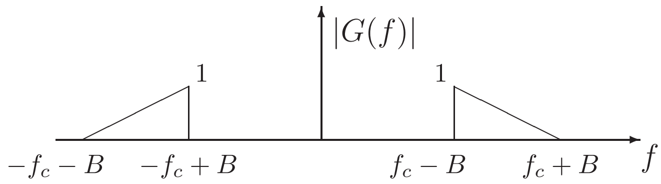
Problems for Quiz 4 (March 29, 2021)

Name:____________________ Student ID:____________________ Score:________________



In the following problems, we will use the shorthand that $\mathcal{H}\{\cdot\}$ and $\mathcal{F}\{\cdot\}$ denote the Hilbert transform and the Fourier transform, respectively. Let $g(t)$ be a real-valued function.

1. (40%) Draw the magnitude $|\hat{G}(f)|$, where $\mathcal{H}\{g(t)\} = \hat{g}(t)$, $\mathcal{F}\{g(t)\} = G(f)$ and $\mathcal{F}\{\hat{g}(t)\} = \hat{G}(f)$.

   Hint: $H_{\text{Hilbert}}(f) = -j\text{sgn}(f)$.

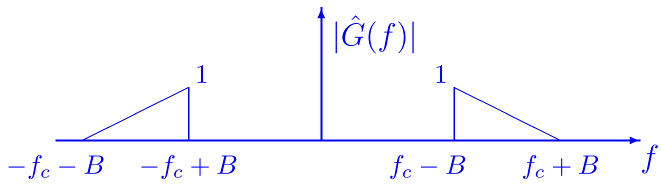
2. (30%) Draw $|P(f)|$, where $p(t) = g(t) + j\frac{1}{2}\hat{g}(t)$.

   Hint: $P(f) = \mathcal{F}\{p(t)\} = \mathcal{F}\{g(t) + j\frac{1}{2}\hat{g}(t)\} = \mathcal{F}\{g(t)\} + j\frac{1}{2}\mathcal{F}\{\hat{g}(t)\} = G(f) + j\frac{1}{2}\hat{G}(f)$

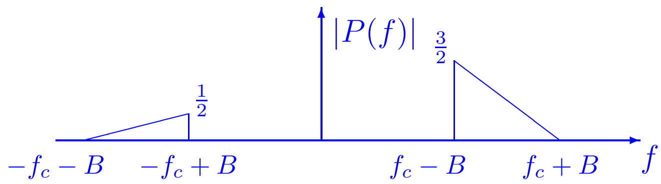
3. (30%) Draw $|Q(f)|$, where $q(t) = g(t) + j\frac{3}{4}\hat{g}(t)$.

Solution.

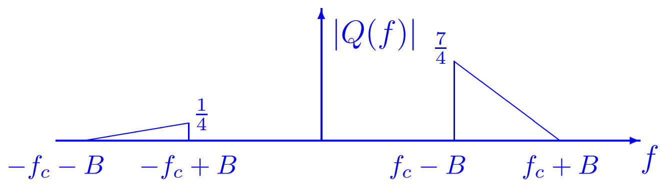
1. Since $|\hat{G}(f)| = |-j\text{sgn}(f)G(f)| = |G(f)|$ for $f \neq 0$, we have:



2. $P(f) = G(f)+j\frac{1}{2}\hat{G}(f) = G(f)+j\frac{1}{2}(-j\text{sgn}(f))G(f) = (1+\frac{1}{2}\text{sgn}(f))G(f) = \begin{cases} \frac{3}{2}G(f), & f > 0 \\ \frac{1}{2}G(f), & f < 0 \end{cases}$



3. $Q(f) = (1 + \frac{3}{4}\text{sgn}(f))G(f) = \begin{cases} \frac{7}{4}G(f), & f > 0 \\ \frac{1}{4}G(f), & f < 0 \end{cases}$



Note: From this problem, you learn that one can use $j\hat{g}(t)$ to shape the relative sizes of the positive spectrum and the negative spectrum. In the extreme case, where $j\hat{g}(t)$ is added to $g(t)$, the negative spectrum can be completely nullified.