Problems for the 10th Quiz

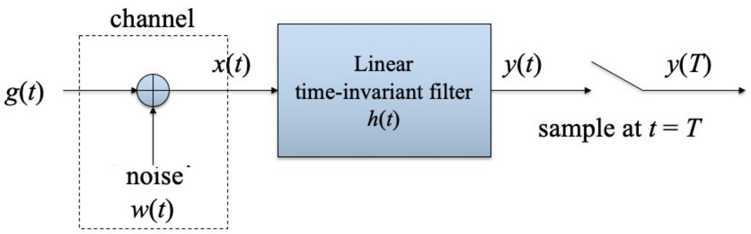
1. (25%) Which adaptive quantization scheme uses **unquantized** samples to adjust the quantization level $\Delta[n]$, AQF or AQB?

   Hint: AQF = Adaptive Quantization Forward and AQB = Adaptive Quantization Backward

   Solution. AQF

2. For the one-shot transmission of message a illustrated below, it can be derived that

$$y(T) = a \cdot \int_{-\infty}^{\infty} H(f)G(f)e^{j2\pi fT}df + \underbrace{\int_{-\infty}^{\infty} w(\tau)h(T-\tau)d\tau}_{=n(t)|_{t=T}}.$$



   (a) (25%) Given that $w(t)$ is WSS with PSD $S_w(f) = J_0 \cdot \delta(f)$, what should be filled inside the parentheses below:

$$E[n^2(T)] = J_0 \cdot (\quad).$$

   Hint: The PSD of $n(t)$ is $S_n(f) = S_w(f)|H(f)|^2$.

   (b) (25%) Continue from (a). Suppose $G(f) = T_b \operatorname{sinc}(T_b f)\, e^{-j\pi f T_\text{b}}$. From the SNR_O formula at the sampler output below,

$$\text{SNR}_\text{O} = \frac{\left|\int_{-\infty}^{\infty} G(f)H(f)e^{j2\pi fT}\text{d}f\right|^2}{\underbrace{\int_{-\infty}^{\infty} S_w(f)|H(f)|^2\text{d}f}_{=E[n^2(T)]}},$$

   is it theoretically possible to find a $H(f)$ such that $\text{SNR}_\text{O} = \infty$? Your answer should be either "YES" or "NO".

   (c) (25%) If your answer is "YES" in (c), what is the value of $H(0)$ that makes $\text{SNR}_\text{O} = \infty$? If your answer is "NO" in (c), what is the value of the maximum SNR_O?

Solution.

   (a) The WSS assumption of $w(t)$ implies

$$E[n^2(T)] = \int_{-\infty}^{\infty} S_n(f)\text{d}f = \int_{-\infty}^{\infty} S_w(f)|H(f)|^2\text{d}f = \int_{-\infty}^{\infty} J_0\,\delta(f)|H(f)|^2\text{d}f = J_0\,|H(0)|^2.$$

   (b) YES.

   Note: Any $H(f)$ satisfying $H(0) = 0$ and $\left|\int_{-\infty}^{\infty} G(f)H(f)e^{j2\pi fT}df\right|^2 \neq 0$ can make $\text{SNR}_\text{O} = \infty$. This indicates if $S_w(f)$ is zero for certain range of f, then $\text{SNR}_\text{O} = \infty$ is achievable.

   (c) $H(0) = 0$