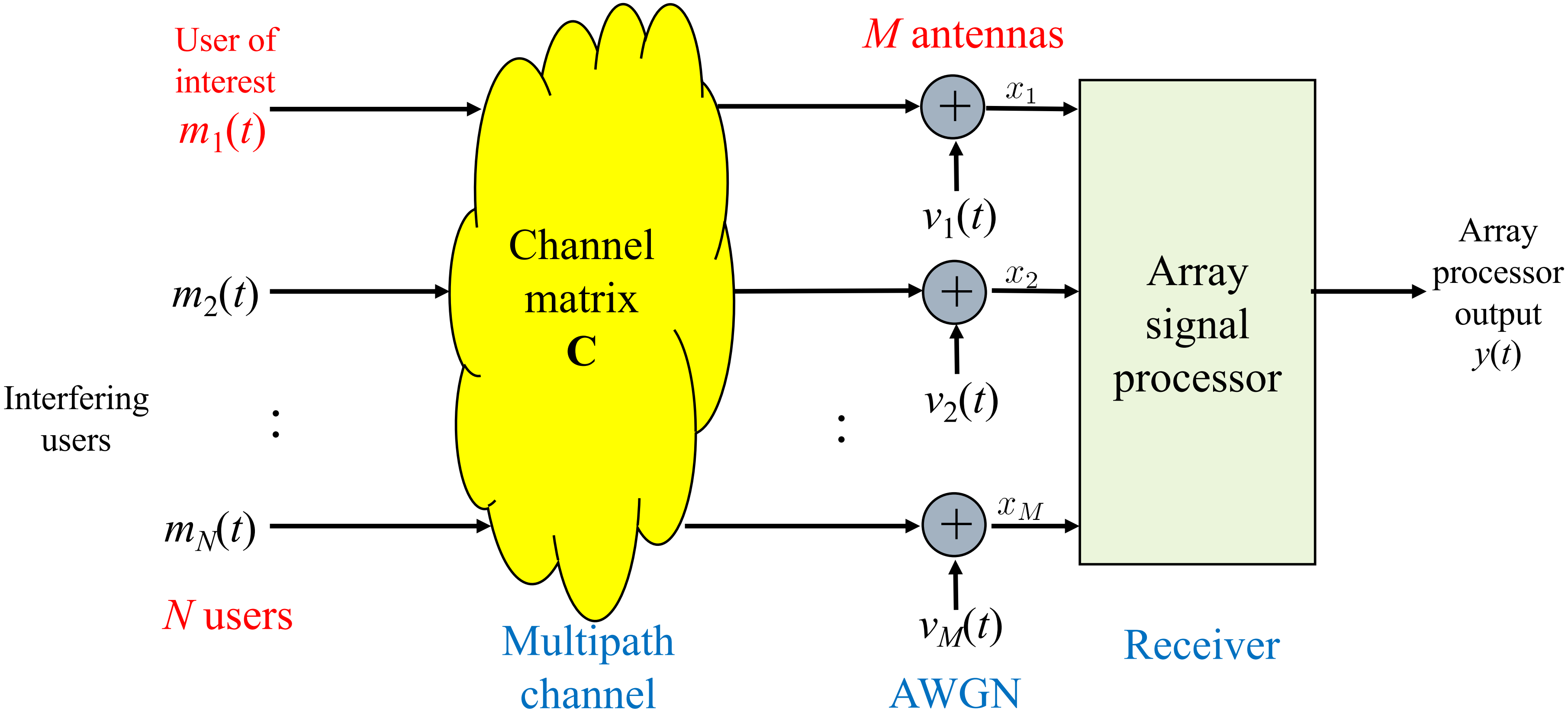
Date	*Standard*	*Rate*	*Technique*	*Frame Size /Look ahead*	*Computation complexity /RAM size*
1965	*G.711*	*64kbps*	*PCM*	*0.125ms/0*	*---/---*
1992	*G.728*	*16kpbs*	*LD-CELP*	*0.625ms/0*	*30 MIPs/---*
1995	*G.723.1*	*5.33/6.4kbps*	*MP-MLQ /ACELP**	*30ms/7.5ms*	*16 MIPs/2200 words*
1995	*G.729*	*8kbps*	*CS-ACELP*	*10ms/5ms*	*20 MIPs/3000 words*
1996	*G.729.A*	*8kbps*	*CS-ACELP*	*10ms/5ms*	*10.5 MIPs/2000 words*

** MP-MLQ for higher bit rate; ACELP for lower bit rate.*

Standard	*Patents*	*Owners*	*Owner List*
G.723.1	*17*	*8*	*AT&T(1), Lucent(3), NTT(3), VoiceCraft(2)*
G.729.A	*20*	*6*	*AT&T(1), France Telecom(1), Lucent(1), NTT(1), Universite DE Sherbrooke(1), VocieCraft(1)*
G.729	*20*	*6*	*AT&T(1), France Telecom(1), Lucent(1), NTT(1), Universite DE Sherbrooke(1), VocieCraft(1)*

Adaptive Antenna Arrays for Wireless Communications

- Antenna arrays can be regarded as space diversity.



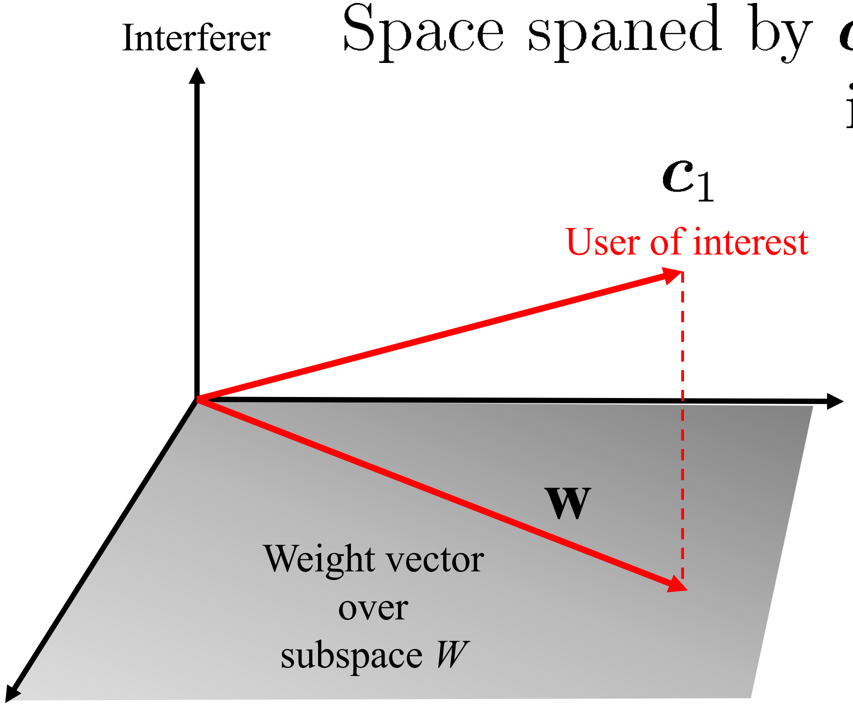
For simplicity, we assume in the sequel that the vectorization of the signals are all real.

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix}_{M\times 1} = \begin{bmatrix} \boldsymbol{c}_1 & \boldsymbol{c}_2 & \cdots & \boldsymbol{c}_N \end{bmatrix}_{M\times N} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_N \end{bmatrix}_{N\times 1} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}_{M\times 1}$$

Give that $\begin{bmatrix} \boldsymbol{c}_1 & \boldsymbol{c}_2 & \cdots & \boldsymbol{c}_N \end{bmatrix}_{M\times N}$ can be perfectly estimated.

Space spaned by $\boldsymbol{c}_2, \cdots, \boldsymbol{c}_N$

i.e., $a_2\boldsymbol{c}_2 + \cdots + a_N\boldsymbol{c}_N$



$\boldsymbol{c}_1 \neq a_2\boldsymbol{c}_2 + \cdots + a_N\boldsymbol{c}_N$
for any $a_2, \cdots, a_N$

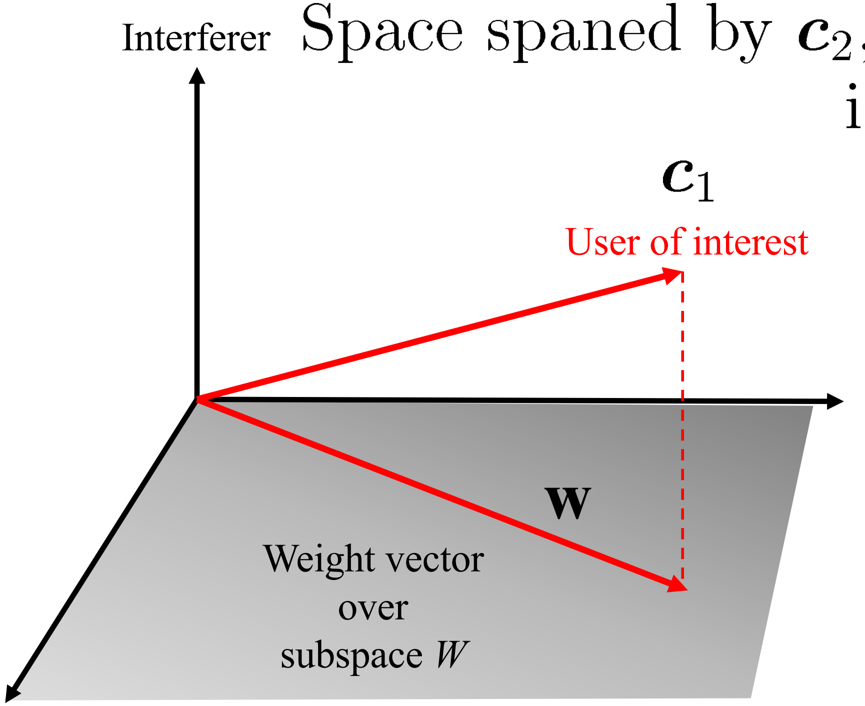
Subspace $\mathcal{W}$ is the space that consists of vectors that are orthogonal to the space spanded by $\boldsymbol{c}_2, \cdots, \boldsymbol{c}_N$.

For any weight vector $\mathbf{w} \in \mathcal{W}$, we have

$$\boldsymbol{w} \cdot \boldsymbol{c}_j = 0 \text{ for } 2 \leq j \leq N.$$

Space spaned by $\boldsymbol{c}_2, \cdots, \boldsymbol{c}_N$

i.e., $a_2\boldsymbol{c}_2 + \cdots + a_N\boldsymbol{c}_N$



$\boldsymbol{c}_1 \neq a_2\boldsymbol{c}_2 + \cdots + a_N\boldsymbol{c}_N$
for any $a_2, \cdots, a_N$

$$\boldsymbol{w}\cdot\begin{bmatrix}x_1\\x_2\\\vdots\\x_M\end{bmatrix}_{M\times 1} = \boldsymbol{w}\cdot\begin{bmatrix}\boldsymbol{c}_1 & \boldsymbol{c}_2 & \cdots & \boldsymbol{c}_N\end{bmatrix}_{M\times N}\begin{bmatrix}m_1\\m_2\\\vdots\\m_N\end{bmatrix}_{N\times 1} + \boldsymbol{w}\cdot\begin{bmatrix}v_1\\v_2\\\vdots\\v_N\end{bmatrix}_{M\times 1}$$

$$= \begin{bmatrix}\boldsymbol{w}\cdot\boldsymbol{c}_1 & 0 & \cdots & 0\end{bmatrix}_{M\times N}\begin{bmatrix}m_1\\m_2\\\vdots\\m_N\end{bmatrix}_{N\times 1} + \boldsymbol{w}\cdot\begin{bmatrix}v_1\\v_2\\\vdots\\v_N\end{bmatrix}_{M\times 1}$$

$$= \boldsymbol{w}\cdot\boldsymbol{c}_1 m_1 + \boldsymbol{w}\cdot\boldsymbol{v}$$

$$\text{The projection that maximizes SNR} = \arg\max_{\boldsymbol{w}\in\mathcal{W}}\frac{E[(\boldsymbol{w}\cdot\boldsymbol{c}_1 m_1)^2]}{E[(\boldsymbol{w}\cdot\boldsymbol{v})^2]}$$

$$= \arg\max_{\boldsymbol{w}\in\mathcal{W}}\frac{(\boldsymbol{w}\cdot\boldsymbol{c}_1)^2 E[m_1^2]}{(\boldsymbol{w}\cdot\boldsymbol{w})(N_0/2)}$$

Cauchy-Schwarz inequality:

$$\frac{(\boldsymbol{w}\cdot\boldsymbol{c}_1)^2 E[m_1^2]}{(\boldsymbol{w}\cdot\boldsymbol{w})(N_0/2)} \le \frac{(\boldsymbol{w}\cdot\boldsymbol{w})(\boldsymbol{c}_1\cdot\boldsymbol{c}_1)E[m_1^2]}{(\boldsymbol{w}\cdot\boldsymbol{w})(N_0/2)} = \frac{(\boldsymbol{c}_1\cdot\boldsymbol{c}_1)E[m_1^2]}{(N_0/2)}$$

with equality holding if, and only if, $\boldsymbol{w} = a\boldsymbol{c}_1$.

Hence, if $\boldsymbol{c}_1 \in \mathcal{W}$,

$$\text{projection maximizing SNR} = \arg\max_{\boldsymbol{w}\in\mathcal{W}} \frac{(\boldsymbol{w}\cdot\boldsymbol{c}_1)^2 E[m_1^2]}{(\boldsymbol{w}\cdot\boldsymbol{w})(N_0/2)} = \boldsymbol{c}_1$$

(Match filter principle)

However, if $\boldsymbol{c}_1 \notin \mathcal{W}$,

we must use the one that is the most alike to the match filter one in order to maximize the output SNR, which is the projection of $\boldsymbol{c}_1$ onto $\mathcal{W}$ that is closest to $\boldsymbol{c}_1$.

Example. What is the dimension of $\mathcal{W}$, if

$\boldsymbol{c}_1, \boldsymbol{c}_2, \cdots, \boldsymbol{c}_N$ are linearly independent?

Answer:

Dimension of $\boldsymbol{c}_2, \cdots, \boldsymbol{c}_N = N - 1$.

Each $\boldsymbol{c}_j$ is $(M \times 1)$ vector.

Dimension of orthogonal space of $\boldsymbol{c}_2, \cdots, \boldsymbol{c}_N = M - (N - 1)$.

Adaptive Antenna Arrays for Wireless Communications

- Adaptive antenna array
  - To adaptively adjust the weights so that the error signal (namely, the difference between resultant signal and reference signal) is essentially zero.
  - Detail theoretical background can be found in Section 4.10.