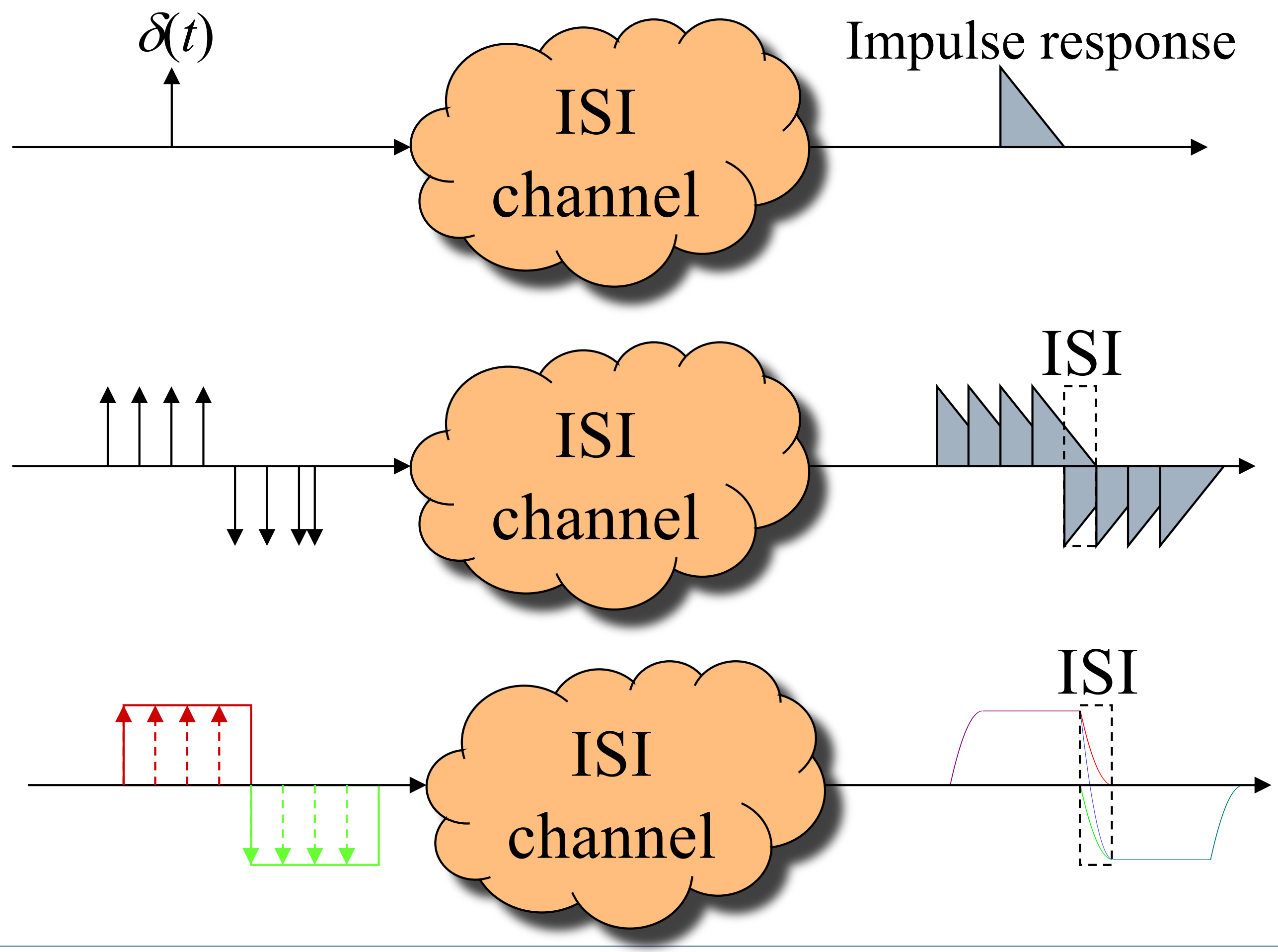
Part 8 Techniques to Compensate for Intersymbol Interference and AWGN

Introduction

- Transmission of *digital data* (bit stream) over a *noisy* baseband channel typically suffers two channel imperfections
  - Intersymbol interference (ISI)
  - Background noise (e.g., AWGN)
- These two interferences/noises often occur simultaneously.
- However, for simplicity, they are often separately considered in analysis.

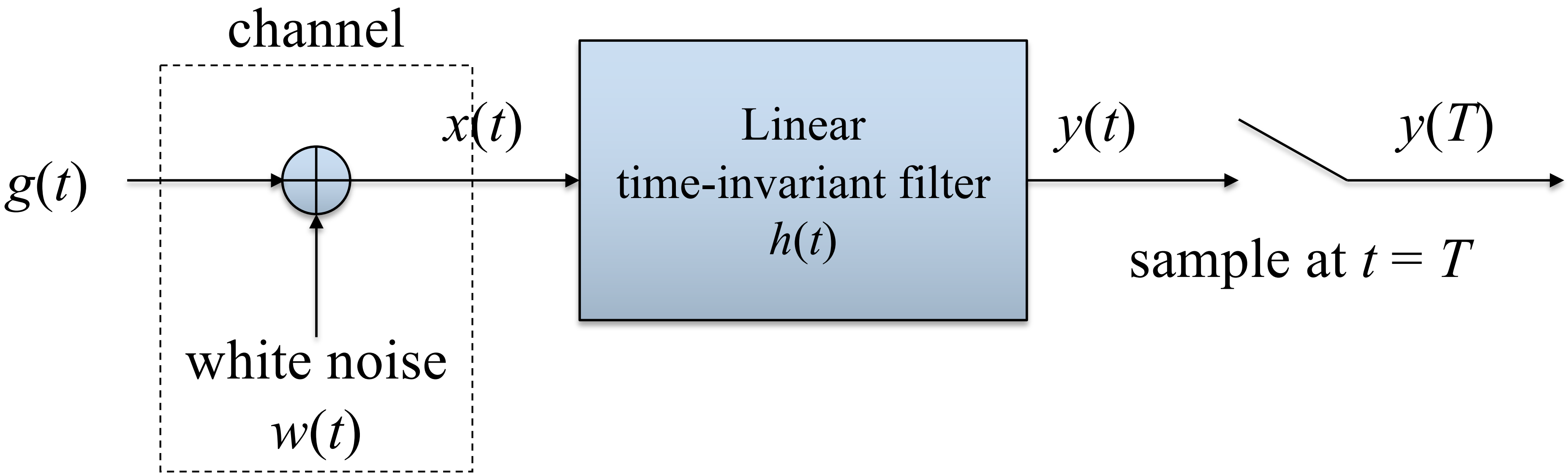
ISI



$\delta(t)$
ISI channel
Impulse response
ISI channel
ISI
ISI channel
ISI

Matched Filter

- Matched filter is a device for the optimal detection of a digital pulse. It is so named because the *impulse response* of the matched filter matches the *pulse shape*.
- System model without ISI



Design Criterion

- To find $h(t)$ such that the output signal-to-noise ratio SNR_O is maximized.

$$x(t) = g(t) + w(t) \text{ for } 0 \le t < T$$

$$\begin{aligned} y(t) &= [g(t) + w(t)] * h(t) \\ &= g(t) * h(t) + w(t) * h(t) \\ &= g_o(t) + n(t) \end{aligned}$$

$$SNR_O = \frac{|g_o(T)|^2}{E[n^2(T)]}$$

Analysis of Matched Filter

$$g_o(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)df$$

$$\Rightarrow |g_o(T)|^2 = \left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df\right|^2$$

With $w(t)$ being white with PSD $N_0/2$,

$$S_N(f) = S_W(f)|H(f)|^2 = \frac{N_0}{2}|H(f)|^2$$

$$\Rightarrow E[n^2(T)] = \int_{-\infty}^{\infty} S_N(f)df = \frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\, df$$

Analysis of Matched Filter

$$\Rightarrow \eta = \frac{\left|\int_{-\infty}^{\infty} G(f)H(f)\exp(j2\pi fT)df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df}$$

Cauchy-Schwarz inequality

$$\begin{aligned}|\langle\psi_1(x),\psi_2(x)\rangle|^2 &= \left|\int_{-\infty}^{\infty}\psi_1(x)\psi_2^*(x)dx\right|^2\\ &\le \left(\int_{-\infty}^{\infty}|\psi_1(x)|^2dx\right)\left(\int_{-\infty}^{\infty}|\psi_2(x)|^2dx\right) = \langle\psi_1(x),\psi_1(x)\rangle\langle\psi_2(x),\psi_2(x)\rangle\end{aligned}$$

with equality holding if, and only if, $\psi_1(x) = k\cdot\psi_2(x)$ for some constant k.

Analysis of Matched Filter

- By Cauchy-Schwarz inequality,

$$\left|\int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\, df \cdot \int_{-\infty}^{\infty} |G(f)\exp(j2\pi fT)|^2\, df$$

$$\Rightarrow \eta \le \frac{\int_{-\infty}^{\infty} |H(f)|^2\, df \cdot \int_{-\infty}^{\infty} |G(f)|^2\, df}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df} = \frac{2}{N_0}\int_{-\infty}^{\infty} |G(f)|^2\, df$$

This is a constant bound, independent of the choice of $h(t)$. Hence, the optimal η is achieved by:

$$\boxed{H(f) = k \cdot G^*(f)\exp(-j2\pi fT)}$$

Analysis of Matched Filter

$$\begin{aligned} h_{\text{opt}}(t) &= \int_{-\infty}^{\infty} k \cdot G^*(f) \exp(-j2\pi fT) \exp(j2\pi ft) df \\ &= k \left(\int_{-\infty}^{\infty} G(f) \exp(j2\pi f(T-t)) df \right)^* \\ &= kg^*(T-t). \end{aligned}$$

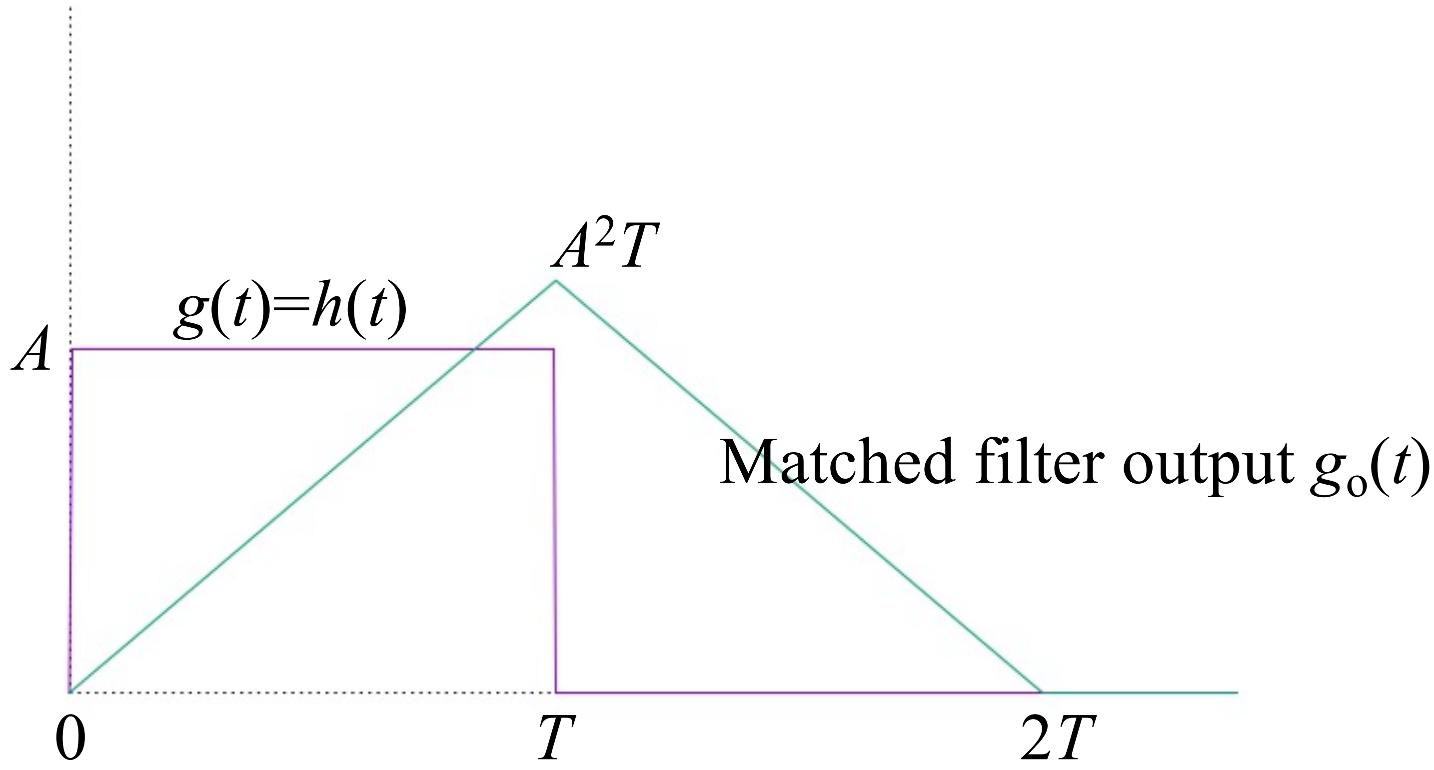
- Hence, under additive white noise, the *optimal received filter* matches the input signal in the sense that it is a time-inversed and delayed version of the complex-conjugated input signal $g(t)$.

Properties of Matched Filter

- The maximum output signal-to-noise ratio only depends on the energy of the input, and has nothing to do with the pulse shape itself.
  - Namely, whether the pulse shape is sinusoidal, rectangular, triangular, etc is irrelevant to the maximum output signal-to-noise ratio, as long as these pulse shapes have the same energy.

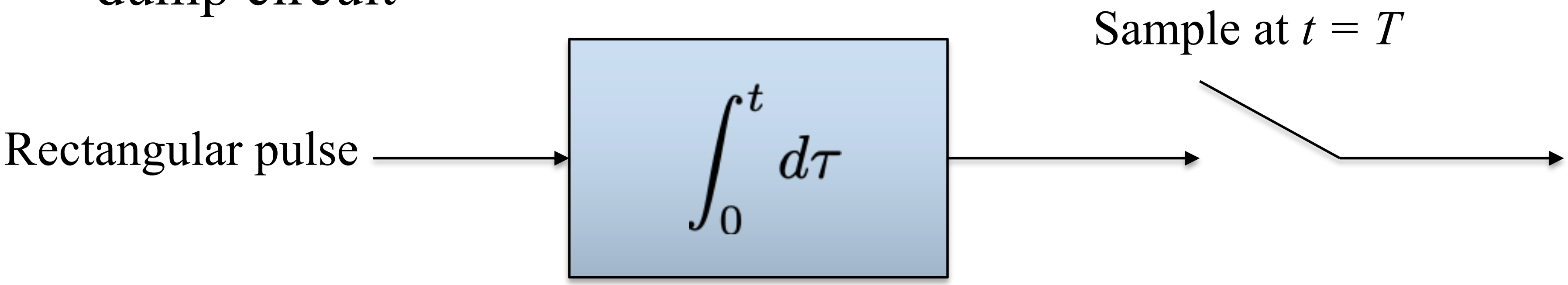
$$\eta_{\max} = \frac{2E_s}{N_0}, \text{ where } E_s = \int_{-\infty}^{\infty} |G(f)|^2 \, df.$$

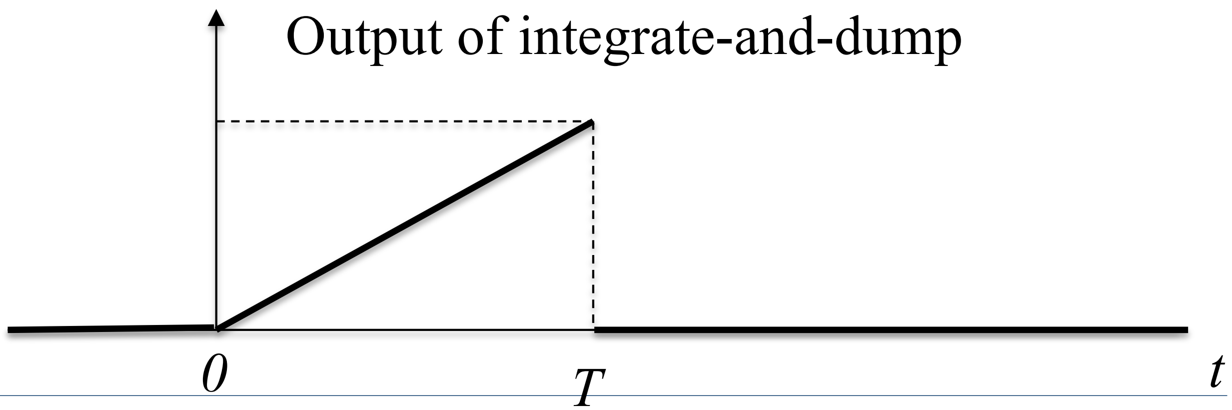
Matched Filter for Rectangular Pulse



Matched Filter for Rectangular Pulse

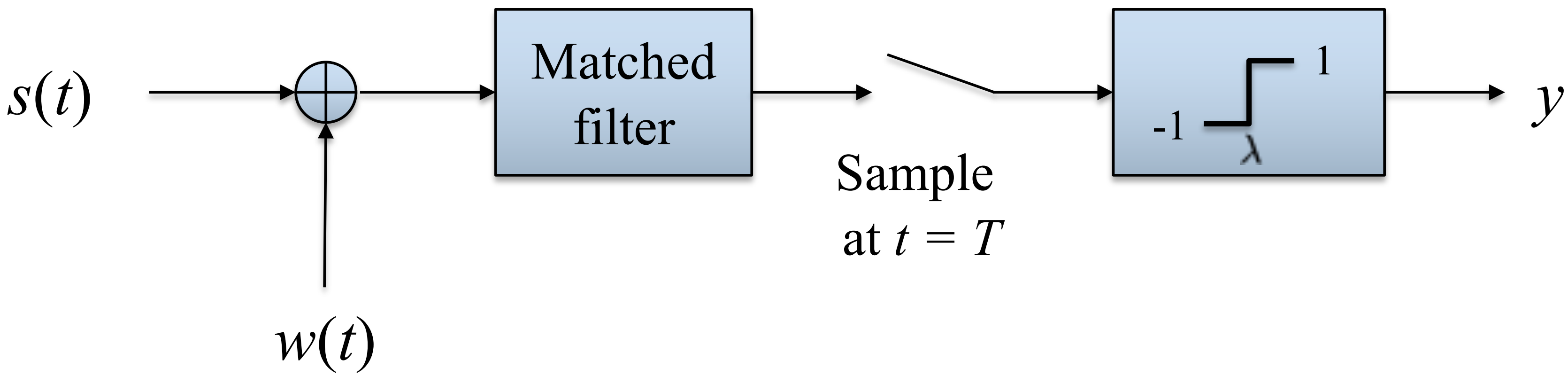
- $h_{opt}(t)$ in this example can be implemented as integrate-and-dump circuit





Error Rate due to Noise

- In what follows, we analyze the error rate of *polar non-return-to-zero* (NRZ) *signaling* in a system with optimal matched filter receiver over AWGN channel.



$s(t) = I \cdot g(t)$, where $I \in \{-1,+1\}$.

$$y(T) = [I \cdot g(t)] * h(t)\Big|_{t=T} + w(t) * h(t)\Big|_{t=T}$$

$$= I \cdot \int_{-\infty}^{\infty} h(\tau) g(T-\tau) d\tau + \int_{-\infty}^{\infty} h(\tau) w(T-\tau) d\tau$$

$$= I \cdot \int_{-\infty}^{\infty} k g^*(T-\tau) g(T-\tau) d\tau + \int_{-\infty}^{\infty} k g^*(T-\tau) w(T-\tau) d\tau$$

$$= I \cdot k \int_{-\infty}^{\infty} |g(\tau)|^2 \, d\tau + k \int_{-\infty}^{\infty} g^*(\tau) w(\tau) d\tau$$

$$= I \cdot kE_g + kn, \text{ where } E_g = \int_{-\infty}^{\infty} |g(\tau)|^2 \, d\tau \text{ and } n = \int_{-\infty}^{\infty} g^*(\tau) w(\tau) d\tau.$$

For notational convenience, we brief $y(T)/k$ by y.

Note: The integration can be taken over $[0,T)$ since $g(t)$ is zero outside this range (as text does). I, however, use the entire real line as the integration range here for convenience.

By AWGN assumption of $w(t)$ and real $g(t)$ assumption,

$n = \int_{-\infty}^{\infty} g^*(\tau)w(\tau)d\tau$ is Gaussian distributed with

$$E[n] = \int_{-\infty}^{\infty} g^*(\tau)E[w(\tau)]d\tau = 0.$$

$$\begin{aligned} E[n^2] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(s)g(t)E[w(s)w(t)]dsdt \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(s)g(t)\frac{N_0}{2}\delta(s-t)dsdt \\ &= \frac{N_0}{2}\int_{-\infty}^{\infty} g^2(s)ds = \frac{N_0}{2}E_g \end{aligned}$$

$$y = I\cdot E_g + n \Rightarrow \begin{cases} \phi_{+1}(y) = Normal(E_g, E_gN_0/2), \text{ if } I = +1; \\ \phi_{-1}(y) = Normal(-E_g, E_gN_0/2), \text{ if } I = -1 \end{cases}$$

Let Ψ be the set, where a decision favoring $+1$ is made.

$$
\begin{aligned}
BER &= \Pr[I=+1]\Pr\{\text{guess}\,(-1)\mid I=+1\}+\Pr[I=-1]\Pr\{\text{guess}\,(+1)\mid I=-1\}\\
&= \Pr[I=+1]\Pr\{y\notin\Psi\mid I=+1\}+\Pr[I=-1]\Pr\{y\in\Psi\mid I=-1\}\\
&= p\left(1-\Pr\{y\in\Psi\mid I=+1\}\right)+(1-p)\Pr\{y\in\Psi\mid I=-1\}\\
&= p+(1-p)\Pr\{y\in\Psi\mid I=-1\}-p\Pr\{y\in\Psi\mid I=+1\}\\
&= p+\int_{\Psi}[(1-p)\phi_{-1}(y)-p\phi_{+1}(y)]dy,\ \text{where } p=\Pr[I=+1].
\end{aligned}
$$

To minimize BER, the optimal set $\Psi_{\text{opt}}=\{y\in\Re:(1-p)\phi_{-1}(y)-p\phi_{+1}(y)<0\}$.

Thus, the optimal decision maker is given by :

$$
d(y)=\begin{cases}+1, & (1-p)\phi_{-1}(y)<p\phi_{+1}(y)\\ -1, & (1-p)\phi_{-1}(y)\ge p\phi_{+1}(y)\end{cases}.
$$

$$\begin{cases} \phi_{+1}(y) = Normal(E_g, E_g N_0 / 2), \text{ if } I = +1; \\ \phi_{-1}(y) = Normal(-E_g, E_g N_0 / 2), \text{ if } I = -1 \end{cases}$$

Let $\mu = E_g$ and $\sigma^2 = E_g N_0 / 2$.

$$\frac{(1-p)}{p} \underset{-1}{\overset{+1}{\lessgtr}} \frac{\phi_{+1}(y)}{\phi_{-1}(y)} = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(y-\mu)^2}{2\sigma^2} \right\}}{\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(y+\mu)^2}{2\sigma^2} \right\}}$$

$$= \exp\left\{ \frac{2\mu y}{\sigma^2} \right\} = \exp\left\{ \frac{2E_g y}{E_g N_0 / 2} \right\} = \exp\left\{ \frac{4y}{N_0} \right\}$$

$$y \underset{-1}{\overset{+1}{\gtrless}} \frac{N_0}{4} \log\left[\frac{(1-p)}{p} \right]$$

This threshold depends on N_0; hence, the best decision relies on the accuracy of N_0 estimate.

Error Rate due to Noise under Uniform Input

- In order to free the system dependence on N_0 estimate, a uniform I is transmitted in which case, $p = ½$.

- The best decision now becomes $y \underset{-1}{\overset{+1}{\gtrless}} 0$.

$$
\begin{aligned}
BER_{\text{opt}} &= \frac{1}{2}\int_0^{\infty}\phi_{-1}(y)dy + \frac{1}{2}\int_{-\infty}^{0}\phi_{+1}(y)dy \\
&= \frac{1}{2}\int_0^{\infty}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(y+\mu)^2}{2\sigma^2}\right\}dy \\
&\quad + \frac{1}{2}\int_{-\infty}^{0}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\}dy
\end{aligned}
$$

$$BER_{\text{opt}} = \frac{1}{2}\int_{\mu}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{y^2}{2\sigma^2}\right\}dy$$

$$+\frac{1}{2}\int_{-\infty}^{-\mu} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{y^2}{2\sigma^2}\right\}dy$$

$$= \int_{\mu}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{y^2}{2\sigma^2}\right\}dy,\ z = \frac{y}{\sqrt{2\sigma^2}}$$

$$= \frac{1}{\sqrt{\pi}}\int_{\mu/\sqrt{2\sigma^2}}^{\infty} \exp\{-z^2\}dz$$

$$= \frac{1}{2}\operatorname{erfc}\left(\frac{\mu}{\sqrt{2\sigma^2}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_g}{N_0}}\right)$$

where $\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}}\int_{u}^{\infty} \exp(-z^2)dz$ is the complementary error function.

Error Function

- Error function $\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-z^2)dz$
- Complementary error function $\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)dz$
- Q-function $Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp\left(-\frac{z^2}{2}\right)dz$

$$\begin{cases} \operatorname{erf}(-u) = -\operatorname{erf}(u) \\ \operatorname{erfc}(u) = 1 - \operatorname{erf}(u) \\ Q(u) = \frac{1}{2}\operatorname{erfc}\left(\frac{u}{\sqrt{2}}\right) \end{cases}$$

Error Function

- Bounds for error function

$$\operatorname{erfc}(x) = \frac{1}{x\sqrt{\pi}} e^{-x^2} \left(1 - \frac{1}{2x^2} + \frac{1 \cdot 3}{2^2 x^4} - \frac{1 \cdot 3 \cdot 5}{2^3 x^6} + \cdots \right)$$

For $x > 0$, $\frac{1}{x\sqrt{\pi}} e^{-x^2} \left(1 - \frac{1}{2x^2} \right) < \operatorname{erfc}(x) < \frac{1}{x\sqrt{\pi}} e^{-x^2}$

(The two bounds are good when x is large.)

Symbol Error Rate

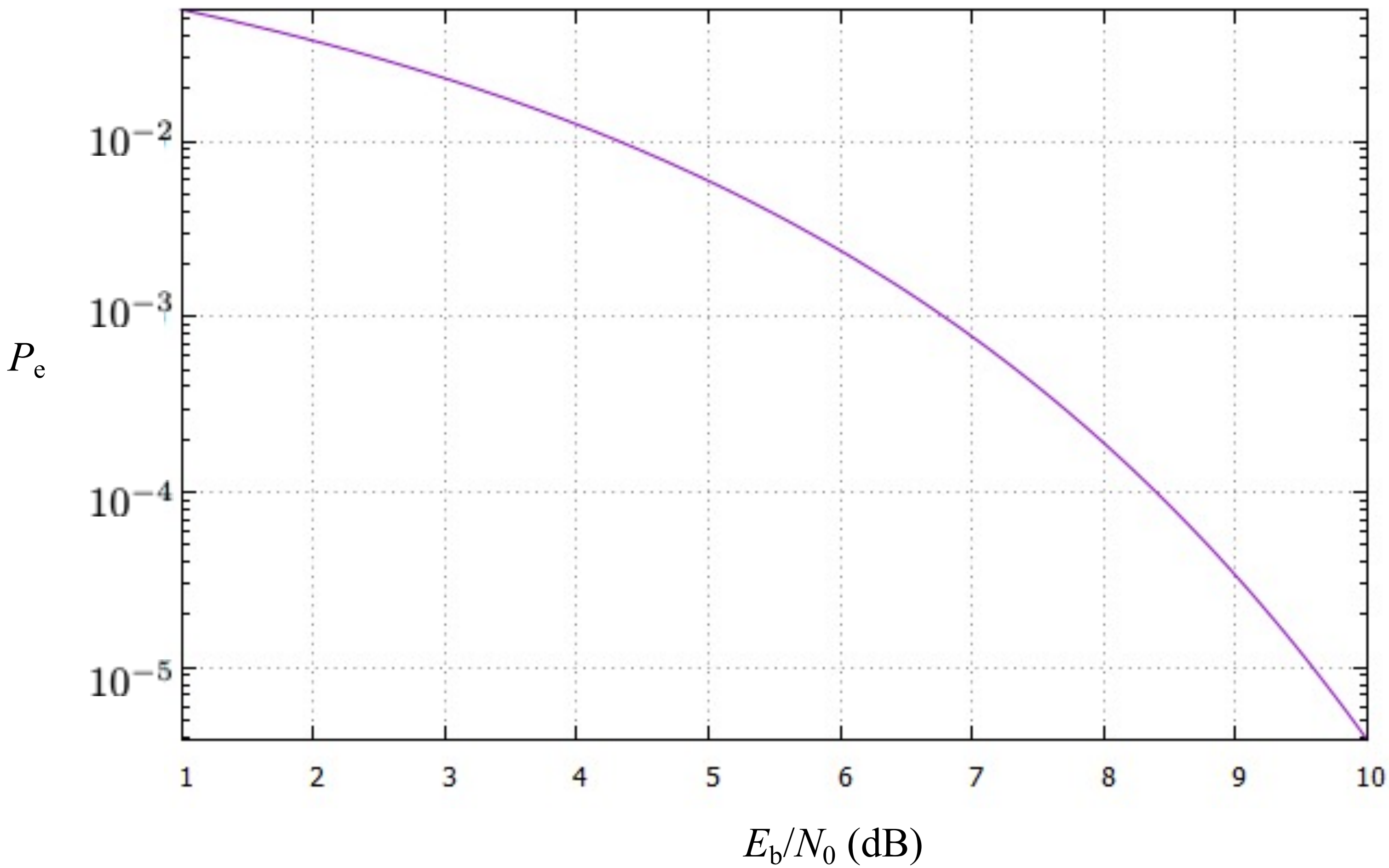
- The optimal BER formula is important in communications:

$$BER_{\text{opt}} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_g}{N_0}}\right) = Q\left(\sqrt{\frac{2E_g}{N_0}}\right)$$

- The best decision is $y \underset{-1}{\overset{+1}{\gtrless}} 0$.

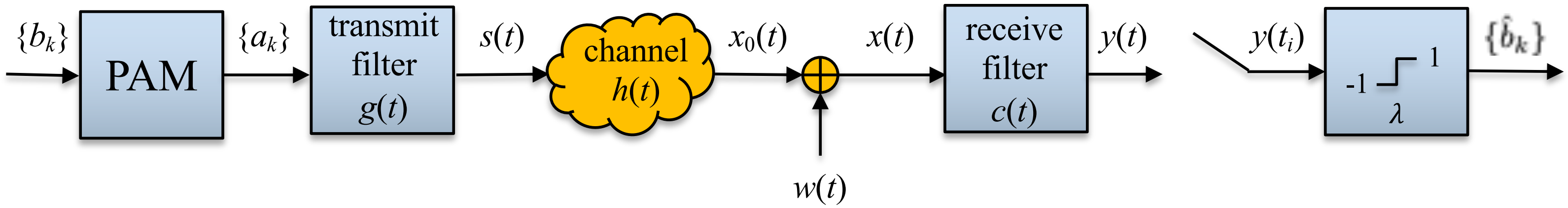
$s(t) = I \cdot g(t)$, where $I \in \{-1,+1\}$.

In this case, $E_b = \int_0^T E[s^2(t)]dt = \int_0^T E[I^2]g^2(t)dt = E_g$



Intersymbol Interference

- The channel is usually *dispersive* in nature.
- In this section, we only consider discrete pulse-amplitude modulation (PAM). Consideration of PDM and PPM will be similar but out of the scope of this section.



$$b_k \in \{0,1\},\ a_k = 2b_k - 1 \text{ and } s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b).$$

Intersymbol Interference

- Notably, in the previous section, we only consider one interval of input.

$$s(t) = I \cdot g(t)$$

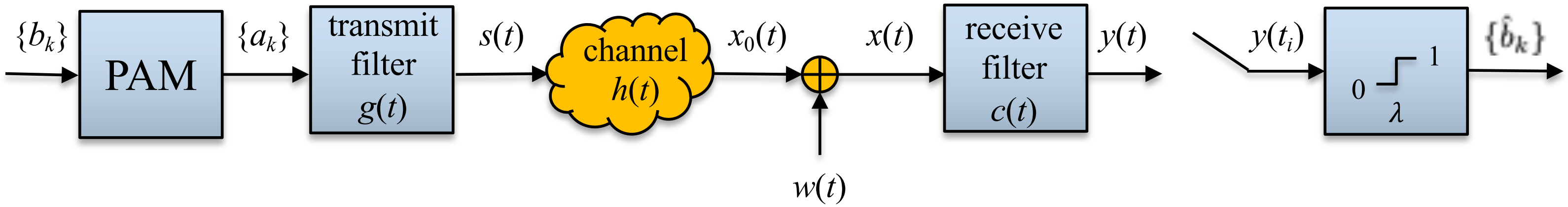
  This is justifiable because of no ISI.

- However, in this section, we have to consider

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b).$$

  since ISI is involved.

- We also assume *perfect synchronization* to simplify the analysis.



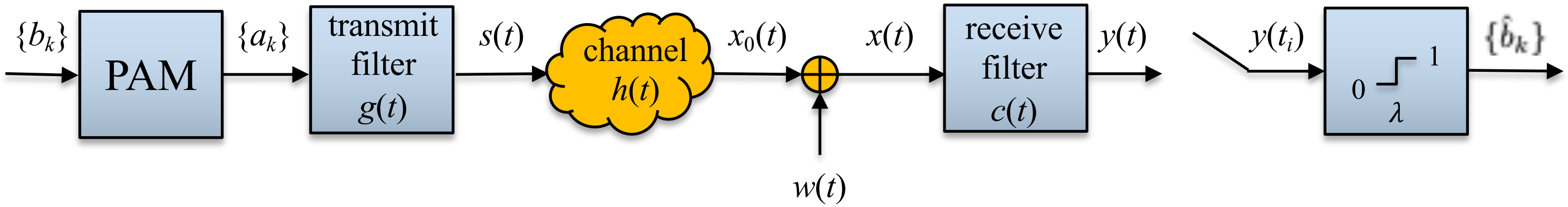
$$s(t)=\sum_{k=-\infty}^{\infty}a_k g(t-kT_b)$$

Information of a_k is carried at $[kT_b, (k+1)T_b)$.
We sample at $iT_b = (k+1)T_b$ to retrieve a_k.

$$x(t)=s(t)*h(t)+w(t)=\sum_{k=-\infty}^{\infty}a_k[g(t-kT_b)*h(t)]+w(t)$$

$$y(t)=x(t)*c(t)=\sum_{k=-\infty}^{\infty}a_k[g(t-kT_b)*h(t)*c(t)]+w(t)*c(t)$$

$$y(iT_b)=\sum_{k=-\infty}^{\infty}a_k[g(t-kT_b)*h(t)*c(t)]\Big|_{t=iT_b}+w(t)*c(t)\Big|_{t=iT_b}$$



$$g(t-kT_b)*h(t)*c(t)=\int_{-\infty}^{\infty}G(f)\exp\{-j2\pi fkT_b\}\cdot H(f)\cdot C(f)\exp\{j2\pi ft\}dt$$

$$=\int_{-\infty}^{\infty}G(f)\cdot H(f)\cdot C(f)\exp\{j2\pi f(t-kT_b)\}dt$$

$$=p(t-kT_b)$$

where $p(t)=\int_{-\infty}^{\infty}G(f)H(f)C(f)\exp\{j2\pi ft\}dt.$

$$\Rightarrow y(t)=\sum_{k=-\infty}^{\infty}a_k p(t-kT_b)+n(t),\text{ where } n(t)=w(t)*c(t)$$

$$\Rightarrow y(iT_b)=\sum_{k=-\infty}^{\infty}a_k p(iT_b-kT_b)+n(iT_b)$$

ISI and Background Noise

- If $H(f)$=1, then the matched filter suffices to eliminate ISI.

$$H(f)=1 \Rightarrow p(t)=\int_{-\infty}^{\infty} G(f)C(f)\exp\{j2\pi ft\}dt.$$

With matched filter $C(f)=G^*(f)\exp\{-j2\pi fT_b\}$, or $c(t)=g^*(T_b-t)$,

$$\begin{aligned} p(iT_b) &= \int_{-\infty}^{\infty} c(\tau)g(iT_b-\tau)d\tau \\ &= \int_{-\infty}^{\infty} g^*(T_b-\tau)g(iT_b-\tau)d\tau \quad (\text{Let } s=T_b-\tau) \\ &= \int_{-\infty}^{\infty} g^*(s)g(s+(i-1)T_b)ds = \begin{cases} 0, & \text{if } i \neq 1 \\ \int_{-\infty}^{\infty} |g(s)|^2 ds, & \text{if } i=1 \end{cases} \end{aligned}$$

provided $g(t)$ is zero outside $[0, T_b)$

$$y((i+1)T_b) = \sum_{k=-\infty}^{\infty} a_k p((i+1-k)T_b) + n((i+1)T_b) = a_i p(T_b) + n((i+1)T_b)$$

As a result, without ISI and additive noise,

$$y((i+1)T_b) = \sum_{k=-\infty}^{\infty} a_k p((i+1-k)T_b) = a_i p(T_b)$$

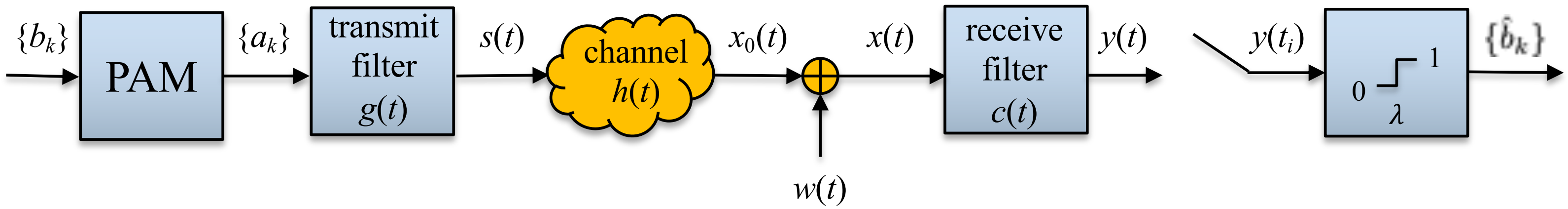
and $\{a_i\}$ can be completely reconstructed by $\{y((i+1)T_b)\}$.

Information of a_i is actually carried during $[iT_b, (i+1)T_b)$.
So, in order to recover a_i, "correlation" (convolution) operation should start at iT_b, and end (i.e., is sampled) at $(i+1)T_b$.
Hence, $y((i+1)T_b)$ is used to reconstruct a_i.

However, this index system requires …, $p(-2T_b)=0$, $p(-T_b)=0$, $p(0)=0$, $p(T_b)=1$, $p(2T_b)=0$, …., which, due to its non-symmetry, may not facilitate the derivation of spectrum condition for $p(t)$. Thus, in what follows, we assume …, $p(-2T_b)=0$, $p(-T_b)=0$, $p(0)=1$, $p(T_b)=0$, $p(2T_b)=0$, …., i.e., the information of a_i is carried during $[(i-1)T_b, iT_b)$.

Nyquist's Criterion for Noiseless Baseband Transmission

- Is it possible to completely eliminate ISI (in principle) by selecting proper $g(t)$ and $c(t)$?



Choose $g(t)$ and $c(t)$ such that $p(t) = \int_{-\infty}^{\infty} G(f)H(f)C(f)\exp(j2\pi ft)dt$

satisfies $p(iT_b) = \begin{cases} 0, & \text{if } i \neq 0 \\ 1, & \text{if } i = 0 \end{cases}$.

Nyquist's Criterion for Noiseless Baseband Transmission

- Let $P(f) = G(f)H(f)C(f)$.
- Sample $p(t)$ with sampling period T_b to produce $P_\delta(f)$.
- From Slide 6-4, we get:

$$P_\delta(f) = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right)$$

- Also from Slide 6-4, we have:

$$P_\delta(f) = \sum_{n=-\infty}^{\infty} p(nT_b)\exp\left(-j2\pi nT_b f\right) = 1$$

$$\left(\text{Because } p(nT_b) = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{if } n \neq 0 \end{cases}\right)$$

Nyquist's Criterion for Noiseless Baseband Transmission

- This concludes that the condition for zero ISI is:

$$\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right) = T_b \qquad \left(\text{Or, } \sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right) = \text{constant.}\right)$$

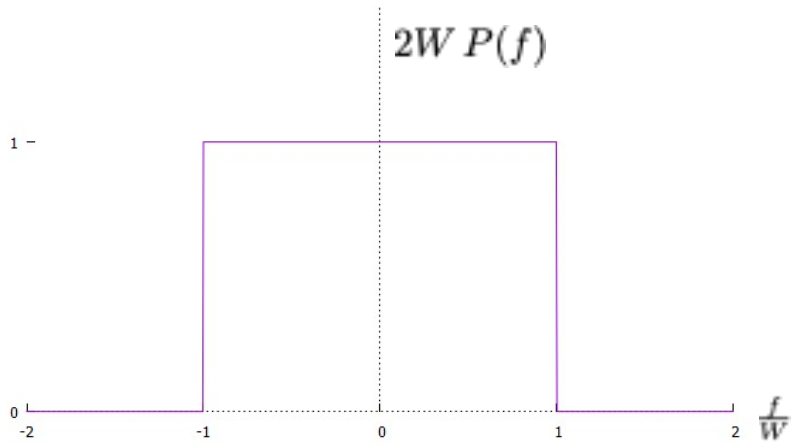
- This is named *Nyquist's criterion*.
  - The overall system frequency function $P(f)$ suffers no ISI for samples taken at interval T_b if it satisfies the above equation.
  - Notably, $P(f)$ represents the overall accumulative effect of transmit filter, channel response, and receive filter.

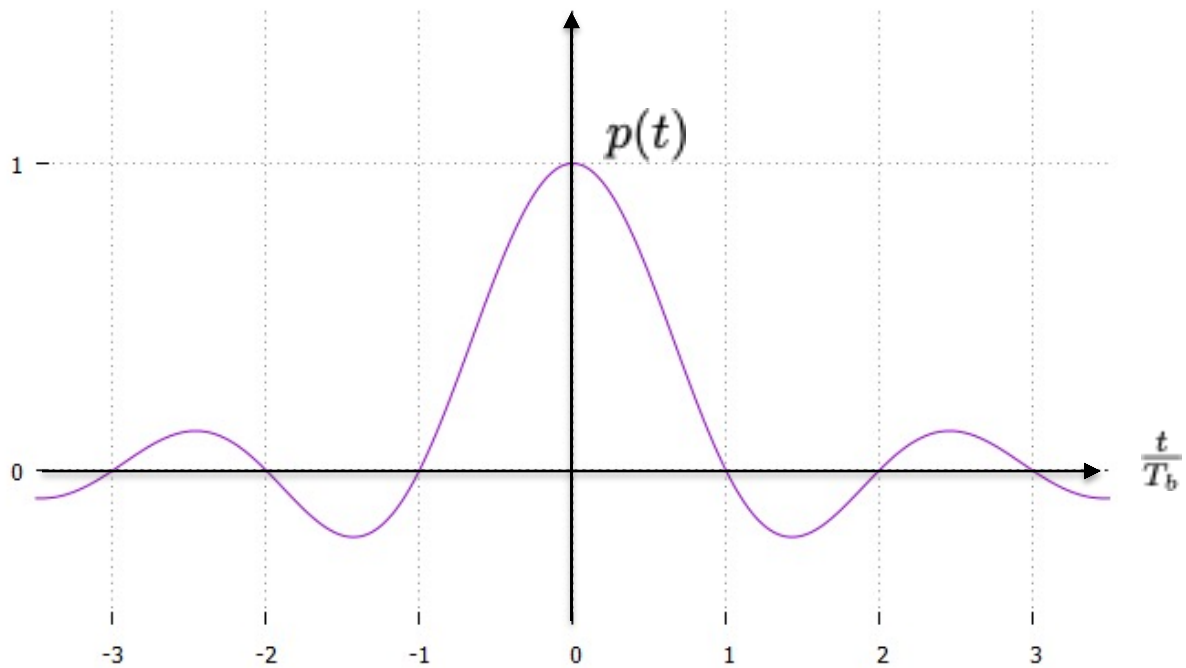
Ideal Nyquist Channel

- The simplest $P(f)$ that satisfies Nyquist's criterion is the rectangular function:

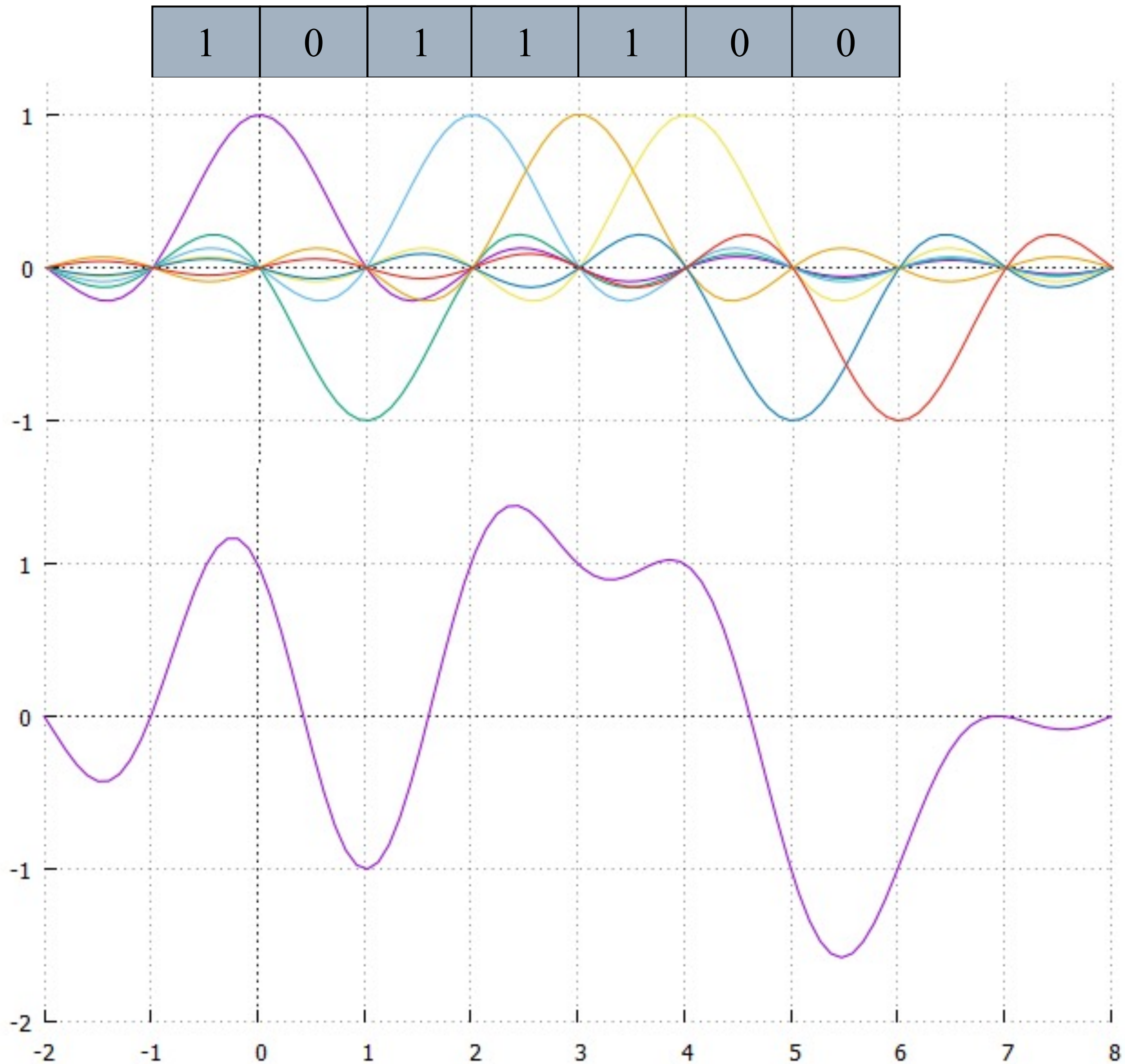
$$P(f) = \begin{cases} T_b, & |f| < W = \dfrac{1}{2T_b} \\ 0, & |f| > W = \dfrac{1}{2T_b} \end{cases} \text{ and } P(-W) + P(W) = T_b.$$

$$\Rightarrow p(t) = \frac{\sin(2\pi W t)}{2\pi W t} = \mathrm{sinc}(2Wt)$$


$2W\,P(f)$
1
0
-2
-1
0
1
2
$\frac{f}{W}$


$p(t)$
1
0
-3
-2
-1
0
1
2
3
$\frac{t}{T_b}$

The information of a_i is carried during $[(i-1)T_b, iT_b)$ and sampled at $t = iT_b$.



Infeasibility of Ideal Nyquist Channel

- Rectangular $P(f)$ is infeasible because:
  - $p(t)$ extends to negative infinity, which means that each a_k has already been transmitted at $t = -\infty$!
  - A system response being flat from $-W$ to W, and zero elsewhere is physically unrealizable.
  - The error margin is quite small, as a slight (erroneous) shift in sampling time (such as, $iT_b+\varepsilon$), will cause a very large ISI.
    - Note that $p(t)$ decays to zero at a very slow rate of $1/|t|$.

Infeasibility of Ideal Nyquist Channel

- Examination of timing error margin
  - Let Δt be the sampling time difference between transmitter and receiver.

$$y(iT_b + \Delta t) = \sum_{k=-\infty}^{\infty} a_k p((i-k)T_b + \Delta t)$$

  - For simplicity, set $i = 0$.

$$y(\Delta t) = \sum_{k=-\infty}^{\infty} a_k p(\Delta t - kT_b)$$

$$= \sum_{k=-\infty}^{\infty} a_k \frac{\sin[2\pi W(\Delta t - kT_b)]}{2\pi W(\Delta t - kT_b)}$$

$$y(\Delta t) = \sum_{k=-\infty}^{\infty} a_k \frac{\sin[2\pi W\Delta t - k\pi]}{2\pi W\Delta t - k\pi}$$

$$= \sum_{k=-\infty}^{\infty} a_k \frac{(-1)^k \sin[2\pi W\Delta t]}{2\pi W\Delta t - k\pi}$$

$$= a_0 \frac{\sin[2\pi W\Delta t]}{2\pi W\Delta t} + \frac{\sin[2\pi W\Delta t]}{\pi} \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{(-1)^k a_k}{2W\Delta t - k}$$

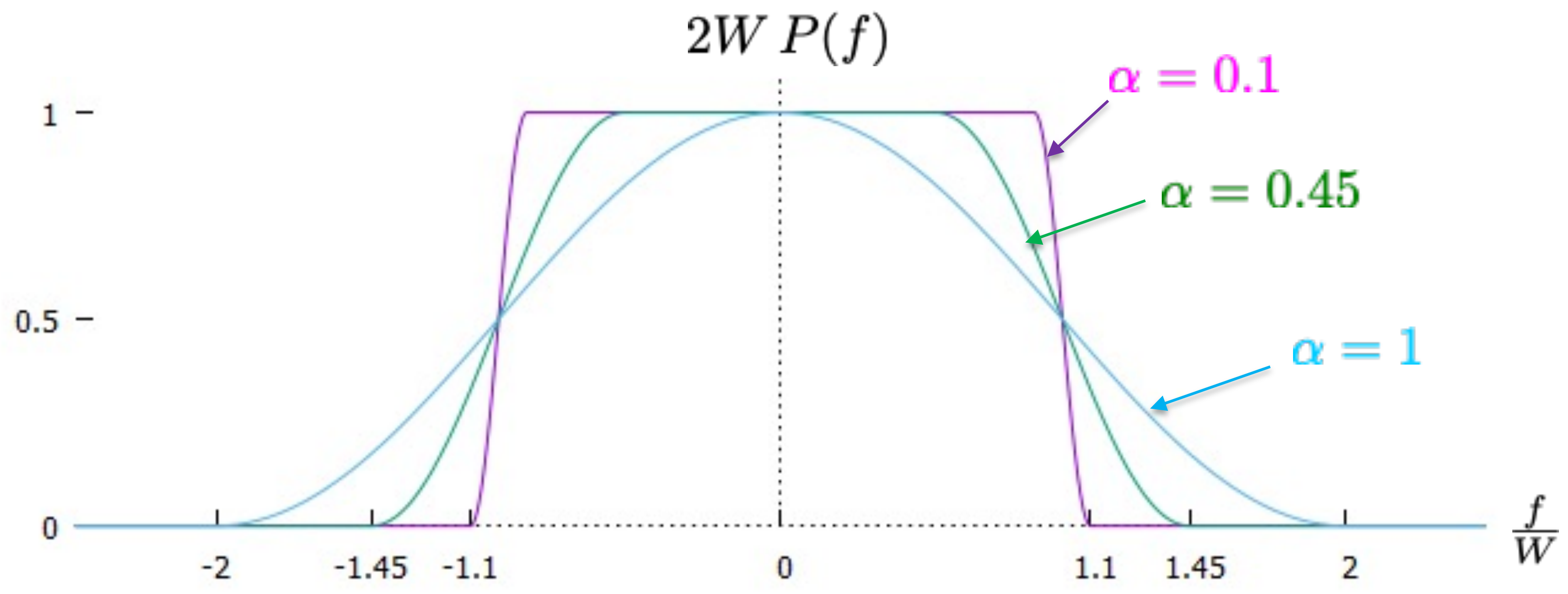
There exists $\{a_k\}$ such that $\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{(-1)^k a_k}{2W\Delta t - k} = \infty$ for any fixed small $\Delta t > 0$.

Question: How to make $p(t)$ decays faster?
Answer: Make $P(f)$ smoother.

Raised Cosine Spectrum

For a nonnegative function $p(t)$,

$$\text{if } \int_{-\infty}^{\infty} t^k p(t)\,dt < \infty, \text{ then } \frac{\partial^k P(f)}{\partial f^k} \text{ exists.}$$



Raised Cosine Spectrum

- We extend the bandwidth of $p(t)$ from W to $2W$, and require that

$$P(f)+P(f-2W)+P(f+2W)=\frac{1}{2W}\quad \text{for } |f|<W.$$

  - So, the price to pay is a larger bandwidth.
  - One of the $P(f)$ that satisfies the above condition is the *raised cosine spectrum*.

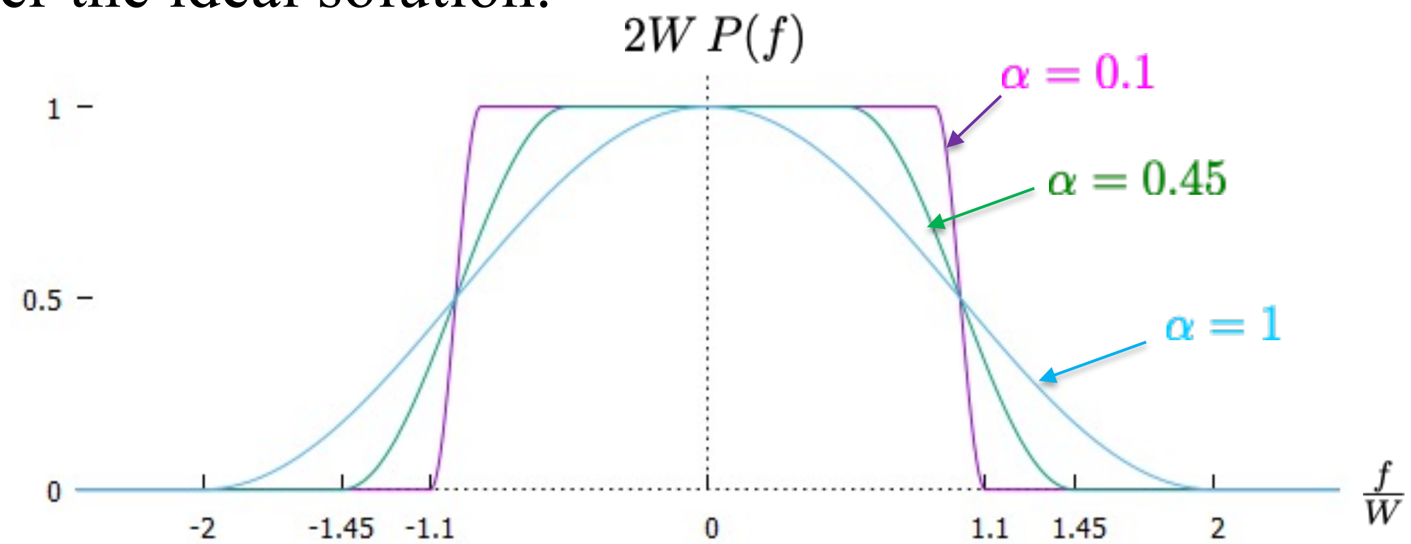
$$P(f)=\begin{cases}\dfrac{1}{2W}, & 0\le |f|<(1-\alpha)W\\[2ex] \dfrac{1}{4W}\left\{1+\cos\left[\dfrac{\pi(|f|-(1-\alpha)W)}{2\alpha W}\right]\right\}, & (1-\alpha)W\le |f|<(1+\alpha)W\\[2ex] 0, & |f|\ge (1+\alpha)W\end{cases}$$

Raised Cosine Spectrum

- The transmission bandwidth of the raised cosine spectrum is equal to:

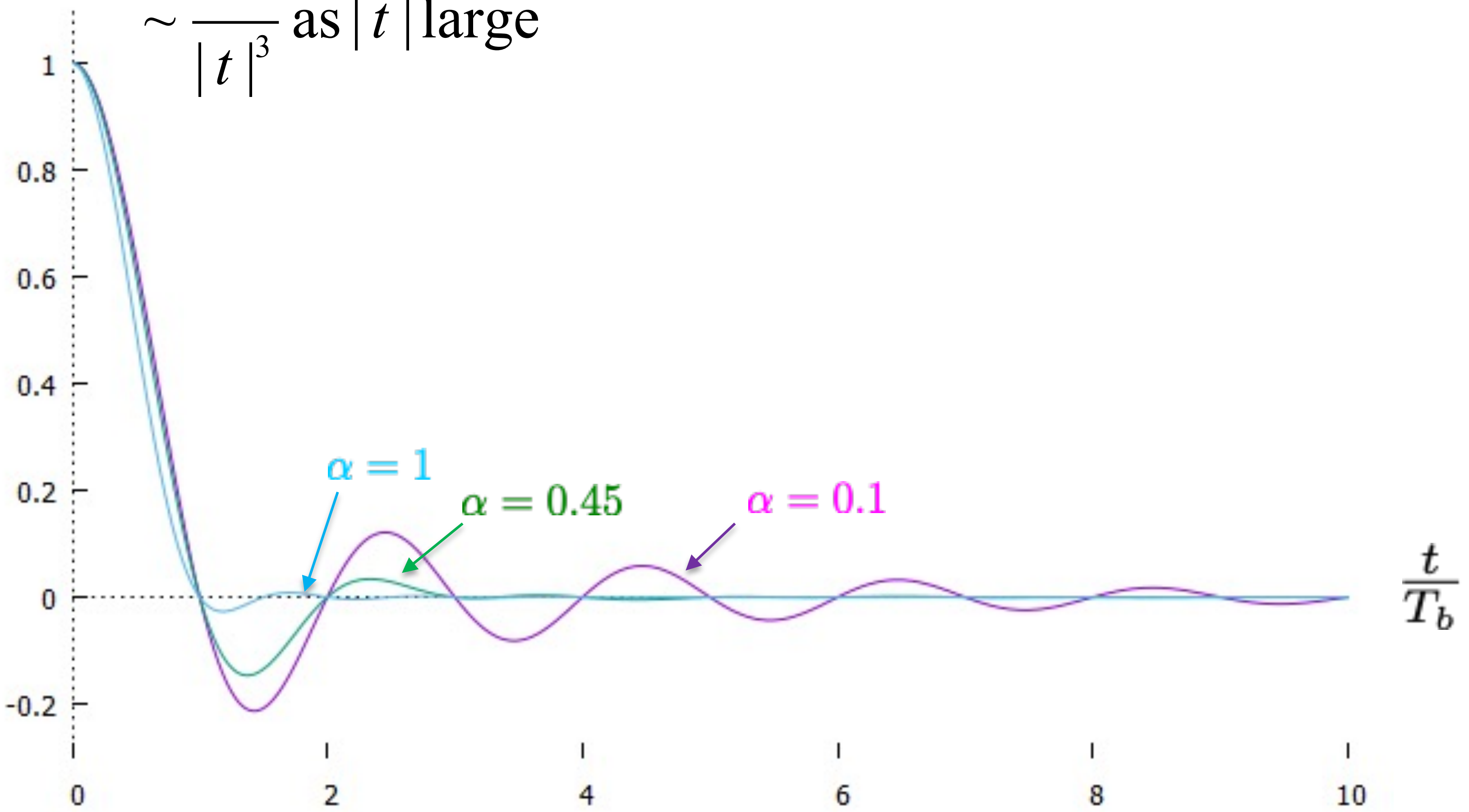
$$B_T = 2W(1+\alpha)$$

where α is the rolloff factor, which is the *excess bandwidth* over the ideal solution.



$$p(t) = \text{sinc}(2Wt)\left(\frac{\cos(2\pi\alpha Wt)}{1-16\alpha^2W^2t^2}\right)$$

$$\sim \frac{1}{|t|^3} \text{ as } |t| \text{ large}$$



Raised Cosine Spectrum

- $p(t) = \text{sinc}(2Wt)\left(\dfrac{\cos(2\pi\alpha Wt)}{1-16\alpha^2W^2t^2}\right)$ consists of two terms:
  - The first term ensures the desired zero crossing of $p(t)$.
  - The second term provides the necessary tail convergence rate of $p(t)$.

- The special case of $\alpha = 1$ is known as the *full-cosine rolloff* characteristic.

$$p(t) = \frac{\text{sinc}(4Wt)}{1-16W^2t^2}$$

Raised Cosine Spectrum

- Useful property of *full-cosine spectrum*.

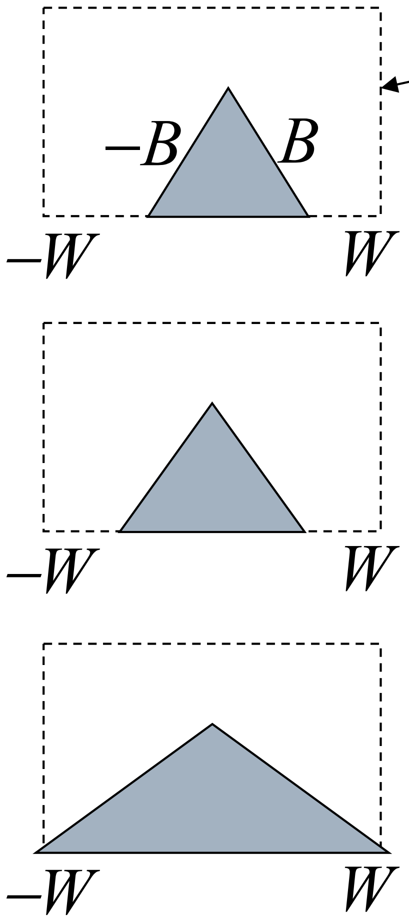
$$p\left(\pm\frac{iT_b}{2}\right)=\begin{cases}1, & i=0\\ \frac{1}{2}, & i=1\\ 0, & i\geq 2\end{cases}$$

  - We have more "zero-crossing" at $\pm 3T_b/2$, $\pm 5T_b/2$, $\pm 7T_b/2,\ldots$ in addition to the desired $\pm T_b$, $\pm 2T_b$, $\pm 3T_b\ldots$
  - This is useful in synchronization. (Think of when "synchronized," the quantity should be small both at $\pm 3T_b/2$, $\pm 5T_b/2$, $\pm 7T_b/2,\ldots$ and at $\pm T_b$, $\pm 2T_b$, $\pm 3T_b\ldots$)
  - However, the price to pay for this excessive synchronization information is to "double the bandwidth."

Correlative-Level Coding

- ISI, when generated in an uncontrolled manner, is an undesirable phenomenon.
- However, ISI may become a friend if it is added to the transmitted signal in a controlled manner.
  - *Known fact*: A signal of bandwidth W can be distortionlessly transmitted using its samples with sampling rate $\geq 2W$.
  - Conversely, in a channel with bandwidth W Hz, the theoretical maximum signal rate is $2W$ symbols per second.

Correlative-Level Coding



A channel with bandwidth W Hz

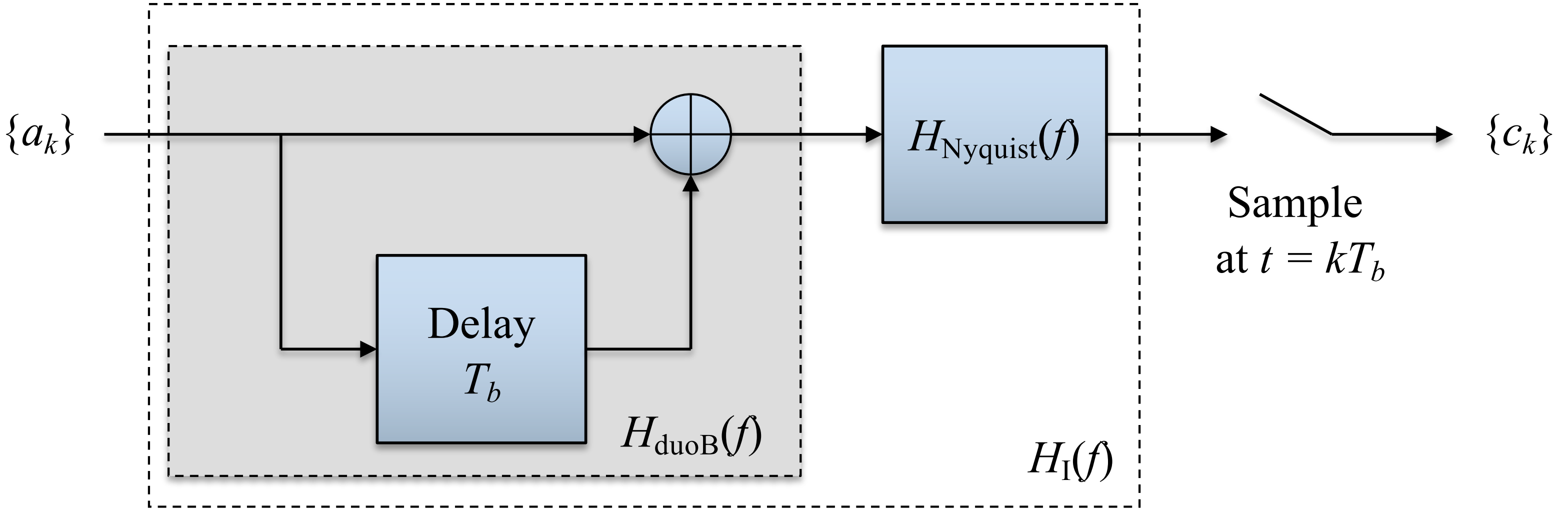
The maximum signal rate is $2W$ samples per second.

Correlative-Level Coding

- Why intentionally adding ISI? Answer: To have better bandwidth efficiency.
  - **Ideal Nyquist pulse shaping** is efficient; it cannot be realized.
  - **Raised cosine pulse shaping** is realizable; it is bandwidth inefficient.
  - By *adding ISI* to the transmitted symbols in a controlled manner, we can achieve the Nyquist rate $2W$ in a channel bandwidth of W Hertz.
    - Correlative-level coding or Partial-response signaling

An Example of Correlative-Level Coding

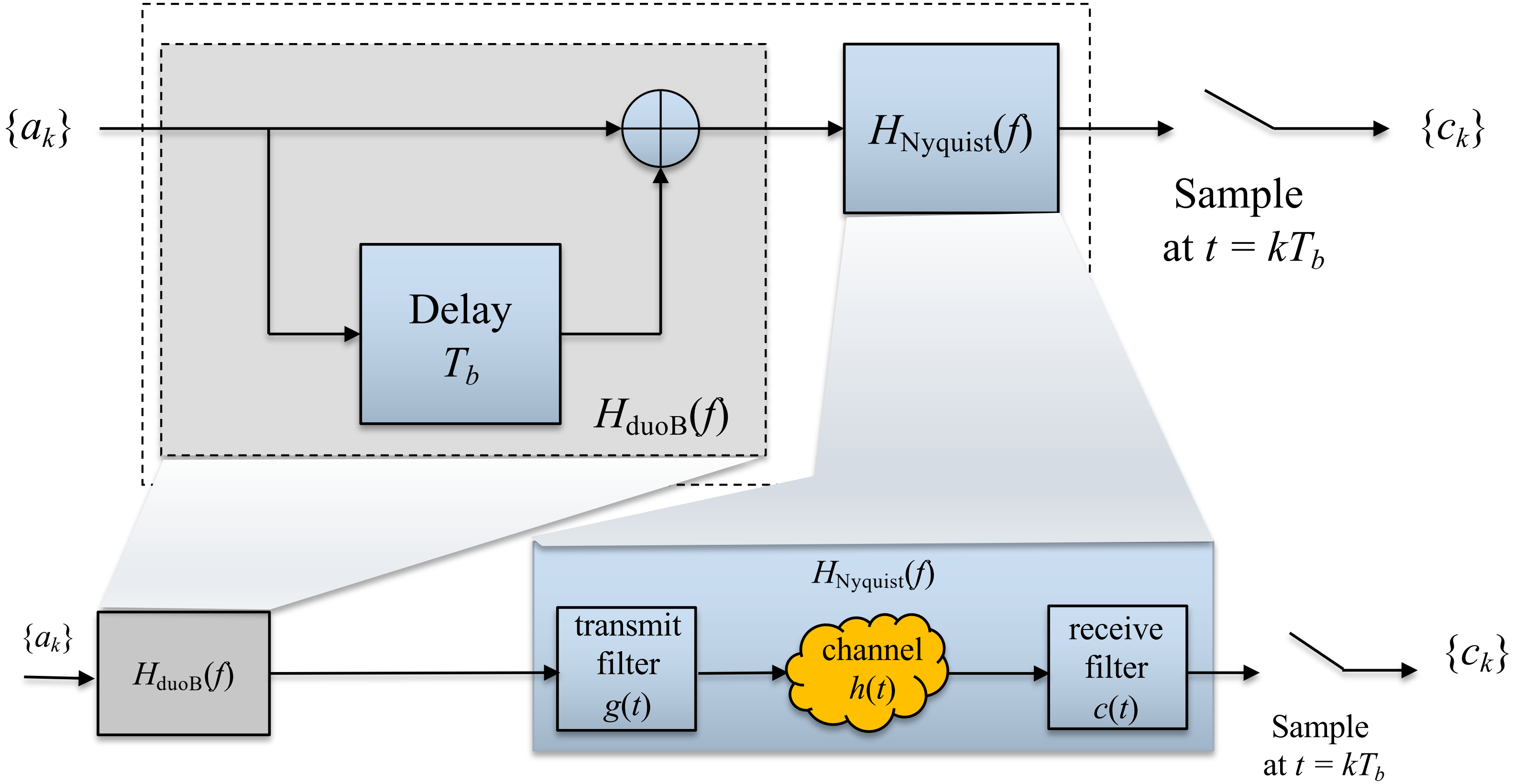
- Duobinary signaling (or *class I partial response*)



$$a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases} \quad \text{where } \{b_k\} \text{ i.i.d.}$$

An Example of Correlative-Level Coding

- Duobinary signaling (or *class I partial response*)



This part can be $H_{\text{Nyquist}}(f)$ or $H_{\text{raised cosine}}(f)$ or any filter that guarantees no ISI.

Duobinary Signaling

- Let us ignore the effect of $H_{\text{Nyquist}}(f)$ first in the block diagram in the previous slide. We directly obtain:

$$c_k = a_k + a_{k-1}$$

$$\Rightarrow H_{DuoB}(f) = 1 + \exp(-j2\pi f T_b)$$

  - Note that c_k has three levels (–2, 0, 2).

- The transfer function of the overall system is thus:

$$\begin{aligned} H_{\text{I}}(f) &= H_{\text{Nyquist}}(f)[1 + \exp(-j2\pi f T_b)] \\ &= H_{\text{Nyquist}}(f)[\exp(j\pi f T_b) + \exp(-j\pi f T_b)] \exp(-j\pi f T_b) \\ &= 2H_{\text{Nyquist}}(f) \cos(\pi f T_b) \exp(-j\pi f T_b) \end{aligned}$$

Duobinary Signaling

- $H_{\text{Nyquist}}(f)$:
  - Give that

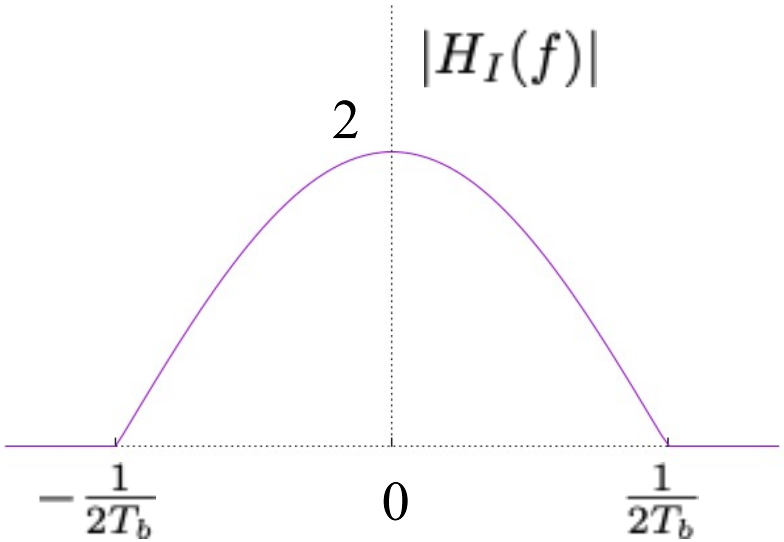
$$H_{\text{Nyquist}}(f) = \begin{cases} 1, & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases}$$

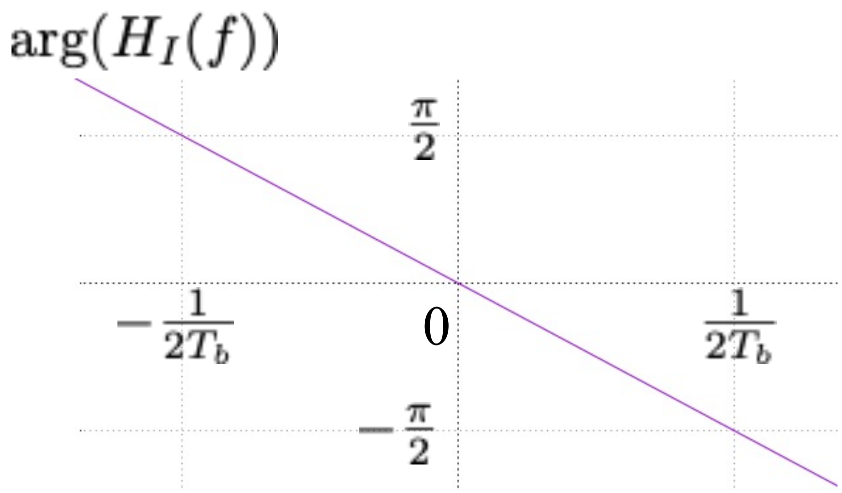
$$\Rightarrow H_{\text{I}}(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases}$$

  - As shown in the next slide, the response $H_{\text{I}}(f)$ is realizable.

Duobinary Signaling

- $H_I(f)$





Duobinary Signaling

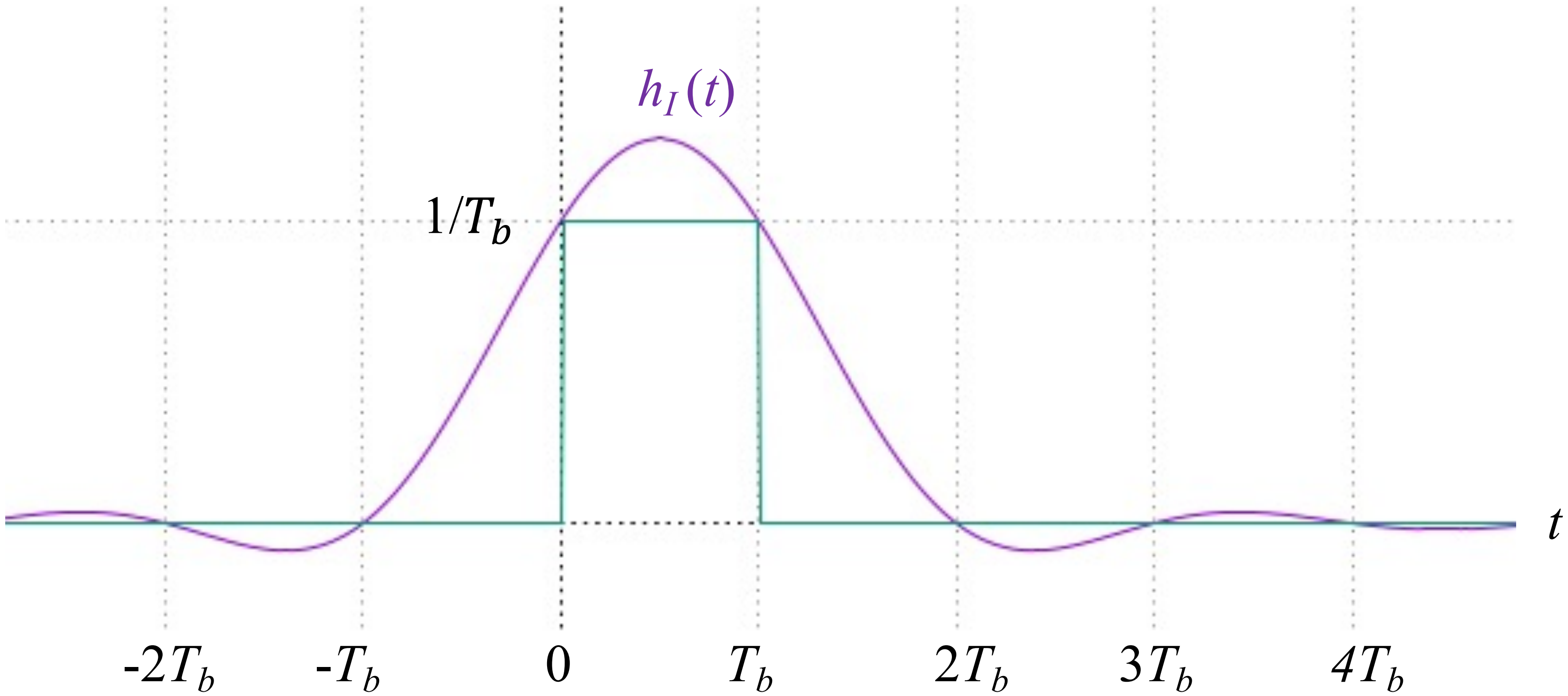
- $h_I(t)$:

$$H_{\mathrm{I}}(f) = H_{\mathrm{Nyquist}}(f)[1 + \exp(-j2\pi f T_b)]$$
$$= H_{\mathrm{Nyquist}}(f) + H_{\mathrm{Nyquist}}(f)\exp(-j2\pi f T_b)$$

$$\Rightarrow h_{\mathrm{I}}(t) = h_{\mathrm{Nyquist}}(t) + h_{\mathrm{Nyquist}}(t - T_b)$$
$$= \left(\mathrm{sinc}\left(\frac{t}{T_b}\right) + \mathrm{sinc}\left(\frac{t-T_b}{T_b}\right)\right)\frac{1}{T_b}$$
$$= \left(\frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin(\pi(t-T_b)/T_b)}{\pi(t-T_b)/T_b}\right)\frac{1}{T_b}$$
$$= \left(\frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b}\right)\frac{1}{T_b}$$
$$= \frac{1}{(T_b - t)}\frac{\sin(\pi t/T_b)}{\pi t/T_b} = \frac{1}{(T_b - t)}\mathrm{sinc}\left(\frac{t}{T_b}\right)$$

Duobinary Signaling

- $h_I(t)$:

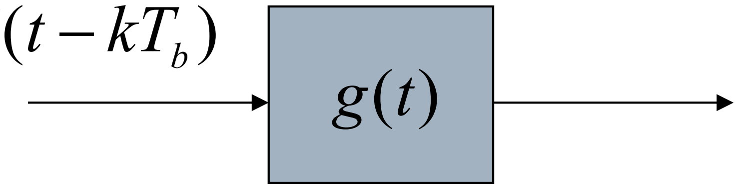


Duobinary Signaling

- **Bandwidth efficiency** of duobinary signaling
  - Example.

Transmitted signal $\sum_{k=-\infty}^{\infty} a_k g(t-kT_b) = \left(\sum_{k=-\infty}^{\infty} a_k \delta(t-kT_b)\right) \star g(t)$

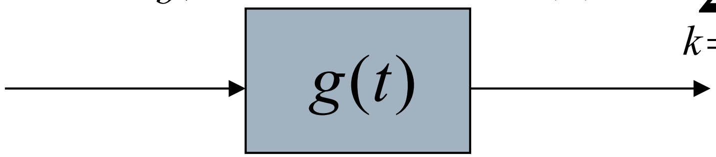
$$\sum_{k=-\infty}^{\infty} a_k \delta(t-kT_b)$$



The input to this filter may not be WSS!
Then, we should use the time-average autocorrelation function.

Duobinary Signaling

$$X(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t - kT_b)$$



$$Y(t) = \sum_{k=-\infty}^{\infty} a_k g(t - kT_b)$$

(to channel)

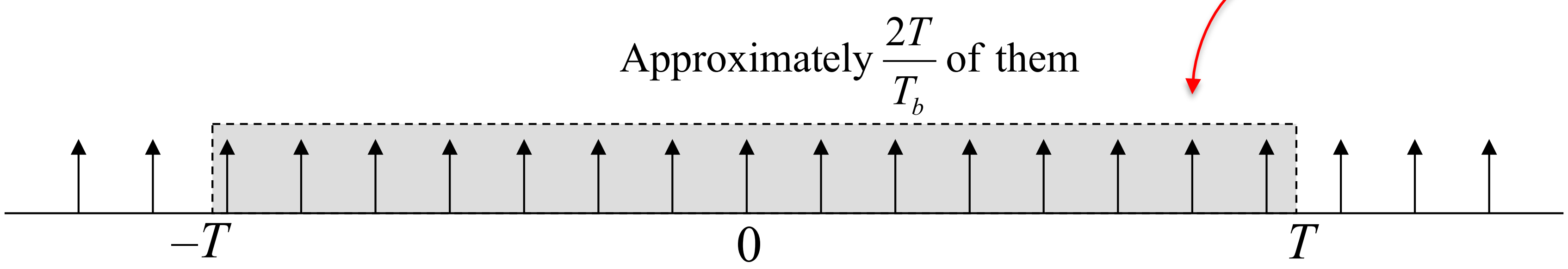
$$\begin{aligned}
\bar{R}_X(\tau) &= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E\left[\left(\sum_{k=-\infty}^{\infty} a_k \delta(t+\tau-kT_b)\right)\left(\sum_{j=-\infty}^{\infty} a_j \delta(t-jT_b)\right)\right] d \\
&= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \sum_{k=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} E[a_k a_j] \delta(t+\tau-kT_b)\delta(t-jT_b) dt
\end{aligned}$$

Assume $E[a_k a_j] = \begin{cases} 1, & k = j; \\ 0, & k \neq j \end{cases}$

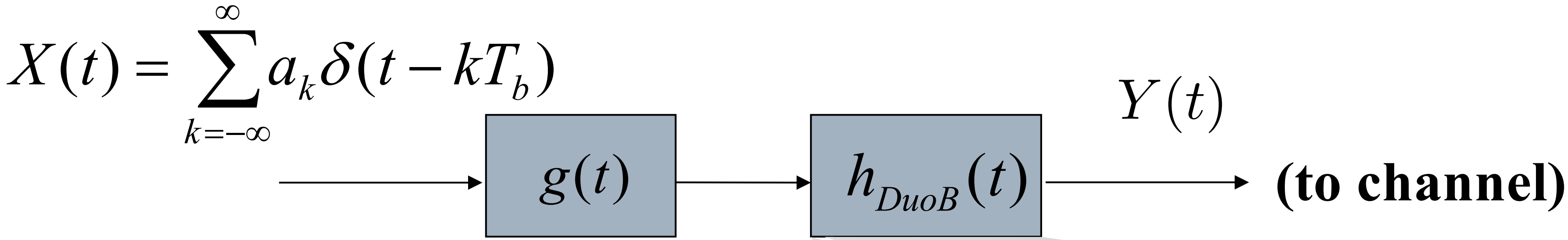
Duobinary Signaling

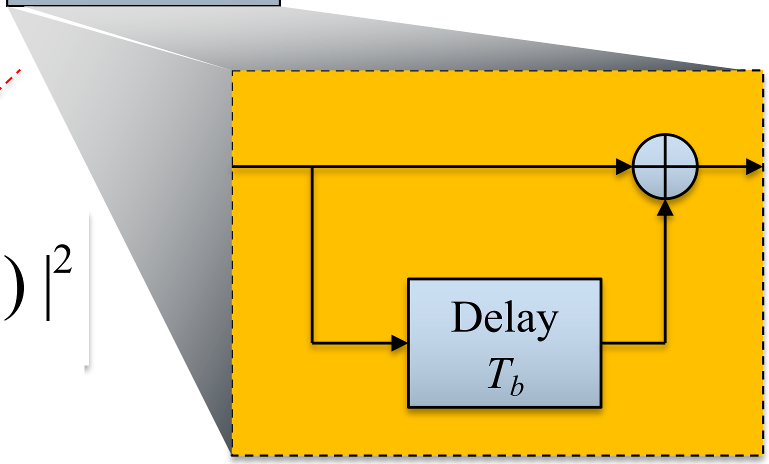
$$
\begin{aligned}
\bar{R}_X(\tau) &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\sum_{k=-\infty}^{\infty}\delta(t+\tau-kT_b)\delta(t-kT_b)dt \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\sum_{k=-\infty}^{\infty}\delta(\tau)\delta(t-kT_b)dt \\
&= \delta(\tau)\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\sum_{k=-\infty}^{\infty}\delta(t-kT_b)dt = \frac{1}{T_b}\delta(\tau)
\end{aligned}
$$

$$
\Rightarrow \quad \bar{S}_Y(f) = \bar{S}_X(f)|G(f)|^2 = \frac{1}{T_b}|G(f)|^2
$$

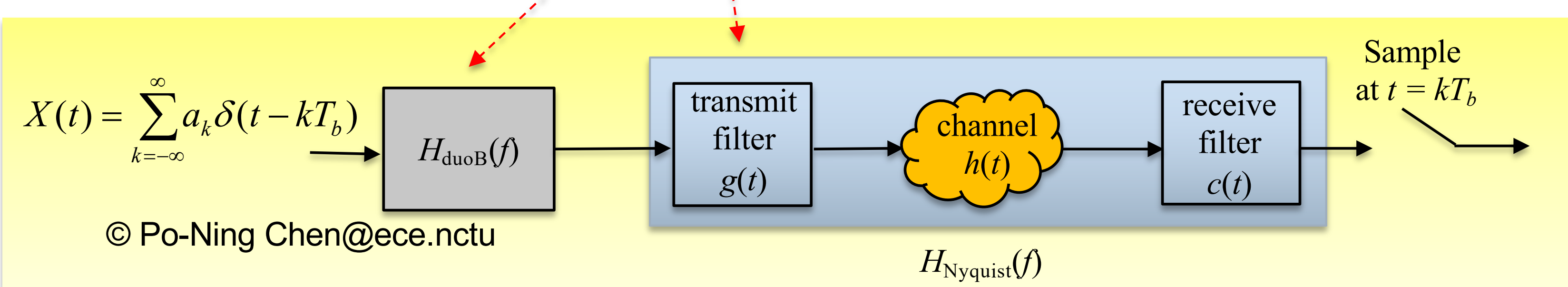


Duobinary Signaling





$$\Rightarrow \bar{S}_Y(f)=\frac{1}{T_b}|G(f)|^2|H_{DuoB}(f)|^2$$



Duobinary Signaling

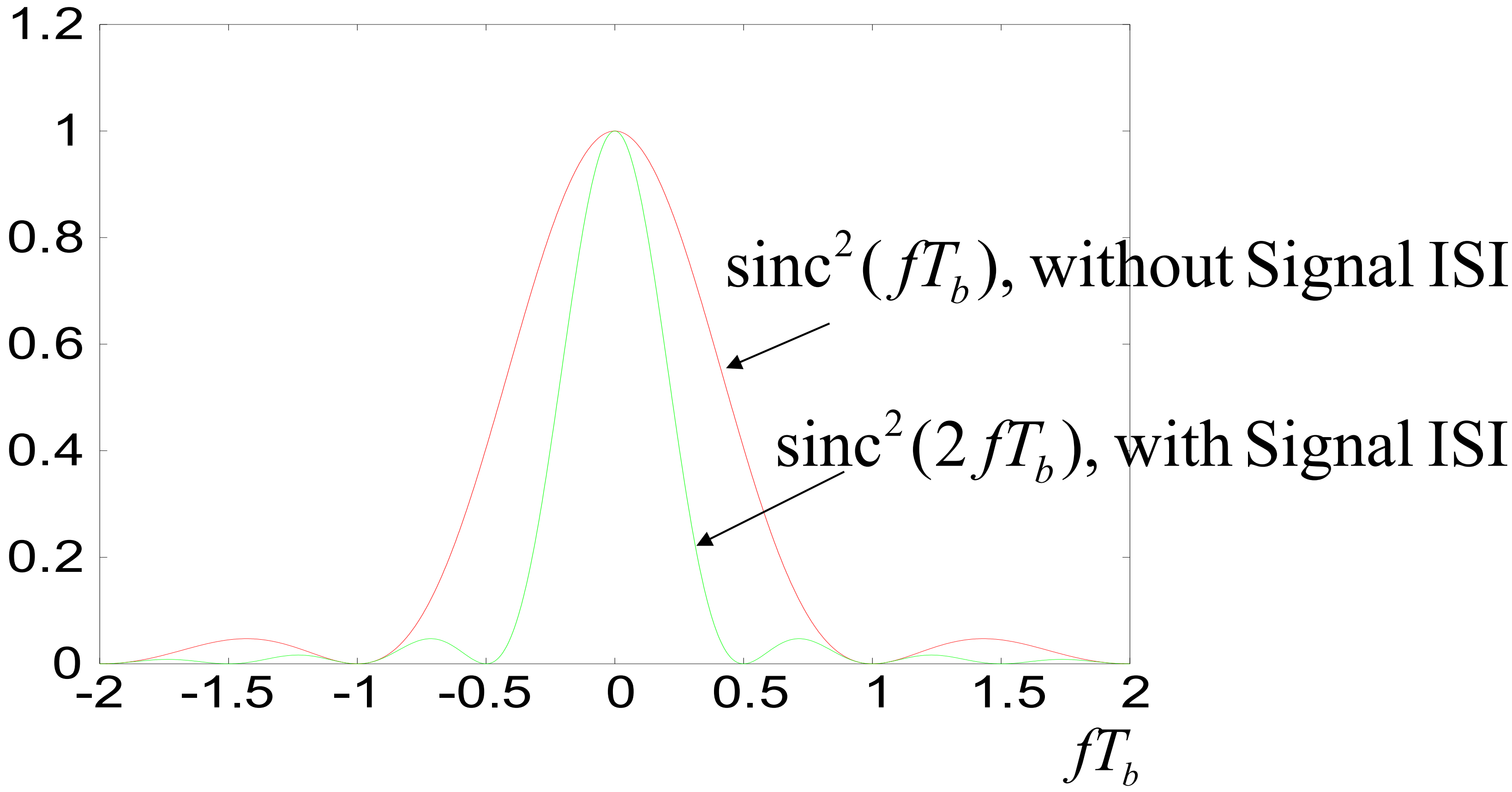
$$H_{DouB}(f) = 2\cos(\pi f T_b)\exp(-j\pi f T_b)$$

$$\text{Assume } g(t) = \begin{cases} 1, & 0 \le t < T_b \\ 0, & \text{otherwise} \end{cases}. \Rightarrow |G(f)|^2 = T_b^2 \operatorname{sinc}^2(fT_b)$$

$$\Rightarrow \frac{\bar{S}_Y(f)}{\bar{S}_Y(0)} = \begin{cases} \operatorname{sinc}^2(fT_b), & \text{No Signal ISI} \\ \cos^2(\pi f T_b)\operatorname{sinc}^2(fT_b), & \text{With Signal ISI} \end{cases}$$

$$= \begin{cases} \operatorname{sinc}^2(fT_b), & \text{No Signal ISI} \\ \operatorname{sinc}^2(2fT_b), & \text{With Signal ISI} \end{cases}$$

Duobinary Signaling



Duobinary Signaling

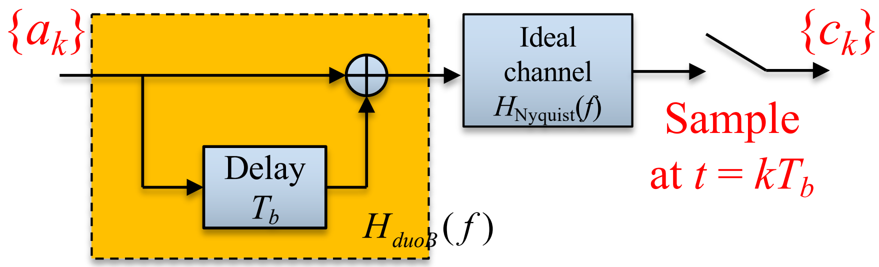
- Conclusions
  - By adding ISI to the transmitted signal in a controlled (and reversible) manner, we can reduce the requirement of bandwidth of the transmitted signal.
  - Hence, in the previous example, $\{c_k\}$ can be transmitted in every $T_b/2$ seconds!
    - Doubling the transmission capacity without introducing additional requirement in bandwidth!
  - *Duobinary signaling* : "Duo" means "doubling the transmission capacity of a straight *binary* system."
  - A larger SNR is required to yield the same error rate because of an increase in the number of signal levels (from –1, +1 to –2, 0, 2). Detailed discussion on error rate impact is omitted here!

Duobinary Signaling

- Conclusions (cont.)
  - The duobinary signaling is also named *class I partial response*.
    - Full response: The transmission wave at each time instance is fully determined by a single information symbol.
    - Partial response: The transmission wave at each time instance is only partially determined by one information.

Decision Feedback for Correlative-Level Coding

- Recovering of $\{a_k\}$ from $\{c_k\}$



$$\hat{a}_k = c_k - \hat{a}_{k-1}$$

  - It requires the previous decision to determine the current symbol.
  - So, the system should feedback the previous decision.
  - Error, therefore, may propagate!

  - How to avoid error propagation? Answer: **Precoding**.

Precoding of Correlative Coding

Without precoding

$\{b_k \in \{0,1\} \text{ i.i.d.}\} \rightarrow a_k = 2b_k - 1 \rightarrow c_k = a_k + a_{k-1}$

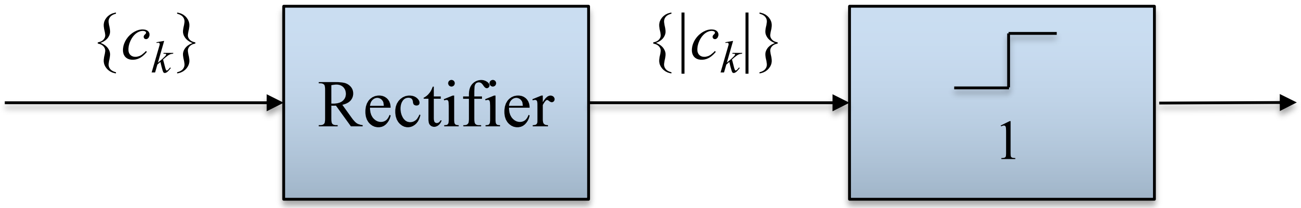
With precoding

$\{b_k \in \{0,1\} \text{ i.i.d.}\} \rightarrow \tilde{b}_k = b_k \oplus \tilde{b}_{k-1} \rightarrow a_k = 2\tilde{b}_k - 1 \rightarrow c_k = a_k + a_{k-1}$

$$
\begin{cases}
c_k = a_k + a_{k-1} \\
\quad = (2\tilde{b}_k - 1) + (2\tilde{b}_{k-1} - 1) \\
\quad = 2\tilde{b}_k + 2\tilde{b}_{k-1} - 2 \\
b_k = \tilde{b}_k \oplus \tilde{b}_{k-1}
\end{cases}
$$

$\tilde{b}_k$	$\tilde{b}_{k-1}$	b_k	c_k
0	0	0	–2
0	1	1	0
1	0	1	0
1	1	0	2

Precoding of Correlative Coding



$$b_k \begin{cases} 1, & |c_k| < 1 \\ 0, & |c_k| > 1 \end{cases}$$

- Final notes
  - The precode must not change the "**duo-** of the transmission capacity of a straight binary system."
  - Hence, $\{\tilde{b}_k\}$ must have the same distribution as $\{b_k\}$ and hence must be i.i.d.

Invariance in Statistics by Precoding

- Uniform i.i.d. of $\{\tilde{b}_k\}$
  - It suffices to show $\Pr\left(\tilde{b}_k \mid \tilde{b}_{k-1}, \tilde{b}_{k-2}, \ldots\right) = \Pr\left(\tilde{b}_k\right)$

$$\tilde{b}_k = b_k \oplus \tilde{b}_{k-1} \Rightarrow \Pr\left(\tilde{b}_k \mid \tilde{b}_{k-1}, \tilde{b}_{k-2}, \ldots\right) = \Pr\left(\tilde{b}_k \mid \tilde{b}_{k-1}\right)$$

$$\begin{cases} \Pr(\tilde{b}_k = 0 \mid \tilde{b}_{k-1} = 0) = \Pr(b_k = 0) = 1/2 \\ \Pr(\tilde{b}_k = 0 \mid \tilde{b}_{k-1} = 1) = \Pr(b_k = 1) = 1/2 \\ \Pr(\tilde{b}_k = 1 \mid \tilde{b}_{k-1} = 0) = \Pr(b_k = 1) = 1/2 \\ \Pr(\tilde{b}_k = 1 \mid \tilde{b}_{k-1} = 1) = \Pr(b_k = 0) = 1/2 \end{cases} \Rightarrow \Pr\left(\tilde{b}_k \mid \tilde{b}_{k-1}, \tilde{b}_{k-2}, \ldots\right) = \Pr\left(\tilde{b}_k\right)$$

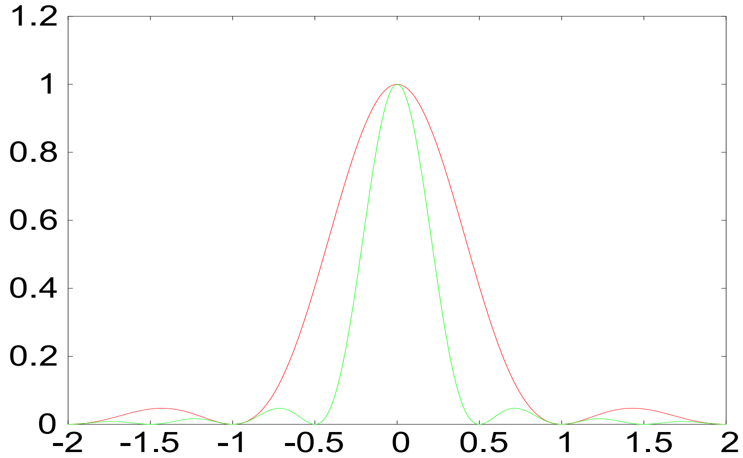
- For uniformity,

$$
\begin{cases}
\Pr(\tilde{b}_k = 0) = \Pr(\tilde{b}_{k-1} = 0)\Pr(\tilde{b}_k = 0 \mid \tilde{b}_{k-1} = 0) \\
\qquad\qquad + \Pr(\tilde{b}_{k-1} = 1)\Pr(\tilde{b}_k = 0 \mid \tilde{b}_{k-1} = 1) \\
\qquad\quad = \Pr(\tilde{b}_{k-1} = 0)\frac{1}{2} + \Pr(\tilde{b}_{k-1} = 1)\frac{1}{2} \\
\qquad\quad = \frac{1}{2} \\
\Pr(\tilde{b}_k = 1) = \Pr(\tilde{b}_{k-1} = 0)\Pr(\tilde{b}_k = 1 \mid \tilde{b}_{k-1} = 0) \\
\qquad\qquad + \Pr(\tilde{b}_{k-1} = 1)\Pr(\tilde{b}_k = 1 \mid \tilde{b}_{k-1} = 1) \\
\qquad\quad = \Pr(\tilde{b}_{k-1} = 0)\frac{1}{2} + \Pr(\tilde{b}_{k-1} = 1)\frac{1}{2} \\
\qquad\quad = \frac{1}{2}
\end{cases}
$$

Q.E.D.

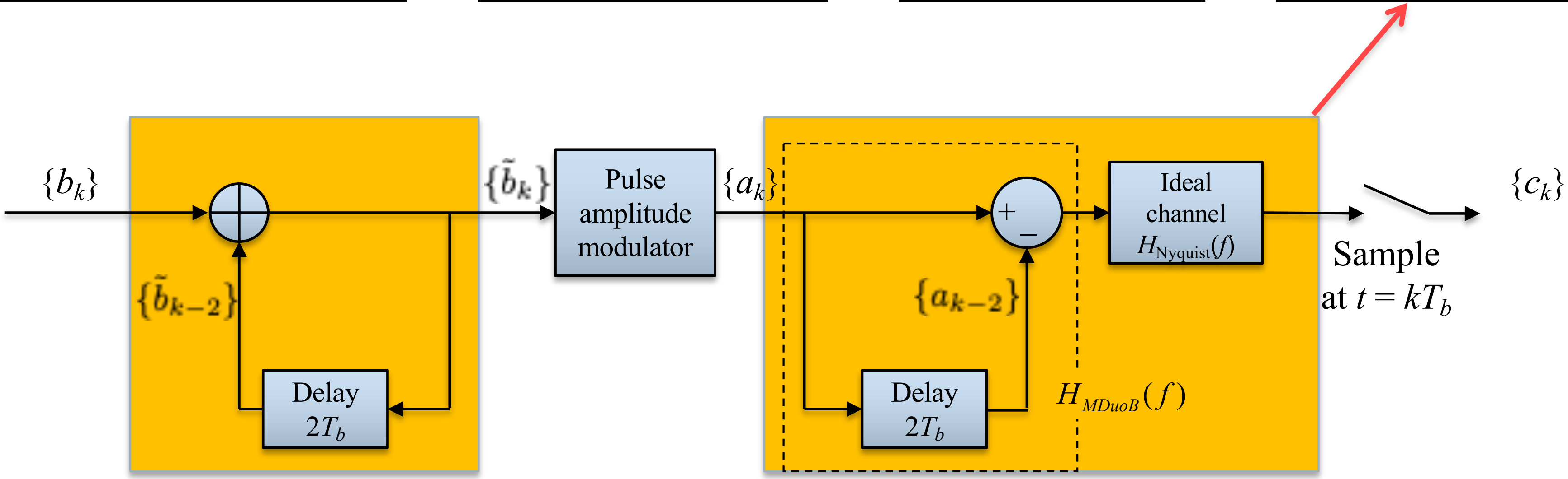
Modified Duobinary Signaling

- The PSD of the signal is nonzero at the origin.
- This is considered to be an **undesirable feature** in some applications, since many communication channels cannot transmit a DC component.
- Solution: Class IV partial response or modified duobinary technique.



Modified Duobinary Signaling

$$\{b_k \in \{0,1\} \text{ i.i.d.}\} \rightarrow \tilde{b}_k = b_k \oplus \tilde{b}_{k-2} \rightarrow a_k = 2\tilde{b}_k - 1 \rightarrow c_k = a_k - a_{k-2}$$



$$\Rightarrow H_{MDuoB}(f) = 1 - \exp(-j4\pi f T_b)$$

Modified Duobinary Signaling

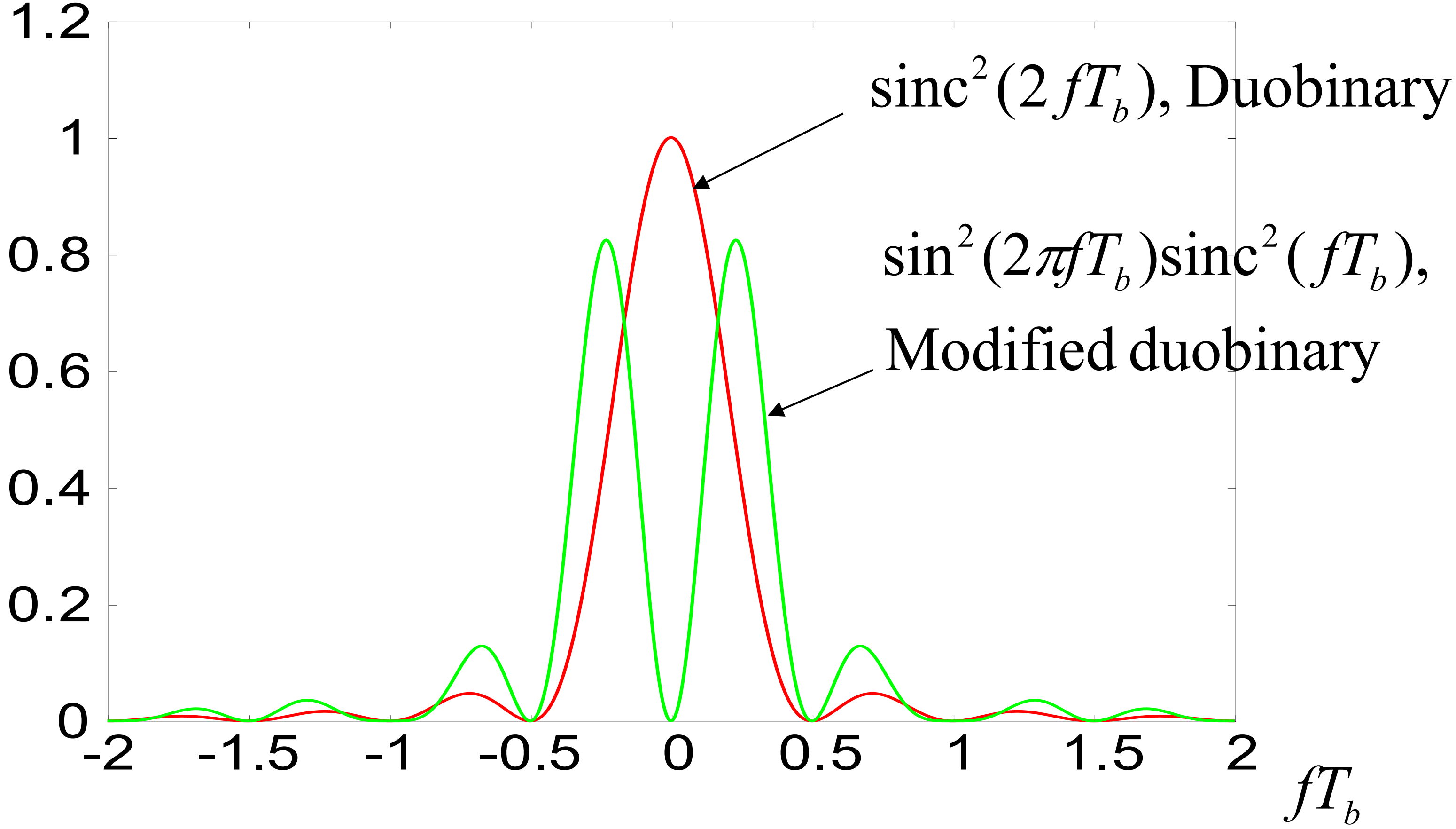
$$\Rightarrow H_{MDuoB}(f) = 1 - \exp(-j4\pi f T_b)$$
$$= [\exp(j2\pi f T_b) - \exp(-j2\pi f T_b)]\exp(-j2\pi f T_b)$$
$$= 2j\sin(2\pi f T_b)\exp(-j2\pi f T_b)$$

$$\text{Assume } g(t) = \begin{cases} 1, & 0 \le t < T_b \\ 0, & \text{otherwise} \end{cases}. \Rightarrow |G(f)|^2 = T_b^2 \operatorname{sinc}^2(fT_b)$$

$$\Rightarrow \begin{cases} \bar{S}_Y(f)/T_b = \operatorname{sinc}^2(2fT_b), & \text{Duobinary} \\ \bar{S}_Y(f)/(4T_b) = \sin^2(2\pi f T_b)\operatorname{sinc}^2(fT_b), & \text{Modified Duobinary} \end{cases}$$

(See Slide 8-59)

Modified Duobinary Signaling



Modified Duobinary Signaling

- Precoding is added to eliminate *error propagation* in decision system.

$$\begin{cases} c_k = a_k - a_{k-2} \\ \quad = (2\tilde{b}_k - 1) - (2\tilde{b}_{k-2} - 1) \\ \quad = 2\tilde{b}_k - 2\tilde{b}_{k-2} \\ b_k = \tilde{b}_k \oplus \tilde{b}_{k-2} \end{cases}$$

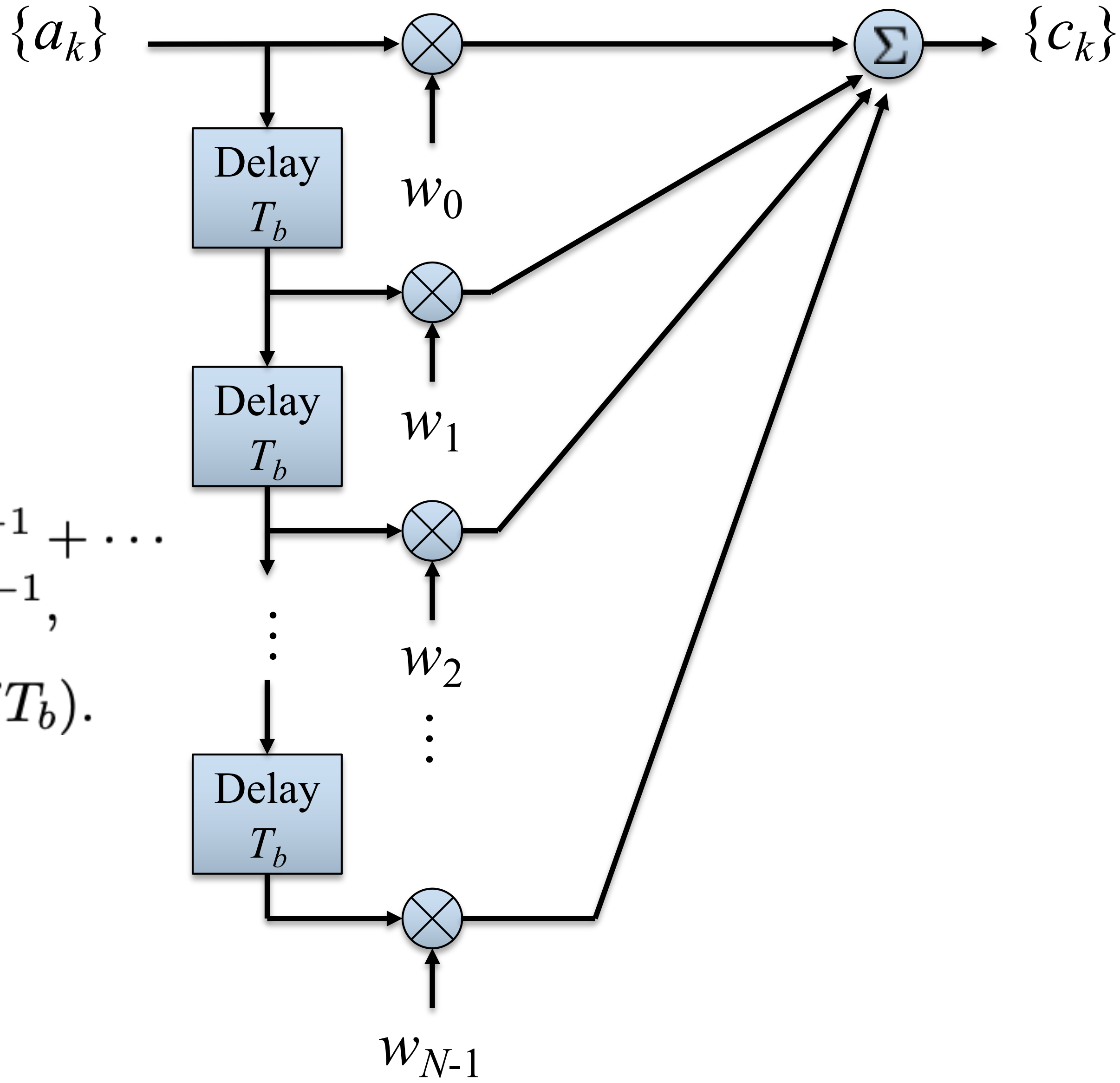
$\tilde{b}_k$	$\tilde{b}_{k-2}$	b_k	c_k
0	0	0	0
0	1	1	–2
1	0	1	2
1	1	0	0

$\{b_k \in \{0,1\} \text{ i.i.d.}\} \rightarrow \tilde{b}_k = b_k \oplus \tilde{b}_{k-2} \rightarrow a_k = 2\tilde{b}_k - 1 \rightarrow c_k = a_k - a_{k-2}$

Generalized Form of Correlative Level Coding (CLC) or Partial Response Signaling

$$H_{\text{CLC}}(f) = w_0 + w_1 z^{-1} + \cdots + w_{N-1} z^{N-1},$$

where $z = \exp(j2\pi f T_b)$.

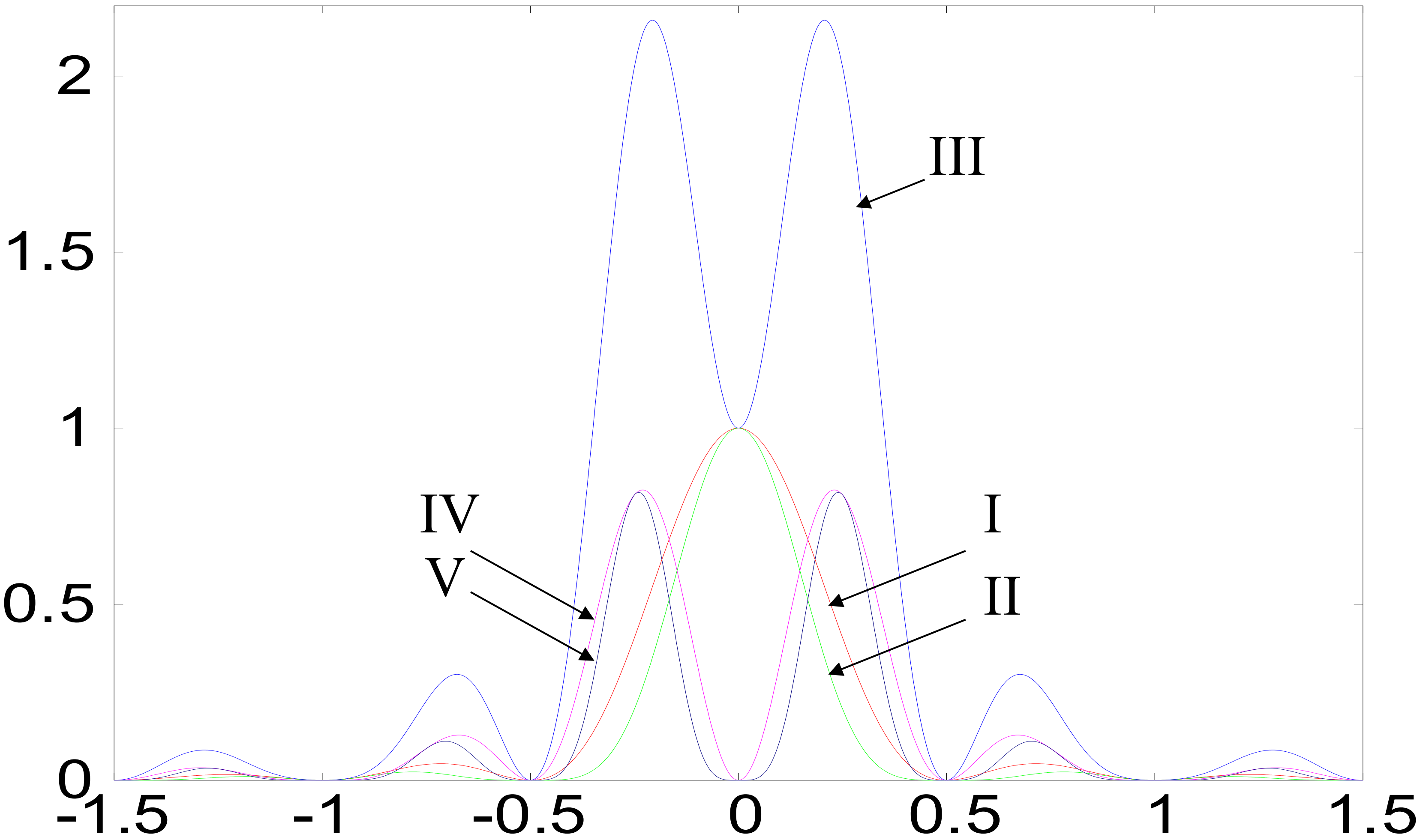


Generalized Form of Correlative-Level Coding or Partial-Response Signaling

Type of Class	N	w_0	w_1	w_2	w_3	w_4	Comments
I	2	1	1				Duobinary coding
II	3	1	2	1			
III	3	2	1	–1			
IV	3	1	0	–1			Modified duobinary coding
V	5	–1	0	2	0	–1	

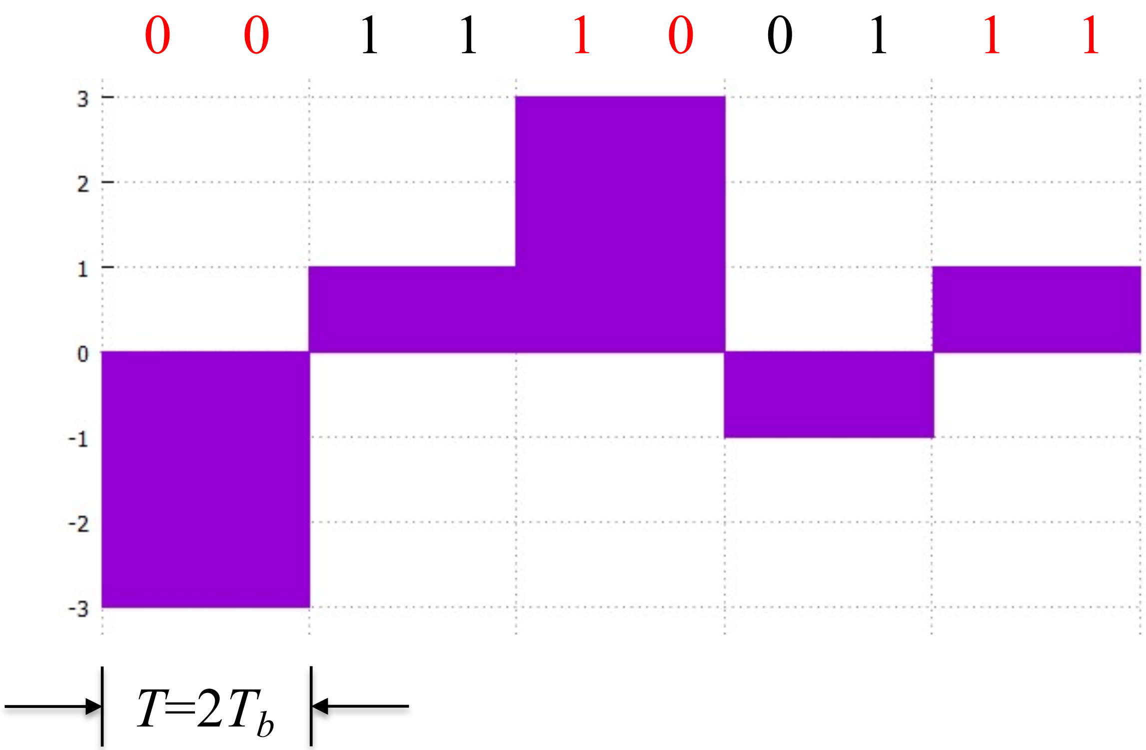
$$\Rightarrow \bar{S}_Y(f) = \frac{|G(f)|^2}{T_b} \times \begin{cases} 4\cos^2(\pi f T_b) & I \\ 16\cos^4(\pi f T_b) & II \\ 4\cos^2(\pi f T_b) + 8\sin^2(2\pi f T_b) & III \\ 4\sin^2(2\pi f T_b) & IV \\ 16\sin^4(2\pi f T_b) & V \end{cases}$$

$$\text{Assume } g(t) = \begin{cases} 1, & 0 \le t < T_b \\ 0, & \text{otherwise} \end{cases} \Rightarrow |G(f)|^2 = T_b^2 \operatorname{sinc}^2(fT_b)$$



Baseband *M*-ary PAM

data stream



Dibit	Amplitude
00	-3
01	-1
11	+1
10	+3

Gray code

Any dibit differs from an adjacent dibit in a single bit position.

Baseband *M*-ary PAM

- For *M*-ary PAM transmission, there are *M* possible symbols with symbol duration *T*.
  - $1/T$ is referred to as the *signaling rate* or *symbol rate* or *symbols per second* or *baud*.

> Baud = the number of times a signal changes state per second

- Some equivalences
  - Each symbol can be equivalently identified with $\log_2 M$ bits.
  - The baud rate $1/T$ can be equivalently transformed to bps as:

$$T = T_b \log_2(M)$$

Baseband M-ary PAM

- Equivalences
  - Virtually fix the symbol error, i.e., fix the level distance (to be 2). For example, (+1, –1) for $M = 2$, and (+3, +1, –1, –3) for $M = 4$. Then, the transmitted power per unit time for M-ary PAM transmission becomes:

$$\frac{E[S^2]}{T} = \frac{\frac{1}{M}\left([-(M-1)]^2 + [-(M-3)]^2 + \cdots + (M-3)^2 + (M-1)^2\right)}{T_b \log_2(M)}$$

$$= \frac{(M^2-1)}{3T_b \log_2(M)} = \left(\frac{1}{T_b}\right)\frac{(M^2-1)}{3\log_2(M)}$$

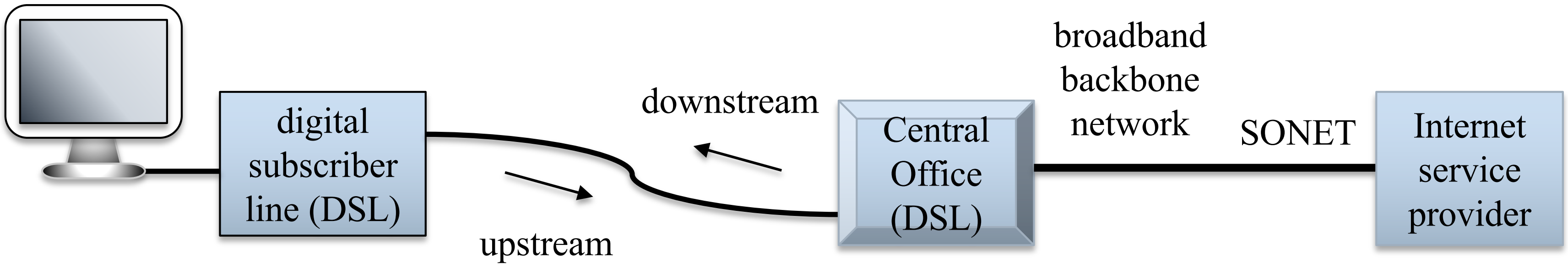
Baseband M-ary PAM

$$\frac{E[S^2]}{T} = \left(\frac{1}{T_b}\right)\frac{(M^2-1)}{3\log_2(M)}$$

For fixed $R_b = 1/T_b$ (bps) and level distance = 2, the transmitted power of an M-ary PAM transmission signal is increased by a factor $M^2/\log_2 M$.

Digital Subscriber Lines (DSL)

- A DSL operates over a local loop (often less than 1.5km) that provides a direct connection between a user terminal (e.g., computer) and a telephone company's *central office* (CO).
  - Since it is a direct connection, no dialup is necessary.
  - The information-bearing signal is kept in the digital domain all the way from the user terminal to an Internet service provider.



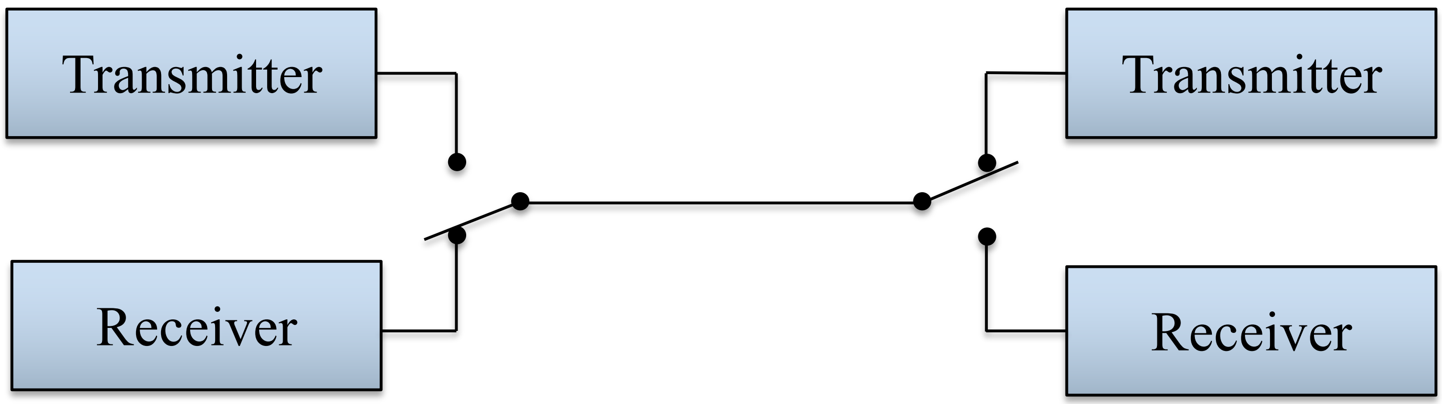
SONET: Synchronous Optical Networking

Digital Subscriber Lines (DSL)

- DSL is intended to provide *high data-rate*, *full-duplex*, *digital* transmission capability using local cost configuration (such as twisted pairs for ordinary telephonic communications).
- One of two possible modes can be used to achieve the full-duplex goal.
  - Time compression multiplexing (TCM) mode
  - Echo-cancellation (EC) mode

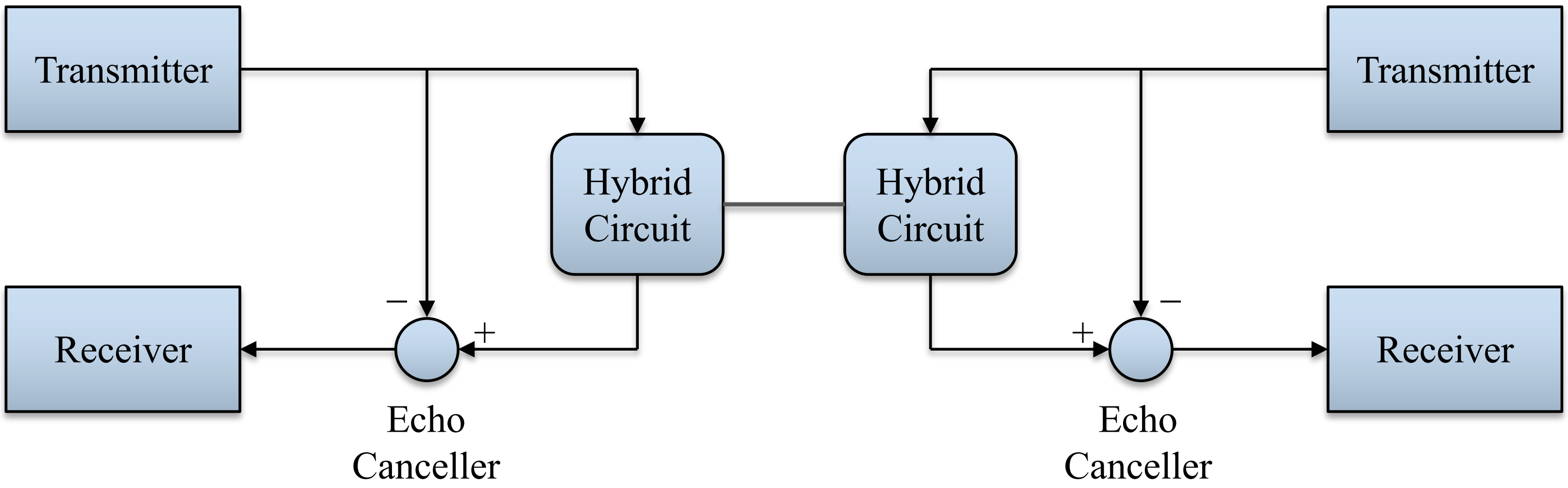
Digital Subscriber Lines (DSL)

- Time-compression multiplexing (TCM) mode
  - A guard time is often inserted between bursts in the two opposite directions of data.
  - The required line rate is slightly greater than twice the data rate.



Digital Subscriber Lines

- Echo-cancellation (EC) mode
  - Support the simultaneous flow of data along the common line in both directions.
  - In this mode, the line rate is the same as the data rate.



Digital Subscriber Lines (DSL)

- Comparison between TCM mode and EC mode
  - EC offers a much better data transmission performance at the expense of higher complexity.
  - However, with the recent advance in VLSI, complexity is no longer a main system concern. So, in North America, the EC mode has been adopted as the basis for designing the transceiver.

Digital Subscriber Lines (DSL)

- Other impairments to DSL
  - *ISI* and *Crosstalk*
    - *The transfer function of a twisted pair line can be approximated by*

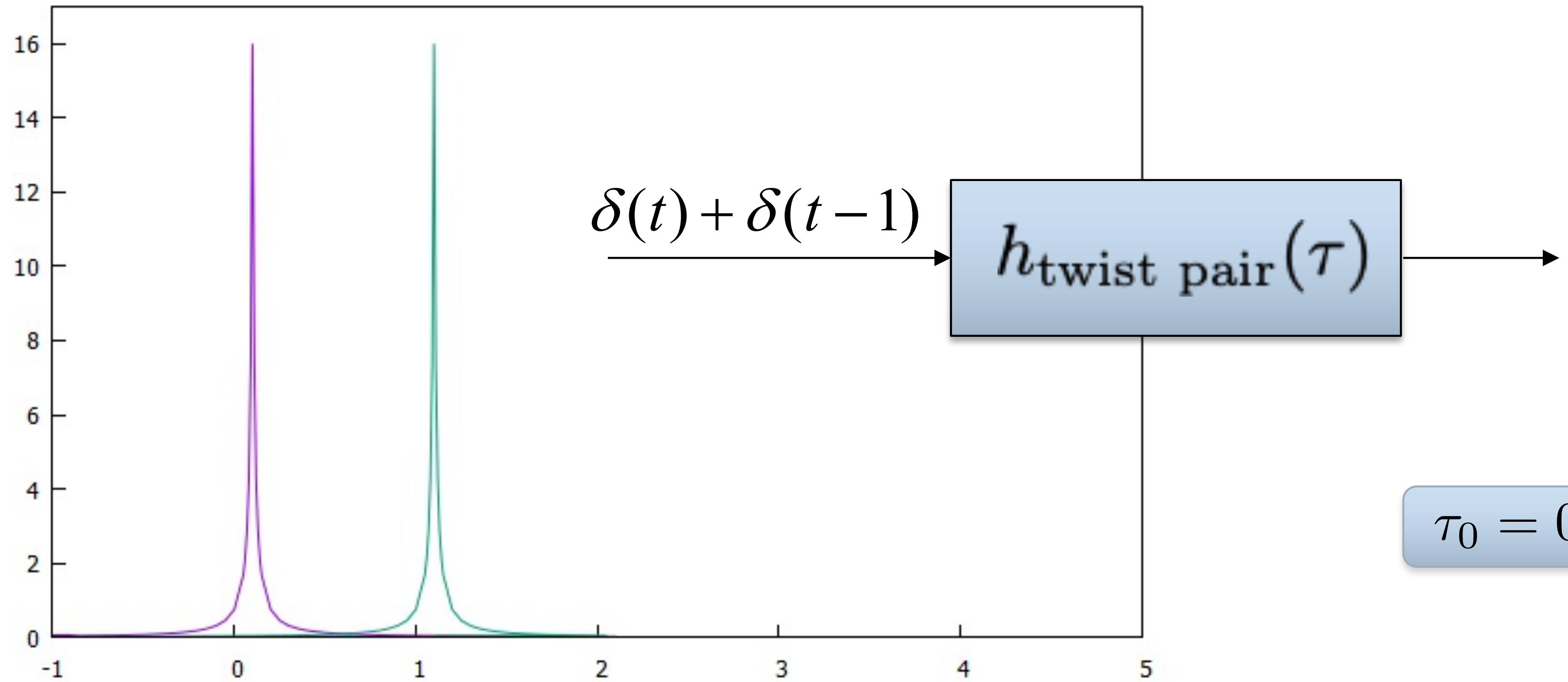
$$|H_{\text{twist pair}}(f)|^2 = \exp(-\alpha\sqrt{|f|})$$

where $\alpha = k\dfrac{l}{l_0}$, k is a physical constant of the twisted pair, and l_0 and l are respectively the reference length and actual length of the twisted pair.

Digital Subscriber Lines (DSL)

- **ISI**

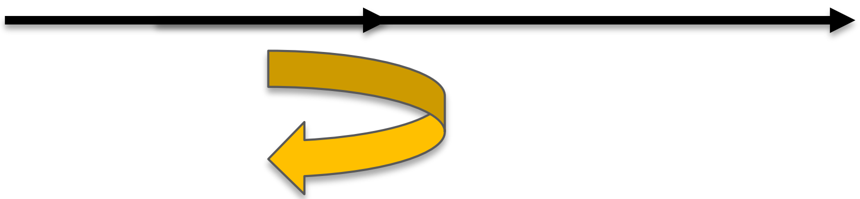
$$\begin{cases} H_{\text{twist pair}} = \exp(-\frac{1}{2}\sqrt{|f|})\exp(-j2\pi f\tau_0) \\ h_{\text{twist pair}}(\tau) = \dfrac{2\pi\sqrt{\tau}\left(\cos\left(\frac{1}{32\pi|\tau|}\right)+\sin\left(\frac{1}{32\pi|\tau|}\right)\right) - {}_1F_2(1;\frac{3}{4},\frac{5}{4};-\frac{1}{4096\pi^2\tau^2})}{16\pi^2\tau^2} \end{cases}$$



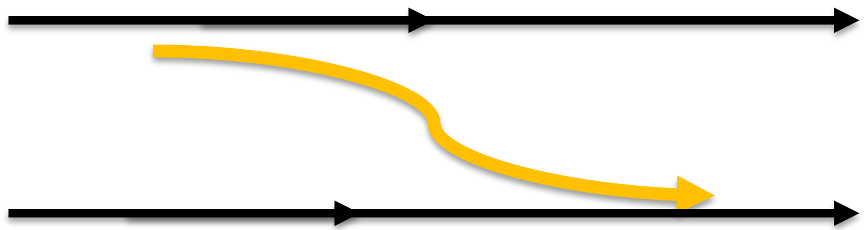
$\tau_0 = 0.1$

Digital Subscriber Lines (DSL)

- Crosstalk
  - Capacitive coupling that exists between adjacent twisted pairs in a cable
    - Near-end crosstalk (NEXT) and Far-end crosstalk (FEXT)



Near-end crosstalk



Far-end crosstalk

Digital Subscriber Lines (DSL)

- Crosstalk (cont.)
  - FEXT suffers the same *line loss* as the signal, whereas NEXT does not.
    - This is close to the phenomenon of *near-far effect* of wireless channel.
  - Accordingly, NEXT will be a more serious problem than FEXT. So, we can ignore the effect of FEXT, and add NEXT filter to the twisted pair channel model.

Digital Subscriber Lines (DSL)

- Other features of DSL channel
  - The PSD of the transmitted signal should be zero at zero frequency because no DC transmission through a *hybrid transformer* is possible.
  - The PSD of the transmitted signal should be low at high frequencies because
    - transmission attenuation in a twisted pair is most severe at high frequency;
    - crosstalk due to capacitive coupling between adjacent twisted pairs increases dramatically at high frequency (recall that the impedance of a capacitor is inversely proportional to frequency).

Digital Subscriber Lines (DSL)

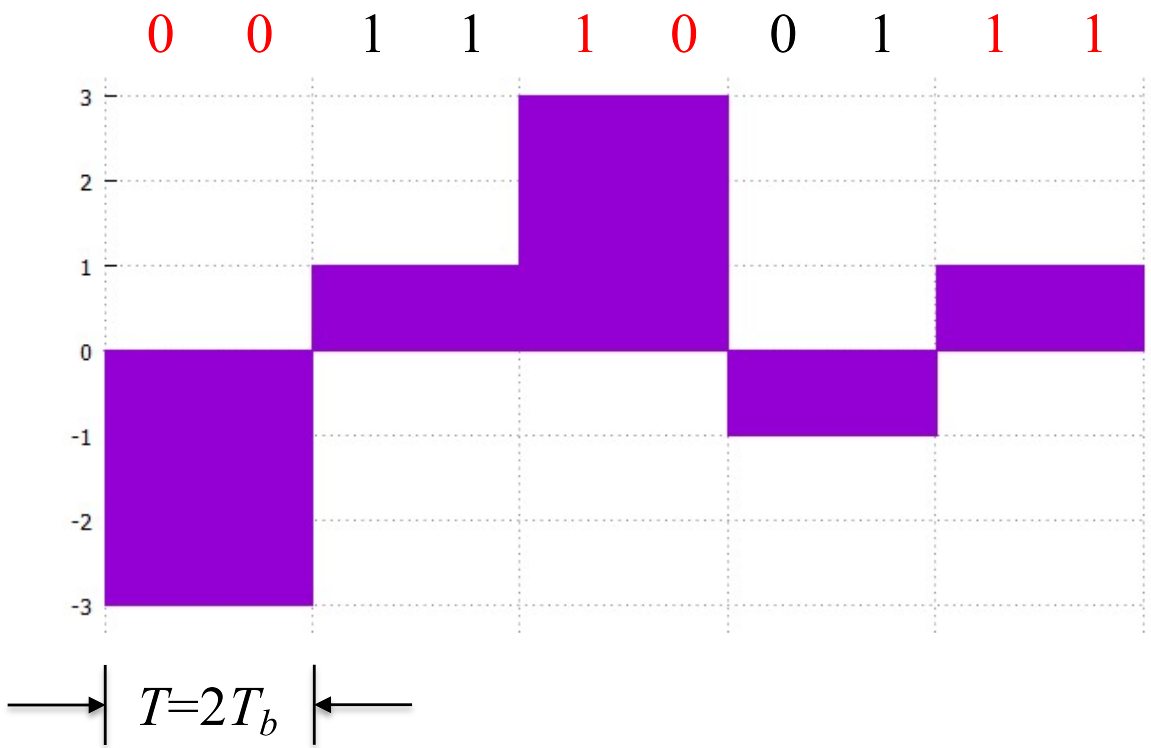
- Possible candidates for line codes that are suitable for DSL
  - Manchester code
    - Zero DC component but large spectrum at high frequency so it is vulnerable to NEXT and ISI.
  - Bipolar return to zero (BRZ) or Alternate mark inversion (AMI) code
    - Successive 1's are represented alternately by positive and negative but equal levels, and 0 is represented by a zero level.
    - Zero DC component. Its NEXT and ISI performance is slightly inferior to the *modified duobinary code* on all digital subscriber loops.

Digital Subscriber Lines (DSL)

- Possible candidates for line codes that are suitable for DSL
  - Modified duobinary code
    - Of no DC component and moderately spectrally efficient. However, its robustness against NEXT and ISI is about 2 to 3 dB poorer than that of (2B1Q) block codes on worst-case subscriber lines.
  - 2B1Q code
    - Two binary bits encoded into one quaternary symbol (four-level PAM signal).
    - Zero DC component, and offers the best performance among all the codes introduced. So, it is adopted as the standard as the North American standard for DSL.

Digital Subscriber Lines (DSL)

- Possible candidates for line codes that are suitable for DSL
  - 2B1Q code



Dibit	Amplitude
00	-3
01	-1
11	+1
10	+3

Digital Subscriber Lines (DSL)

- 2B1Q code (cont.)
  - With 2B1Q line coding, adaptive equalizer and echo cancellation, it is possible to achieve BER = 10^{-7} operating full duplex at 160 kb/s.

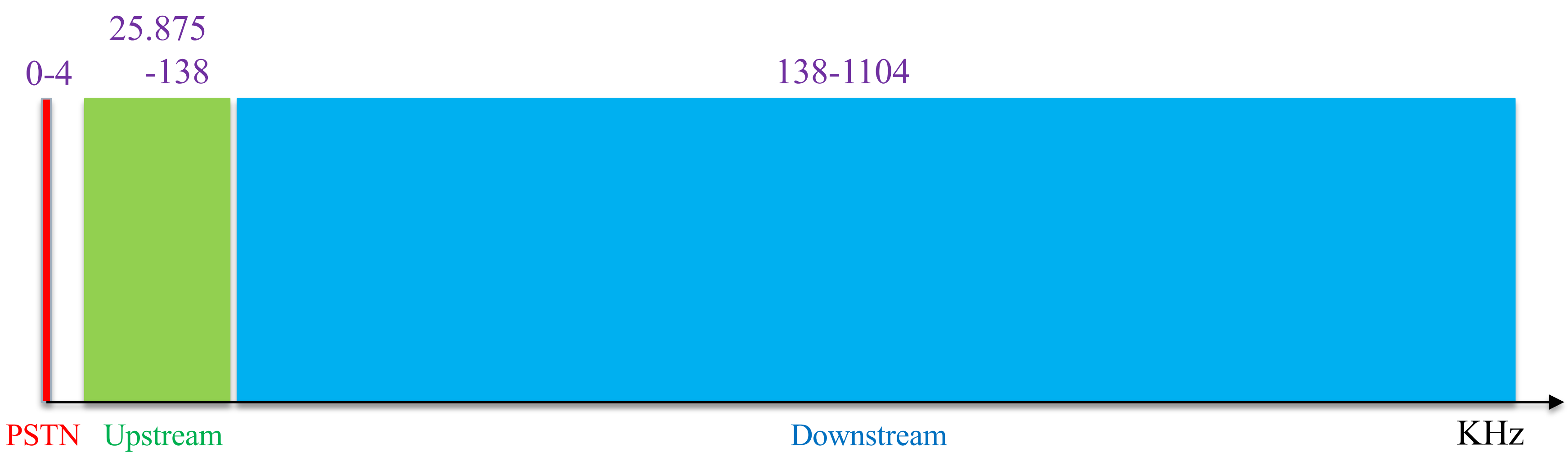
Asymmetric Digital Subscriber Lines

- ADSL is targeted to simultaneously support three services at a single twisted-wire pair
  - Data transmission downpstream at 9 Mbps
  - Data transmission upstream at 1Mpbs
  - Plain old telephone service (POTS)

- Some notes
  - It is named *asymmetric* because the downstream bit rate is much higher than the upstream bit rate.
  - The actually achievable bit rates depend on the length of the twisted pair used to do the transmission.

Asymmetric Digital Subscriber Lines

- Frequency-division multiplexing (FDM) technique is used to combine analog voice and DSL data.
- Upstream and downstream data transmission are placed in different frequency band to avoid crosstalk.



Asymmetric Digital Subscriber Lines

- Various applications can be applied to asymmetric transmissions, such as video-on-demand (VoD).
  - For example
    - Downstream = 1.544 Mbps (DS1) for video data
    - Upstream = 160 kbps for real-time control commands.

Optimal Linear Receiver

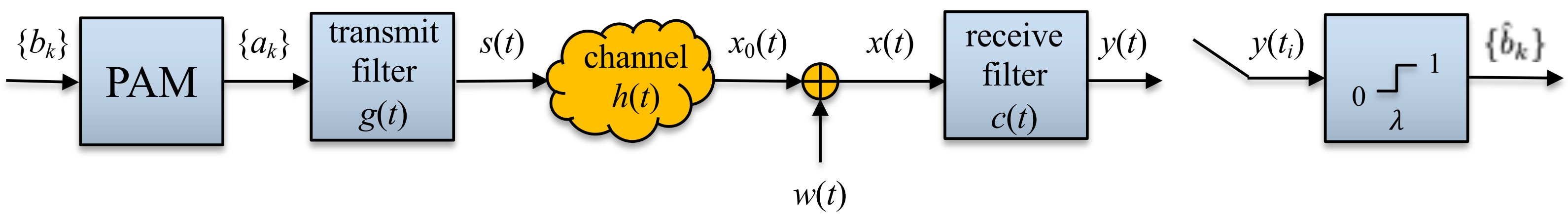
- Zero-forcing equalizer
  - A receiver design is to use a *zero-forcing equalizer* followed by a decision-making device.
  - The design objective of a zero-forcing equalizer is to force the ISI to "zero" at all sampling instances $t = kT_b$ for $k \neq 0$, provided that "the channel noise $w(t)$ is zero."

Optimal Linear Receiver

- Zero-forcing equalizer (cont.)
  - This reduces to *Nyquist's criterion*.

$$\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right) = T_b \quad \text{or} \quad p(nT_b) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

where $P(f) = G(f)H(f)C(f)$.



Optimal Linear Receiver

- Zero-forcing equalizer (cont.)
  - A serious consequence of the ignorance of $w(t)$ in the design of a zero-forcing equalizer is the performance degradation due to *noise enhancement*.

Optimal Linear Receiver

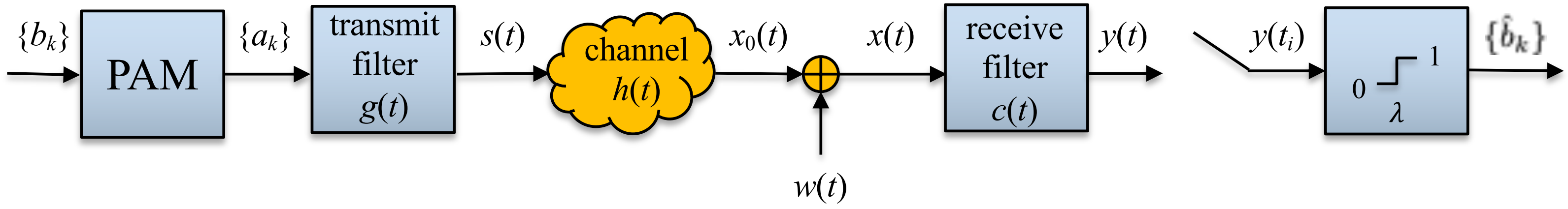
- Example of *noise enhancement*
  - Suppose that the receiver filter is a tapped-delay-line equalizer, which is of the form

$$c(t) = \sum_{k=0}^{\infty} c_k \delta(t - kT_b)$$

  - Assume ideally that $G(f) = 1$. Hence, Nyquist's criterion becomes:

$$p(nT_b) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

where $P(f) = H(f)C(f)$.



$$p(t) = \int_{-\infty}^{\infty} h(\tau) c(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) \left(\sum_{k=0}^{\infty} c_k \delta(t-\tau-kT_b) \right) d\tau$$

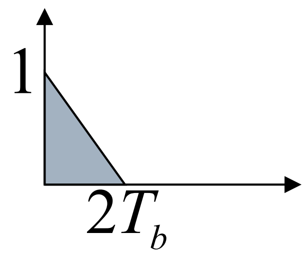
$$= \sum_{k=0}^{\infty} c_k \int_{-\infty}^{\infty} h(\tau) \delta(t-\tau-kT_b) d\tau$$

$$= \sum_{k=0}^{\infty} c_k h(t-kT_b)$$

$$p_n = p(nT_b) = \sum_{k=0}^{\infty} c_k h((n-k)T_b) = \sum_{k=0}^{\infty} c_k h_{n-k} = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$

It is reasonable to assume that $h_n = 0$ for $n < 0$, and $h_0 = 1$.

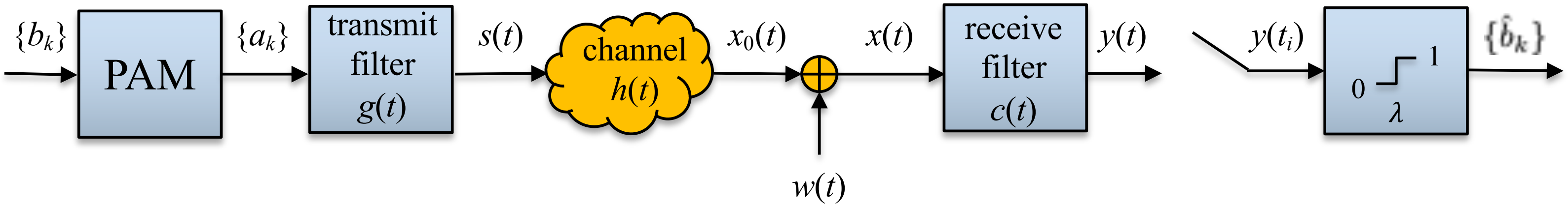
$$\Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ h_1 & 1 & 0 & \cdots & 0 \\ h_2 & h_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_N & h_{N-1} & h_{N-2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{N-1} \\ c_N \end{bmatrix} \text{ for arbitrary } N > 0.$$

Suppose 

$$h(\tau) = \begin{cases} 1 - |\tau|/(2T_b), & 0 \le \tau < 2T_b \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow h_0 = 1, h_1 = \frac{1}{2}, \text{ and } h_n = 0 \text{ for } n \ne 0,1.$$

$$\Rightarrow c_n = (-1)^n 2^{-n} \text{ for } (N \ge)\ n \ge 0, \text{ and zero, otherwise.}$$



The above $c(t)$ can successfully remove ISI, provided $w(t) = 0$. Now, add the additive white Gaussian noise $w(t)$, which also passes the filter $c(t)$.

At any time instance nT_b, the sampled noise becomes

$$\int_{-\infty}^{\infty} w(\tau)c(nT_b - \tau)d\tau = \int_{-\infty}^{\infty} w(\tau)\sum_{k=0}^{\infty} c_k \delta(nT_b - kT_b - \tau)d\tau$$

$$= \sum_{k=0}^{\infty} c_k \int_{-\infty}^{\infty} w(\tau)\delta(nT_b - kT_b - \tau)d\tau = \sum_{k=0}^{\infty} c_k w(nT_b - kT_b) = \sum_{k=0}^{\infty} c_k w_{n-k}$$

The sampled noise variance then becomes:

$$\mathrm{Var}\left[\sum_{k=0}^{\infty} c_k w_{n-k}\right] = \sum_{k=0}^{\infty} c_k^2 \mathrm{Var}[w_{n-k}] = \sigma_w^2 \sum_{k=0}^{\infty} 2^{-2k} = \frac{4}{3}\sigma_w^2 > \sigma_w^2$$

- An easier way to interpret the noise enhancement phenomenon
  - Nyquist's criterion requires that:

$$\sum_{n=-\infty}^{\infty} P\left(f-\frac{n}{T_b}\right)=\sum_{n=-\infty}^{\infty} H\left(f-\frac{n}{T_b}\right)C\left(f-\frac{n}{T_b}\right)=T_b$$

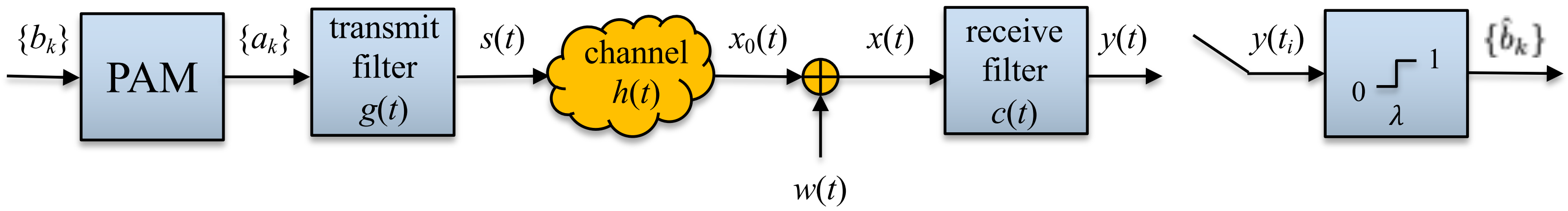
  - A sufficient condition for Nyquist's criterion is that:

$$H(f)C(f)=\text{Raised Cosine Spectrum}$$

  - When $H(f)$ is very small at some frequency range, $C(f)$ has to be very large at the same frequency range in order to "equalize" the spectrum.
  - Thus, the noise spectrum $S_W(f)|C(f)|^2$ after passing through $C(f)$ will be "enhanced."

Optimal Linear Receiver

- To alleviate noise enhancement phenomenon, it is better to simultaneously consider the ISI and channel noise.
- An approach of this kind is to use the *mean-square error criterion*, and find a balanced solution to the problem of reducing the effects of both channel noise and intersymbol interference.



$$\begin{cases} y(t) = c(t) * x(t) = \int_{-\infty}^{\infty} c(\tau)x(t-\tau)d\tau \\ x(t) = \sum_k a_k q(t - kT_b) + w(t) \\ q(t) = g(t) * h(t) \end{cases}$$

$$\Rightarrow y(iT_b) = \sum_k a_k \int_{-\infty}^{\infty} c(\tau)q(iT_b - \tau - kT_b)d\tau + \int_{-\infty}^{\infty} c(\tau)w(iT_b - \tau)d\tau = \xi_i + n_i$$

For perfect receiver, $y(iT_b) = a_i$.

So, the error $e_i = (\xi_i + n_i) - a_i$.

The mean squared error criterion then wishes to minimize :

$$J_i = E[e_i^2] = E[\{(\xi_i + n_i) - a_i\}^2].$$

$$= E[\xi_i^2] + E[n_i^2] + E[a_i^2] + 2E[\xi_i n_i] - 2E[n_i a_i] - 2E[\xi_i a_i]$$

1st term

For i.i.d. $\{a_k\}$, where $a_k = \pm 1$,

$$E[\xi_i^2] = \sum_k \sum_l E[a_k a_l] \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\tau_1) c(\tau_2) q(iT_b - kT_b - \tau_1) q(iT_b - lT_b - \tau_2) d\tau_1 d\tau_2$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(\tau_1) c(\tau_2) R_q(\tau_1, \tau_2; i) d\tau_1 d\tau_2$$

where $R_q(\tau_1, \tau_2; i) = \sum_k q(iT_b - kT_b - \tau_1) q(iT_b - kT_b - \tau_2)$

Observe that $R_q(\tau_1,\tau_2;i) = \sum_{k=-\infty}^{\infty} q(iT_b - kT_b - \tau_1)q(iT_b - kT_b - \tau_2)$ is invariant with respect to i, and under certain condition, it is only a function of $\tau_1 - \tau_2$. We can then re-express it as $R_q(\tau_1 - \tau_2)$.

$$
\begin{aligned}
E[\xi_i^2] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\tau_1)c(\tau_2)R_q(\tau_1-\tau_2)d\tau_1\,d\tau_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\tau_1)c(\tau_2)\left(\int_{-\infty}^{\infty} S_q(f)e^{\imath 2\pi f(\tau_1-\tau_2)}df\right)d\tau_1\,d\tau_2 \\
&= \int_{-\infty}^{\infty} S_q(f)\left(\int_{-\infty}^{\infty} c(\tau_1)e^{-\imath 2\pi(-f)\tau_1}d\tau_1\right)\left(\int_{-\infty}^{\infty} c(\tau_2)e^{-\imath 2\pi f\tau_2}d\tau_2\right)df \\
&= \int_{-\infty}^{\infty} S_q(f)C(-f)C(f)df \\
&= \int_{-\infty}^{\infty} S_q(f)|C(f)|^2 df.
\end{aligned}
$$

2nd term

Assume white $w(t)$ with PSD $N_0/2$.

$$E[n_i^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\tau_1)c(\tau_2)E[w(iT_b-\tau_1)w(iT_b-\tau_2)]\,d\tau_1\,d\tau_2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(\tau_1)c(\tau_2)\frac{N_0}{2}\delta(\tau_1-\tau_2)\,d\tau_1\,d\tau_2 = \frac{N_0}{2}\int_{-\infty}^{\infty} c^2(\tau_1)\,d\tau_1$$

$$= \frac{N_0}{2}\int_{-\infty}^{\infty}|C(f)|^2 df$$

$$\begin{aligned}
&= \frac{N_0}{2}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} C(f_1)e^{\imath 2\pi f_1\tau}df_1\right)\left(\int_{-\infty}^{\infty} C(f_2)e^{\imath 2\pi f_2\tau}df_2\right)d\tau \\
&= \frac{N_0}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(f_1)C(f_2)\left(\int_{-\infty}^{\infty} e^{-\imath 2\pi[-(f_1+f_2)]\tau}d\tau\right)df_1 df_2 \\
&= \frac{N_0}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(f_1)C(f_2)\delta(-f_1-f_2)df_1 df_2 \\
&= \frac{N_0}{2}\int_{-\infty}^{\infty} C(f_1)C(-f_1)df_1
\end{aligned}$$

3rd term

For i.i.d. $\{a_k\}$ where $a_k = \pm 1$, $E[a_i^2] = 1$.

4th and 5th term

By independence of $\{a_k\}$ and $w(t)$, and zero mean of n_i,

$$E[\xi_i n_i] = E[\xi_i]E[n_i] = 0 \text{ and } E[n_i a_i] = E[n_i]E[a_i] = 0.$$

6th term

$$E[\xi_i a_i] = \sum_k E[a_k a_i] \int_{-\infty}^{\infty} c(\tau) q(iT_b - kT_b - \tau) d\tau = \int_{-\infty}^{\infty} c(\tau) q(-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} C(f_1) e^{\imath 2\pi f_1 \tau} df_1 \right) \left(\int_{-\infty}^{\infty} Q(f_2) e^{\imath 2\pi f_2 (-\tau)} df_2 \right) d\tau$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(f_1) Q(f_2) \left(\int_{-\infty}^{\infty} e^{-\imath 2\pi (f_2 - f_1)\tau} d\tau \right) df_1 df_2$$

$$
\begin{aligned}
E[\xi_i a_i] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(f_1)Q(f_2)\delta(f_2 - f_1)df_1 df_2 \\
&= \int_{-\infty}^{\infty} C(f)Q(f)df \\
&= \int_{-\infty}^{\infty} \left[C_\mathrm{r}(f)Q_\mathrm{r}(f) - C_\mathrm{i}(f)Q_\mathrm{i}(f)\right] df
\end{aligned}
$$

where the last step follows from the observation that $E[\xi_i a_i]$ must be a real number, and $C_\mathrm{r}(f)$ and $C_\mathrm{i}(f)$ are respectively the real and imaginary parts of $C(f)$, i.e., $C(f) = C_\mathrm{r}(f) + \imath C_\mathrm{i}(f)$, and similarly $Q(f) = Q_\mathrm{r}(f) + \imath Q_\mathrm{i}(f)$.

Substitute all six terms into J_i.

$$
J_i = \int_{-\infty}^{\infty} \underbrace{\left[\left(S_q(f) + \frac{N_0}{2}\right)|C(f)|^2 - 2Q_\mathrm{r}(f)C_\mathrm{r}(f) + 2Q_\mathrm{i}(f)C_\mathrm{i}(f)\right]}_{A(f)} df + 1
$$

$$
\begin{aligned}
A(f) &= \left(S_q(f)+\frac{N_0}{2}\right)|C(f)|^2 - 2Q_{\rm r}(f)C_{\rm r}(f) + 2Q_{\rm i}(f)C_{\rm i}(f) \\
&= \left(S_q(f)+\frac{N_0}{2}\right)C_{\rm r}^2(f) - 2Q_{\rm r}(f)C_{\rm r}(f) \\
&\quad + \left(S_q(f)+\frac{N_0}{2}\right)C_{\rm i}^2(f) + 2Q_{\rm i}(f)C_{\rm i}(f) \\
&= \left(S_q(f)+\frac{N_0}{2}\right)\left[C_{\rm r}(f) - \frac{Q_{\rm r}(f)}{(S_q(f)+N_0/2)}\right]^2 - \frac{Q_{\rm r}^2(f)}{(S_q(f)+N_0/2)} \\
&\quad + \left(S_q(f)+\frac{N_0}{2}\right)\left[C_{\rm i}(f) + \frac{Q_{\rm i}(f)}{(S_q(f)+N_0/2)}\right]^2 - \frac{Q_{\rm i}^2(f)}{(S_q(f)+N_0/2)}
\end{aligned}
$$

$$\Rightarrow C(f) = \frac{Q^*(f)}{S_q(f)+N_0/2}$$ for MMSE equalizer.

An equalizer that is so designed is referred to as the *minimum-mean square error* (MMSE) equalizer.

MMSE Equalizer

- Summary
  - The MMSE equalizer can be viewed as the concatenation of two filters:
    - A matched filter $Q^*(f)$ to $Q(f) = G(f)H(f)$
    - An equalizer whose frequency response is the inverse of $S_q(f) + N_0/2$.

MMSE Equalizer

- Property of $S_q(f)$
  - The text wrote that $S_q(f) = \frac{1}{T_b}\sum_k \left| Q\left(f + \frac{k}{T_b} \right) \right|^2$, which is periodic with period $1/T_b$. This implies that $R_q(\tau)$ consists of a series of pulse train with width T_b, which is **not entirely** true.

$$R_q(\tau_1 - \tau_2) = \sum_k q(kT_b - \tau_1) q(kT_b - \tau_2)$$

$$S_q(f) = \int_{-\infty}^{\infty} R_q(\tau)\exp(-j2\pi f\tau)d\tau = \int_{-\infty}^{\infty}\left(\sum_k q(kT_b-\tau)q(kT_b)\right)\exp(-j2\pi f\tau)d\tau$$

$$= \sum_k q(kT_b)\int_{-\infty}^{\infty} q(kT_b-\tau)\exp(-j2\pi f\tau)d\tau$$

$$= \sum_k q(kT_b)\int_{-\infty}^{\infty} q(v)\exp(-j2\pi f(kT_b-v))dv$$

$$= \sum_k q(kT_b)\exp(-j2\pi fkT_b)\int_{-\infty}^{\infty} q(v)\exp(j2\pi fv)dv$$

$$= Q^*(f)\sum_k q(kT_b)\exp(-j2\pi fkT_b)$$

$q(v)$ is real $\Leftrightarrow Q^*(f) = Q(-f)$

$$= Q^*(f)\int_{-\infty}^{\infty}\left(\sum_k q(t)\delta(t-kT_b)\right)\exp(-j2\pi ft)dt$$

$$= Q^*(f)\cdot\frac{1}{T_b}\sum_k Q\left(f+\frac{k}{T_b}\right)$$

Realization of MMSE Equalizer

- One can approximate $1/[S_q(f) + N_0/2]$ by a periodic function with:

$$S_q(f) = Q^*(f) \cdot \frac{1}{T_b} \sum_k Q\left(f + \frac{k}{T_b} \right) \approx \frac{1}{T_b} \sum_k \left| Q\left(f + \frac{k}{T_b} \right) \right|^2 \equiv \tilde{S}_q(f)$$

- Since $\Theta_q(f) = 1/[\tilde{S}_q(f) + N_0/2]$ is periodic with period $1/T_b$, we obtain by Fourier series that

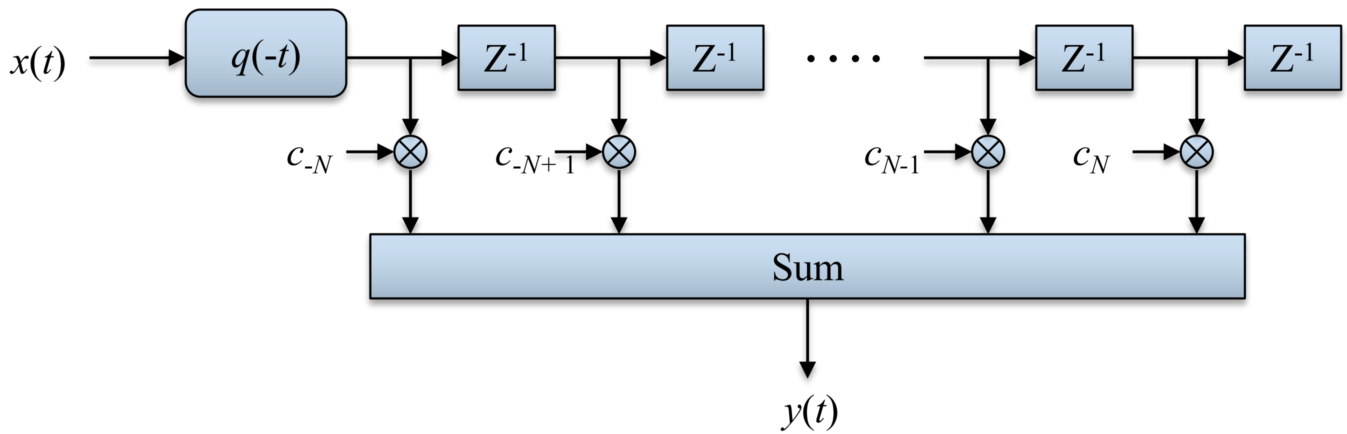
$$\Theta_q(f) = \sum_{k=-\infty}^{\infty} c_k \exp(j2\pi k f T_b)$$

where $c_k = T_b \int_{-1/(2T_b)}^{1/(2T_b)} \Theta_q(f) \exp(-j2\pi k f T_b) df.$

Realization of MMSE Equalizer

- We can approximate $\Theta_q(f)$ by its main $2N+1$ terms as:

$$\Theta_q(f) \approx \sum_{k=-N}^{N} c_k \exp(j2\pi k f T_b) \Rightarrow \theta_q(\tau) \approx \sum_{k=-N}^{N} c_k \delta(t + kT_b)$$



One can therefore approximate $1/[S_q(f) + N_0/2]$ by a transversal tapped-delay-line equalizer.

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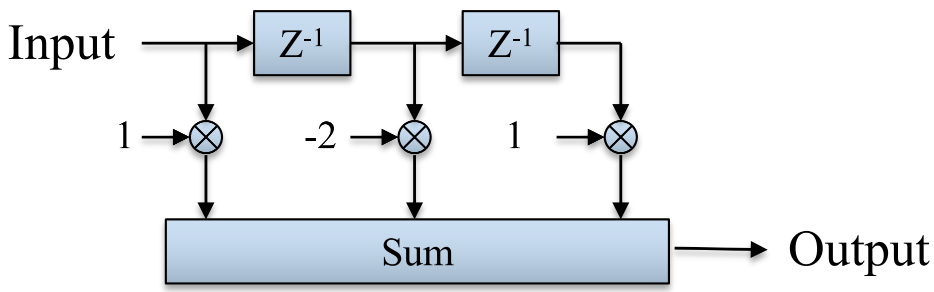


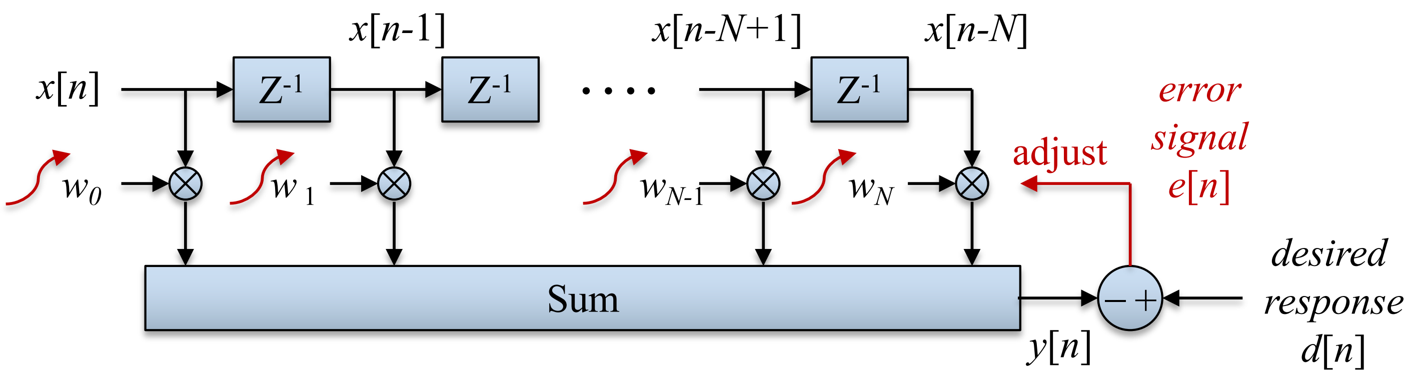
One can therefore approximate $1/[S_q(f) + N_0/2]$ by a transversal tapped-delay-line equalizer.

Realization of MMSE Equalizer

- Final notes
  - In a real-life telecommunication environment, the channel is usually time-varying.
  - Therefore, an *adaptive receiver* that provides the adaptive realization of both the matched filter and the equalizer in a combined manner is usually necessary.

Adaptive Equalization

- The equalizer is adjusted under the guidance of a *training sequence* transmitted through the channel.



Adaptive Equalization

- Least-mean-square (LMS) algorithm

$$e[n] = d[n] - y[n] = d[n] - \sum_{k=0}^{N} w_k x[n-k]$$

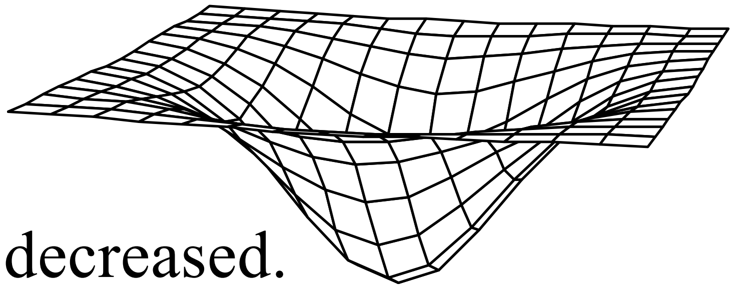
- Design objective
  - To find the filter coefficients w_0, w_1, …, w_N so as to minimize *index of performance* J:

$$J = e^2[n]$$

Adaptive Equalization

- To minimize J, we should update w_i toward the bottom of the J-bowel.

$$g_i \equiv \frac{\partial J}{\partial w_i}$$



  - So, when $g_i > 0$, w_i should be decreased.
  - On the contrary, w_i should be increased if $g_i < 0$.
  - Hence, we may define the update rule as:

$$\hat{w}_{i,\text{next}} = \hat{w}_{i,\text{current}} - \frac{1}{2}\mu \cdot g_i$$

where μ is a chosen constant step size, and ½ is included only for convenience of analysis.

Adaptive Equalization

$$J = \left(d[n] - \sum_{k=0}^{N} w_k x[n-k] \right)^2$$

$$= d^2[n] - 2\sum_{k=0}^{N} w_k d[n] x[n-k] + \sum_{k=0}^{N}\sum_{j=0}^{N} w_k w_j x[n-k] x[n-j]$$

$$g_i = \frac{\partial J}{\partial w_i} = -2d[n]x[n-i] + 2\sum_{k=0}^{N} w_k x[n-k] x[n-i]$$

$$= -2x[n-i]\left(d[n] - \sum_{k=0}^{N} w_k x[n-k] \right)$$

$$= -2x[n-i]e[n]$$

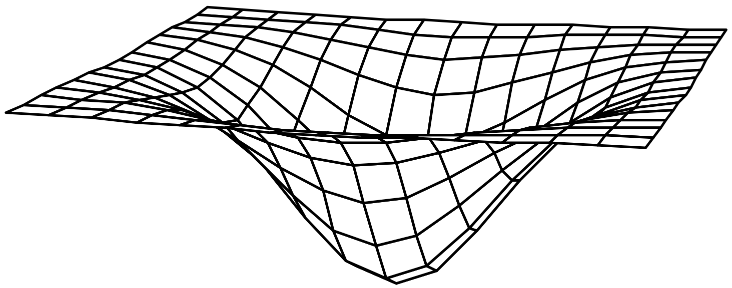
Adaptive Equalization

$$\Rightarrow \text{Repeat} \begin{cases} e[n] = d[n] - \displaystyle\sum_{k=0}^{N} w_{k,\text{current}} x[n-k] \\ \text{For } 0 \le i \le N, w_{i,\text{next}} = w_{i,\text{current}} + \mu \cdot x[n-i]e[n] \\ \text{For } 0 \le i \le N, w_{i,\text{current}} = w_{i,\text{next}} \end{cases}$$

- Some notes on LMS algorithm
  - There is no guarantee that the algorithm converges to a local minimum (could converge to a saddle point).
  - There is even no guarantee that the algorithm converges.

Adaptive Equalization

- Some notes on LMS algorithm (cont.)
  - If μ is too large, high excess mean-square error may occur.
  - If μ is too small, a *slow rate of convergence* may arise.



Operation of the Equalizer

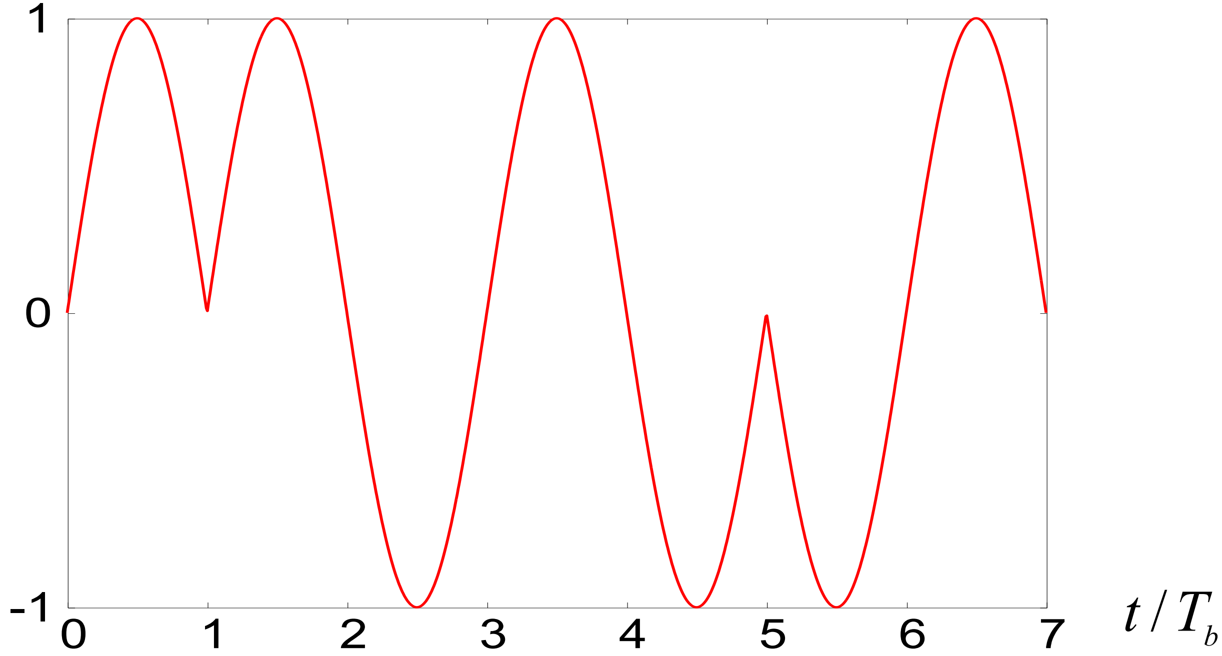
- Two modes of operations for adaptive equalizer
  - Training mode
  - Decision-directed mode

Decision-Directed Mode

- In normal operation, the decisions made by the receiver are correct with high probability.
- Under such premise, we can use the previous decisions to *calibrate* or *track* the tap coefficients.
- In this mode,
  - if μ is too large, high excess mean-square error may occur.
  - if μ is too small, a *too-slow tracking* may arise.

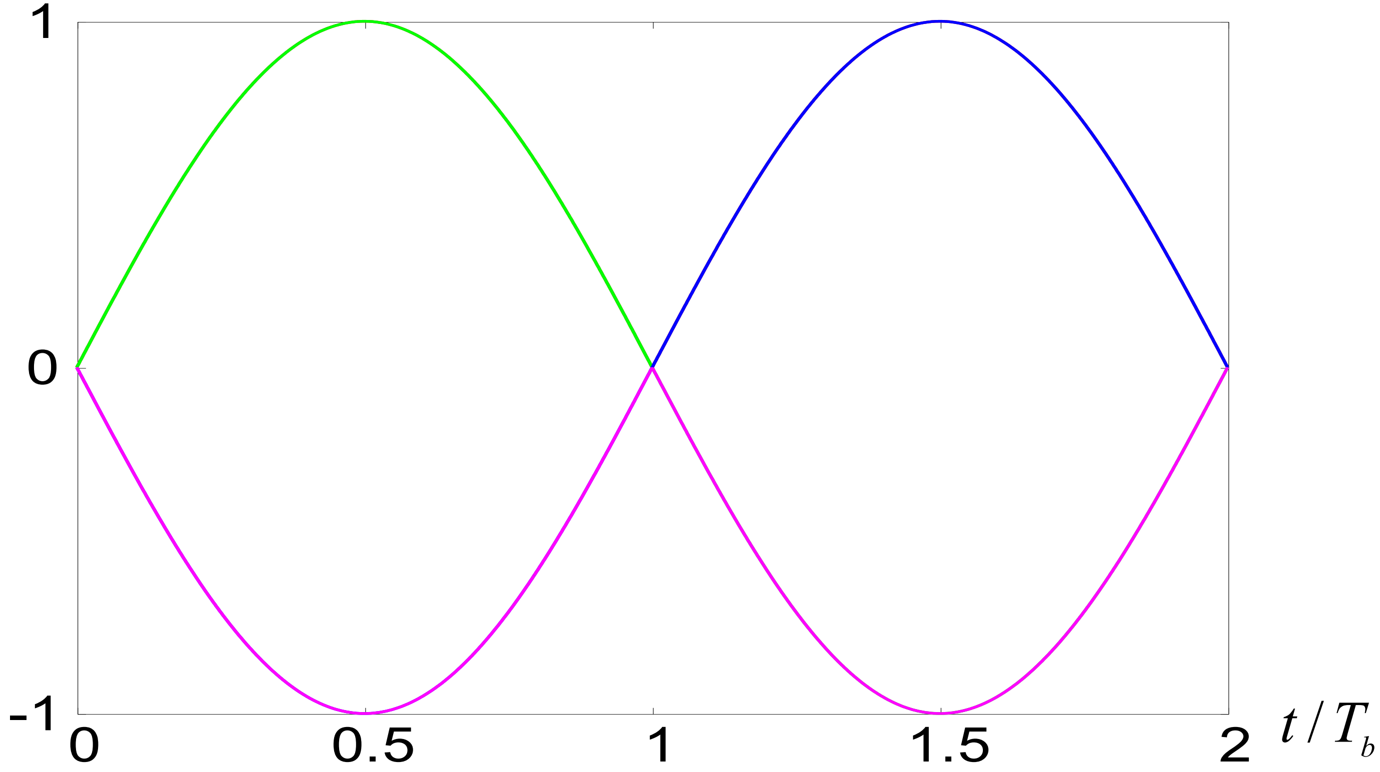
Eye Patterns

- A good tool to examine ISI is the eye pattern.
- Eye pattern: The synchronized superposition of all possible realizations of the signal viewed within a particular signaling interval.



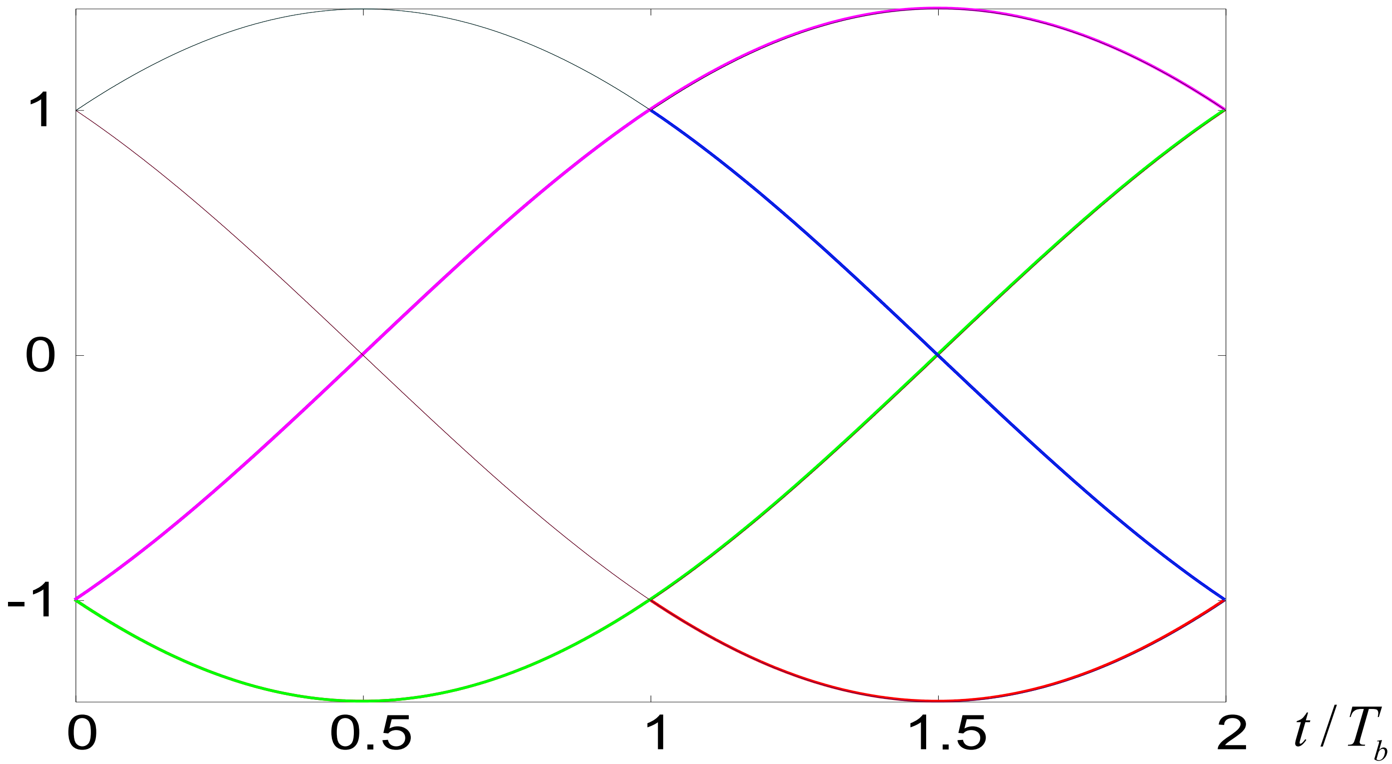
Eye Patterns

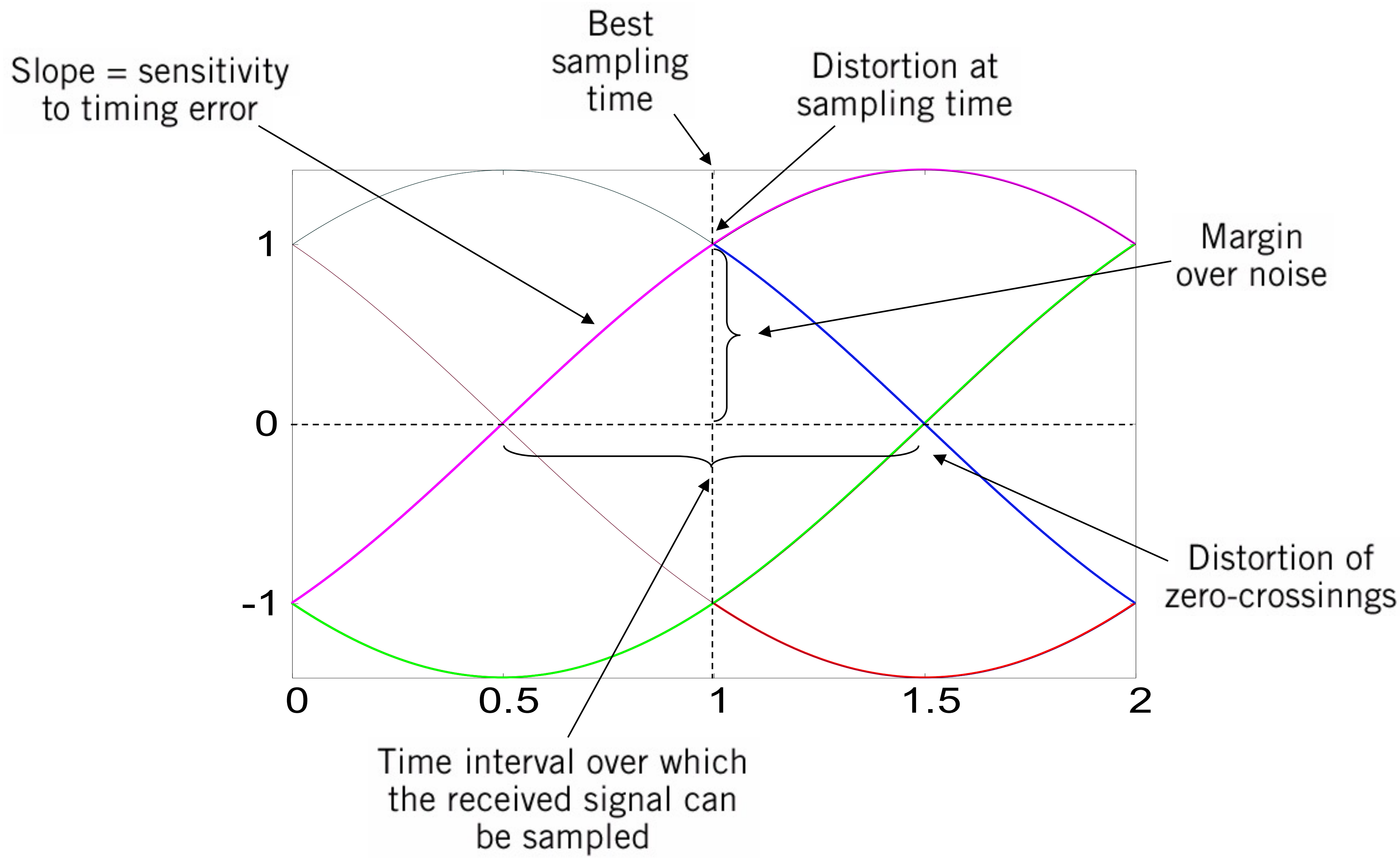
- The eye pattern for pulse shaping function $p(t)$ that is half-cycle sine wave with duration T_b, and with error-free ± 1 transmission.



Eye Patterns

- The eye pattern for pulse shaping function $p(t)$ that is half-cycle sine wave with duration $2T_b$, and with error-free ± 1 transmission.

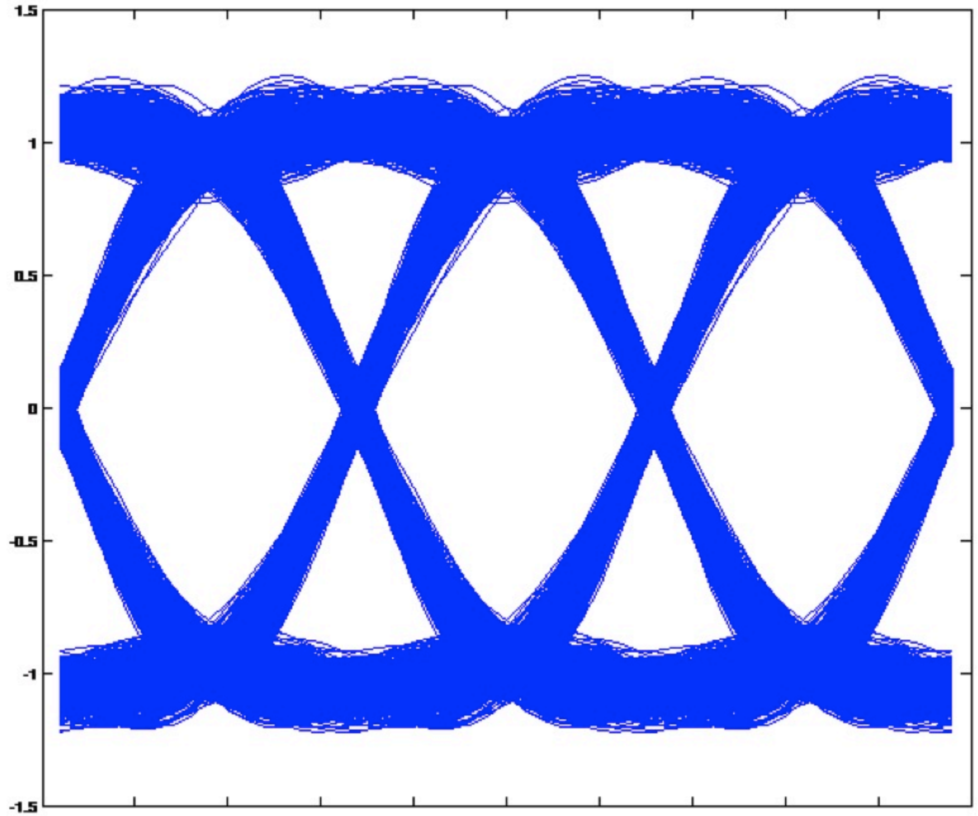




Best
sampling
time
Slope = sensitivity
to timing error
Distortion at
sampling time
Margin
over noise
Distortion of
zero-crossinngs
Time interval over which
the received signal can
be sampled
1
0
-1
0
0.5
1
1.5
2

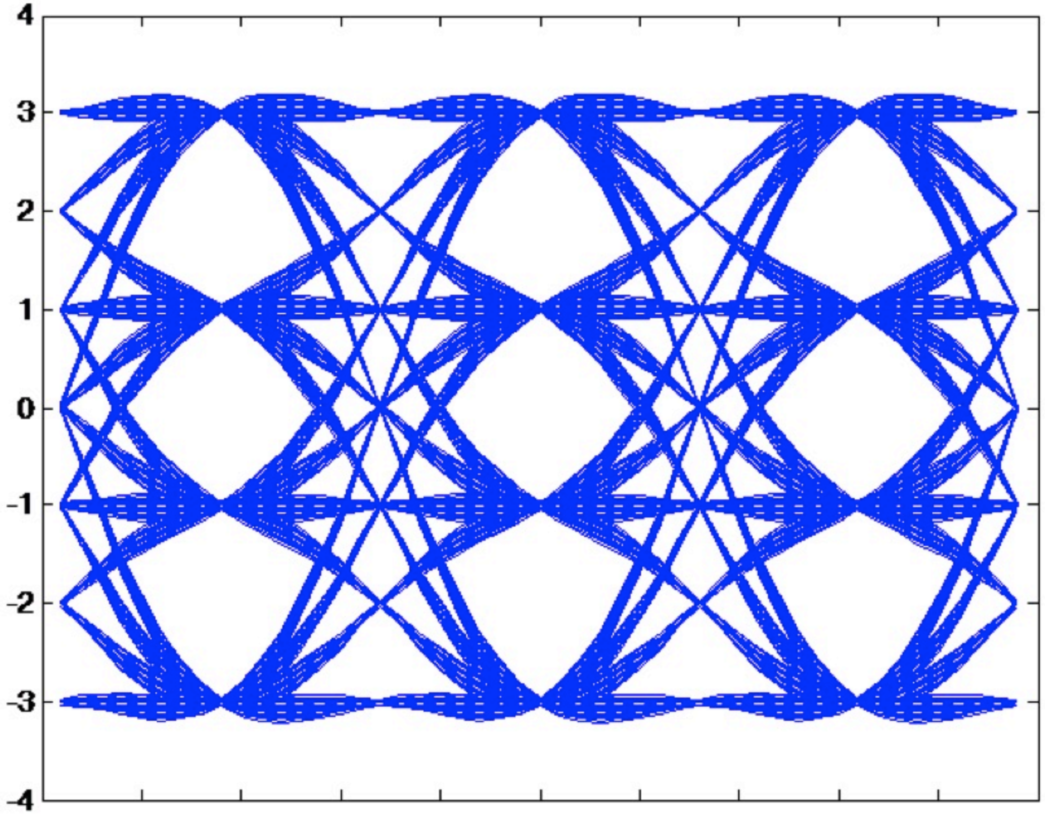
Eye Patterns

- The eye pattern for error-free ± 1 transmission but insufficient transmission bandwidth.



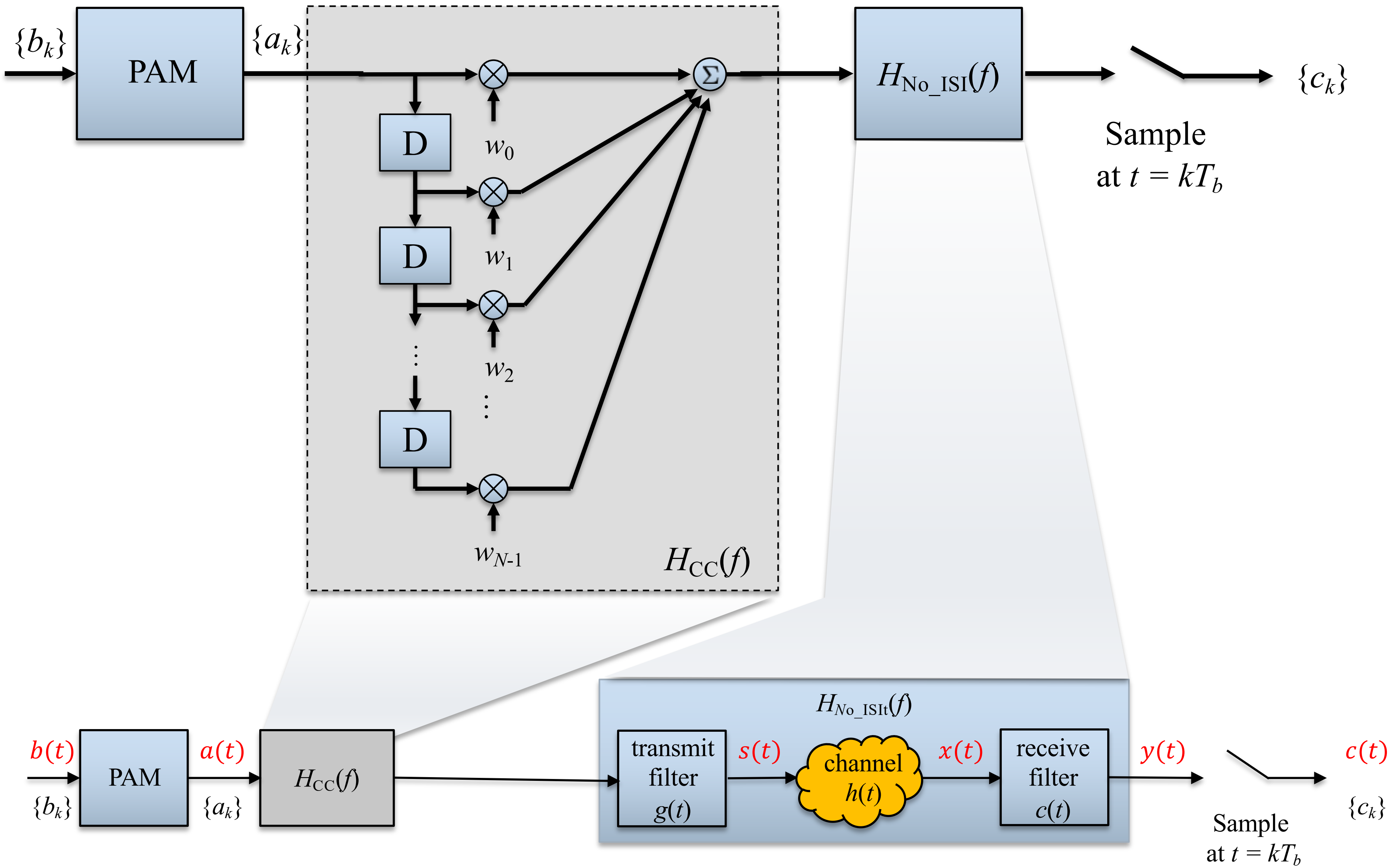
Eye Patterns

- The eye pattern for error-free 4PAM transmission but insufficient transmission bandwidth.



Summary

- ISI and background noise
- Matched filter
- Nyquist's criterion (Raised cosine spectrum)
- Correlative level coding (Duobinary and modified duobinary)
- DSL and ADSL
- Optimal linear receiver and MMSE equalizer



{bk}
PAM
{ak}
D
D
D
w0
w1
w2
wN-1
Σ
HCC(f)
HNo_ISI(f)
{ck}
Sample
at t = kTb
b(t)
PAM
a(t)
{bk}
{ak}
HCC(f)
HNo_ISIt(f)
transmit
filter
g(t)
s(t)
channel
h(t)
x(t)
receive
filter
c(t)
y(t)
c(t)
{ck}
Sample
at t = kTb