Problems for the 5th Quiz (April 8, 2021)

Name:________________ Student ID:________________ Score:____________

1. The modulated signal for DSB-C is given by $s(t) = [1 + k_a\, m(t)]\cos(2\pi f_c t)$.

   (a) (40%) Find the Fourier transform of $s(t)$.
   Hint:
   $$\begin{aligned} S(f) &= \mathcal{F}\{\cos(2\pi f_c t) + k_a\, m(t)\cos(2\pi f_c t)\} \\ &= \mathcal{F}\{\cos(2\pi f_c t)\} + k_a\,\mathcal{F}\{m(t)\} \star \mathcal{F}\{\cos(2\pi f_c t)\} = \cdots \end{aligned}$$
   where $\star$ denotes the convolution operation; $\mathcal{F}\{\cdot\}$ denotes the Fourier transform operation; $\mathcal{F}\{m(t)\} = M(f)$; and $\mathcal{F}\{\cos(2\pi f_c t)\} = \frac{1}{2}(\delta(f - f_c) + \delta(f + f_c))$.

   (b) (30%) Now suppose $m(t) = \cos(2\pi f_m t)$. What is the range of the modulation index k_a over which no overmodulation for DSB-C occurs?
   Note: k_a must be a positive number.

   (c) (30%) Will the answer be the same as (b) if one requires $|k_a m(t)| \le 1$ to ensure no overmodulation for DSB-C?

Solution.

(a)

$$\begin{aligned} S(f) &= \mathcal{F}\{\cos(2\pi f_c t) + k_a\, m(t)\cos(2\pi f_c t)\} \\ &= \mathcal{F}\{\cos(2\pi f_c t)\} + k_a\,\mathcal{F}\{m(t)\} \star \mathcal{F}\{\cos(2\pi f_c t)\} \\ &= \frac{1}{2}(\delta(f - f_c) + \delta(f + f_c)) + k_a\, M(f) \star \frac{1}{2}(\delta(f - f_c) + \delta(f + f_c)) \\ &= \frac{1}{2}(\delta(f - f_c) + \delta(f + f_c)) + \frac{k_a}{2}(M(f - f_c) + M(f + f_c)) \end{aligned}$$

(b) Overmodulation for DSB-C occurs when $1 + k_a m(t) < 0$. Thus, no overmodulation dictates

$$0 \le 1 + k_a m(t) = 1 + k_a \cos(2\pi f_m t)$$

which implies that when $\min_t\{1 + k_a\cos(2\pi f_m t\} = 1 - k_a \ge 0$, no overmodulation occurs. Hence, the answer of this problem is $0 < k_a \le 1$.

(c) The problem statement may be too conservative. So I provide two "correct" solutions.

It is clear that

$$1 \ge |k_a m(t)| = |k_a \cos(2\pi f_m t)|$$

dictates $k_a \le 1$. Hence, the answer is the same as (b).

It is clear that

$$1 > |k_a m(t)| = |k_a \cos(2\pi f_m t)|$$

dictates $k_a < 1$. Hence, the answer is a little different from (b).

Note: When $m(t)$ is balanced in the sense that

$$\max_t m(t) + \min_t m(t) = 0,$$

the criterion in (b) and the criterion in (c) give the same answer. This justifies the adoption of $|km(t)| \le 1$ as the design criterion of DSB-C in practice.