Quiz 1 (March 8, 2021) : Introduction to Communication Systems

Name:__________________ Student ID:__________________ Score:__________________

1. (a) (40%) Give the conditions on the mean function $\mu_X(t)$ and the autocorrelation function $R_X(t_1, t_2)$, which define a wide-sense stationary (WSS) process $X(t)$.

   (b) (30%) Name a reason why $g(\tau) = \tau^3$ cannot be the autocorrelation function $R_X(\tau)$ of some WSS process $X(t)$.

   (c) (30%) Fix a complex WSS process $X(t)$ with mean function $\mu_X(t)$, autocorrelation function $R_X(t_1, t_2)$ and autocovariance function $C_X(t_1, t_2)$. Given that $R_X(3, 3.2) = 2+j$ and $\mu_X(3) = 1$, what is $C_X(2.7, 2.5)$?

   Hint: Autocorrelation function equals sum of autocovariance function and product of the mean function.

Solution.

(a) $\mu_X(t)$ is a constant and $R_X(t_1, t_2) = R_X(t_1 - t_2)$ is only a function of time difference $(t_1 - t_2)$.

(b) $g(\tau)$ cannot be the autocorrelation function of some WSS process because it is not conjugate symmetric.

Note: Alternatively, you may say $g(\tau)$ cannot be the autocorrelation function of some WSS process because the absolute value of its real part does not peak at the origin.

(c) For a WSS process, $C_X(t_1 - t_2) = R_X(t_1 - t_2) - |\mu_X|^2$. With $t_1 = 2.7$ and $t_2 = 2.5$, we obtain

$$\begin{aligned} C_X(2.7, 2.5) = C_X(0.2) &= R_X(0.2) - 1 \\ &= R_X^*(-0.2) - 1 \\ &= 2 - j - 1 \\ &= 1 - j \end{aligned}$$