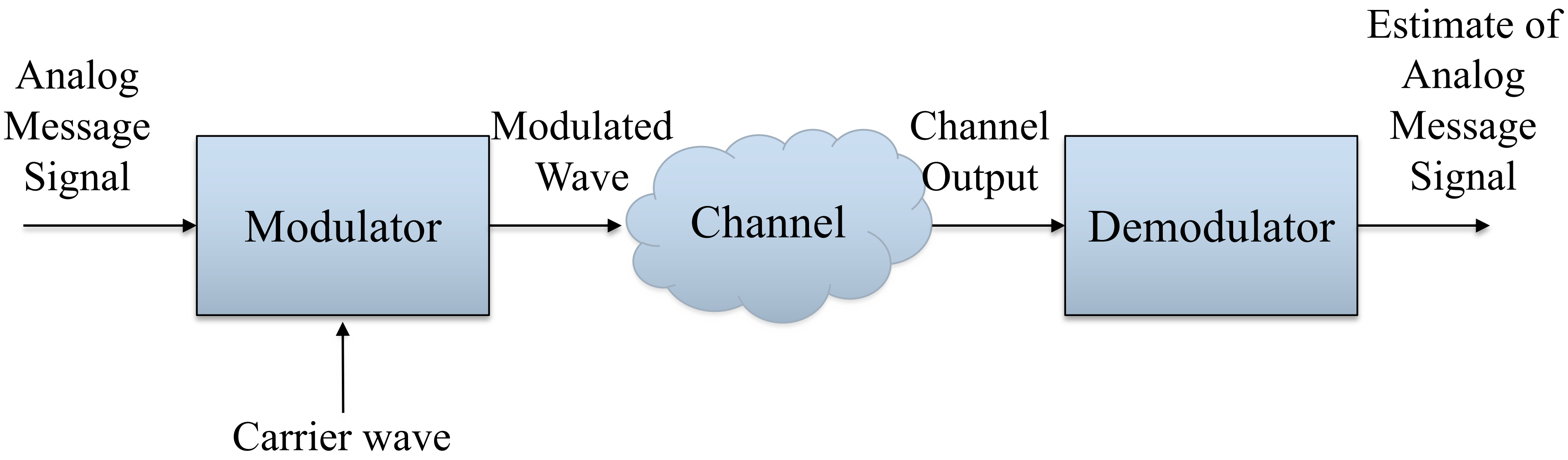
Part 4 Noisefree Analog Modulation and Demodulation

Introduction

- Analog communication system
  - The most common carrier is the sinusoidal wave.



Introduction

- Modulation
  - A process by which *some characteristic of a carrier* is varied in accordance with a *modulating wave* (baseband signal).
- Sinusoidal Continuous-Wave (CW) modulation
  - Amplitude modulation
  - Angle modulation

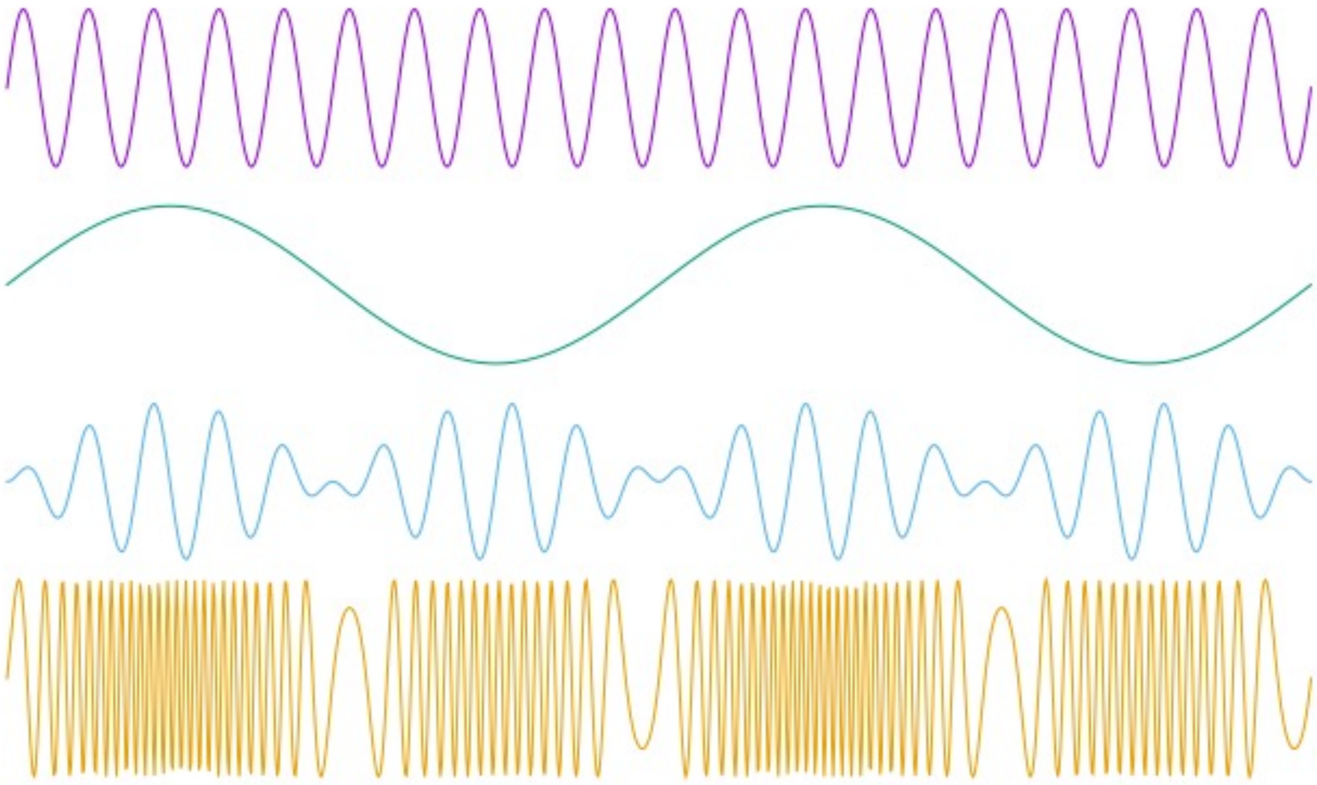
Introduction

Sinusoidal Carrier

Baseband Signal

Amplitude Modulation

Frequency Modulation



Double-Sideband with Carrier (DSB-C) or simply Amplitude Modulation (AM)

Carrier $c(t) = A_c \cos(2\pi f_c t)$

Baseband $m(t)$

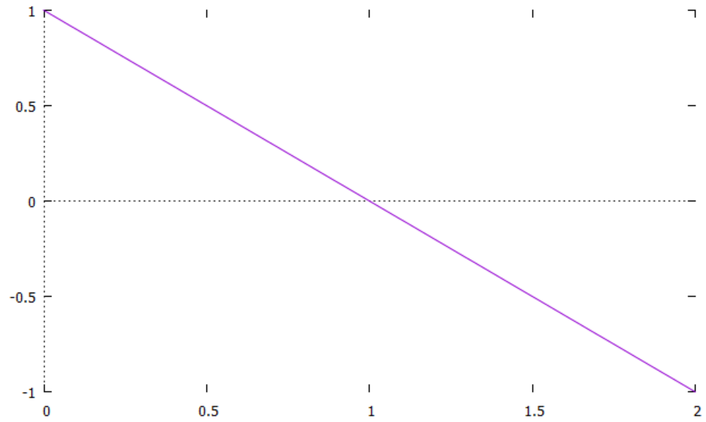
Modulated Signal $s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$,

where k_a is amplitude sensitivity or modulation index.

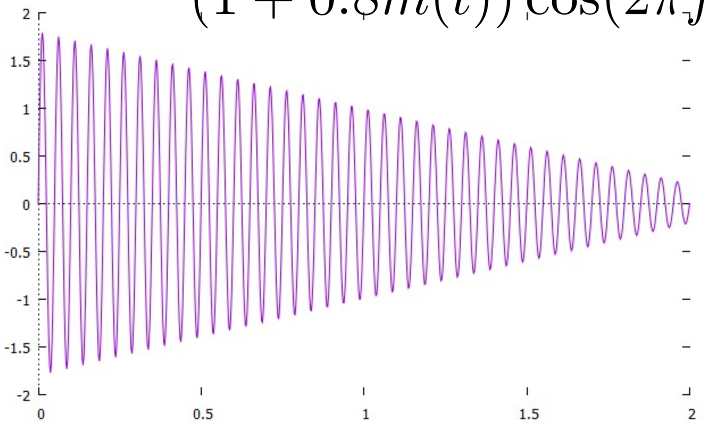
- Two required conditions on amplitude sensitivity
  - $1 + k_a m(t) \geq 0$, which is ensured by $|k_a m(t)| \leqq 1$.
    - The case of $|k_a m(t)| > 1$ is called *overmodulation*.
    - The value of $|k_a m(t)|$ is sometimes represented by "percentage" (because it is limited by 1), and is named $(|k_a m(t)| \times 100)\%$ modulation.
  - $f_c >> W$, where W is the message bandwidth.
    - Violation of this condition will cause **nonvisualized envelope**.

Overmodulation

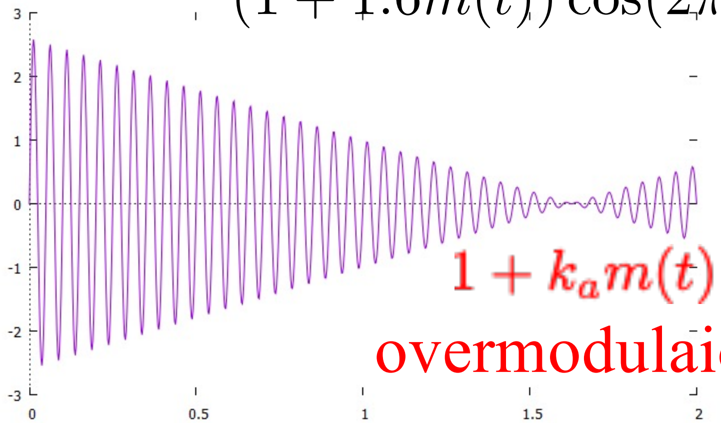
$m(t) = 1 - t$



$(1 + 0.8m(t))\cos(2\pi f_c t)$



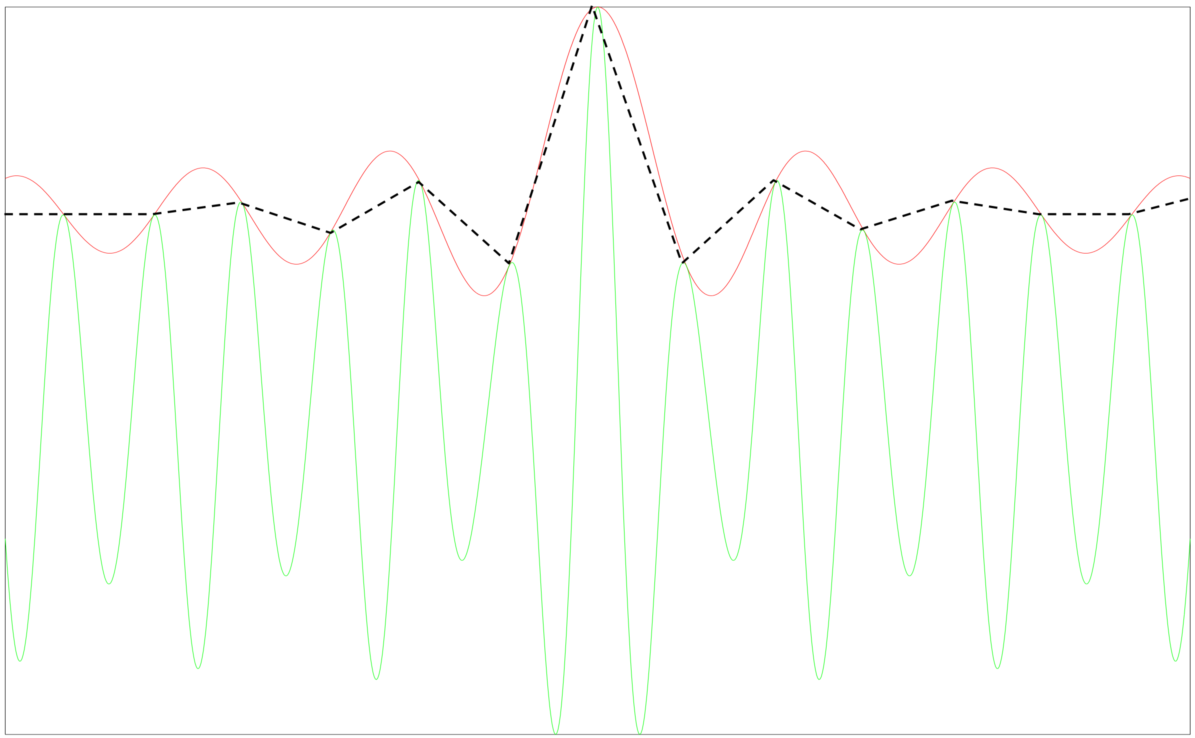
$(1 + 1.6m(t))\cos(2\pi f_c t)$



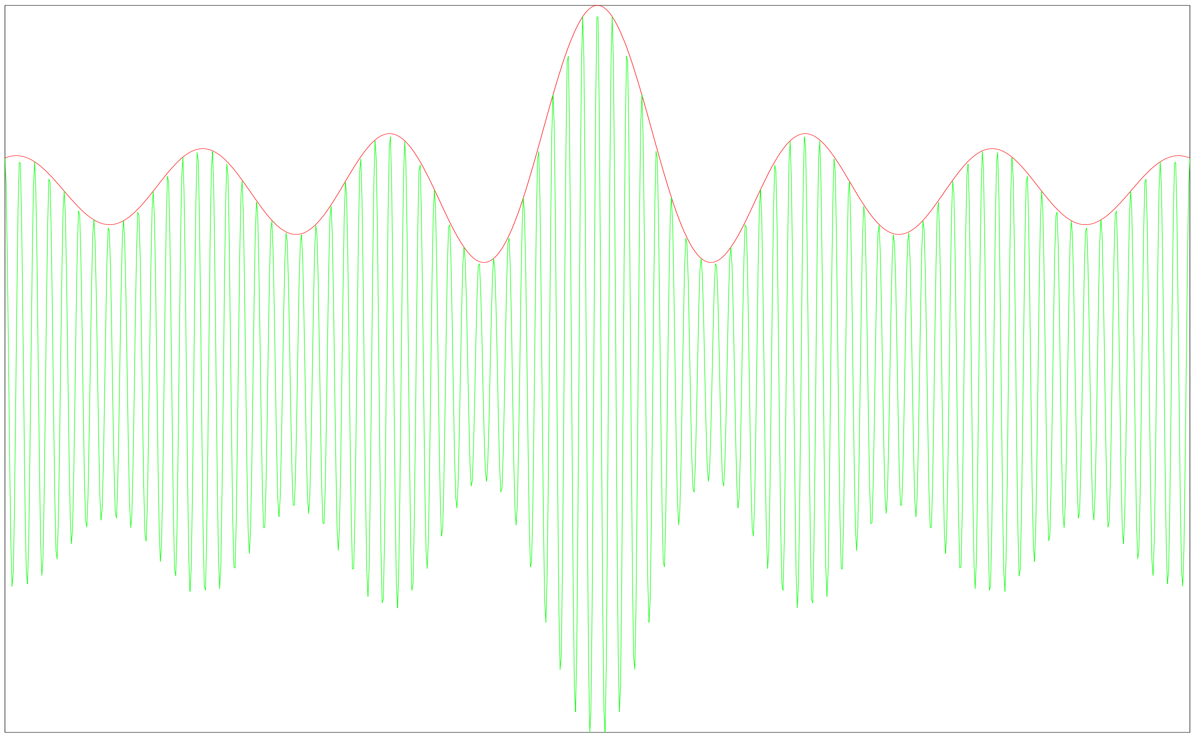
$1 + k_a m(t) < 0$

overmodulaion

Non-Visualized Envelope



Visualized Envelope

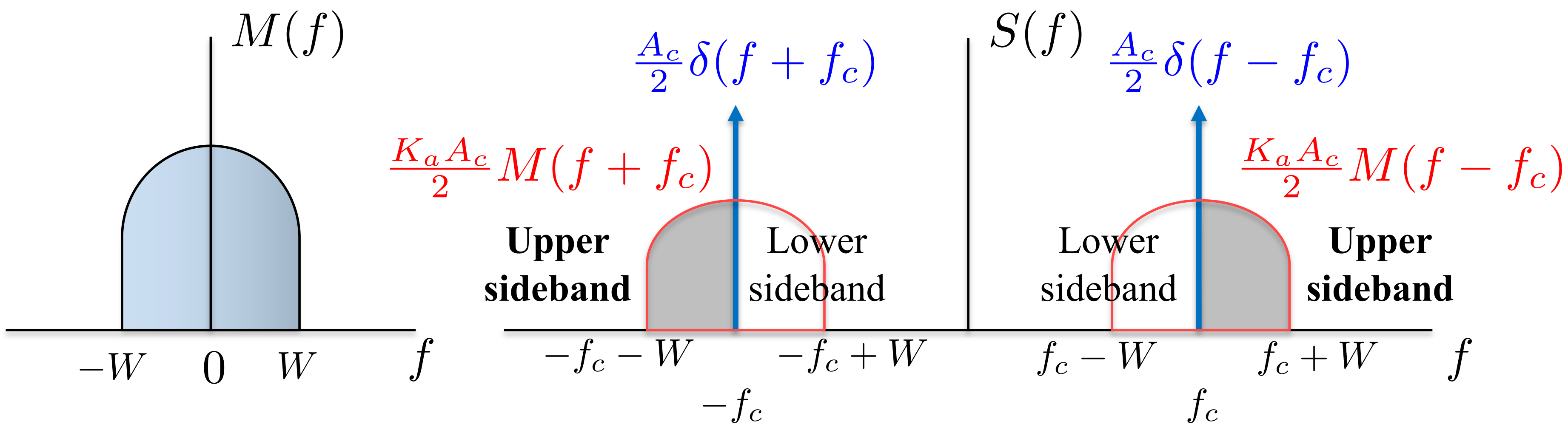


Transmission Bandwidth

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

$$\Rightarrow S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

Transmission bandwidth $B_T = 2W$.

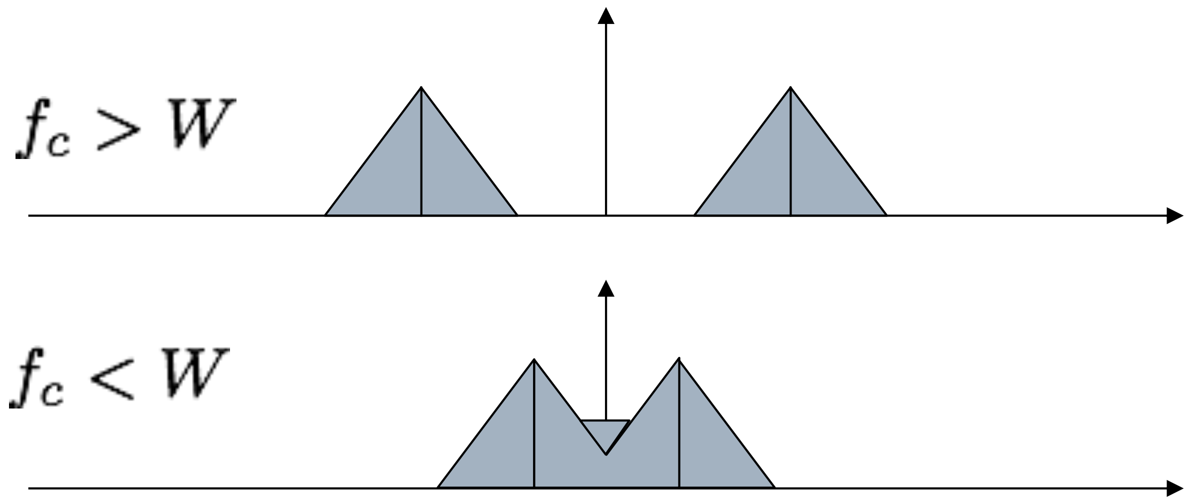


Transmission Bandwidth

- Transmission bandwidth of an AM wave
  - For positive frequencies, the highest frequency component of the AM wave equals $f_c + W$, and the lowest frequency component equals $f_c - W$.
  - The difference between these two frequencies defines the transmission bandwidth B_T for an AM wave.

Transmission Bandwidth

- The condition of $f_c > W$ ensures that the sidebands do not overlap.



Negative Frequency

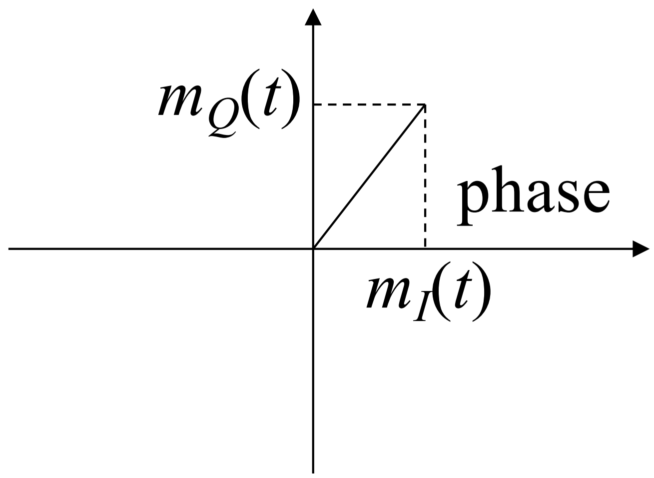
- Operational meaning of "negative frequency"
  - If time-domain signal is *real-valued*, the *negative frequency spectrum* is simply a (complex-conjugate) mirror of the *positive frequency spectrum*.
  - We may then define a one-sided spectrum as

$$S_{\text{one-sided}}(f) = 2S(f) \quad \text{for} \quad f \geq 0.$$

  - Hence, if only real-valued signal is considered, it is unnecessary to introduce "negative frequency."

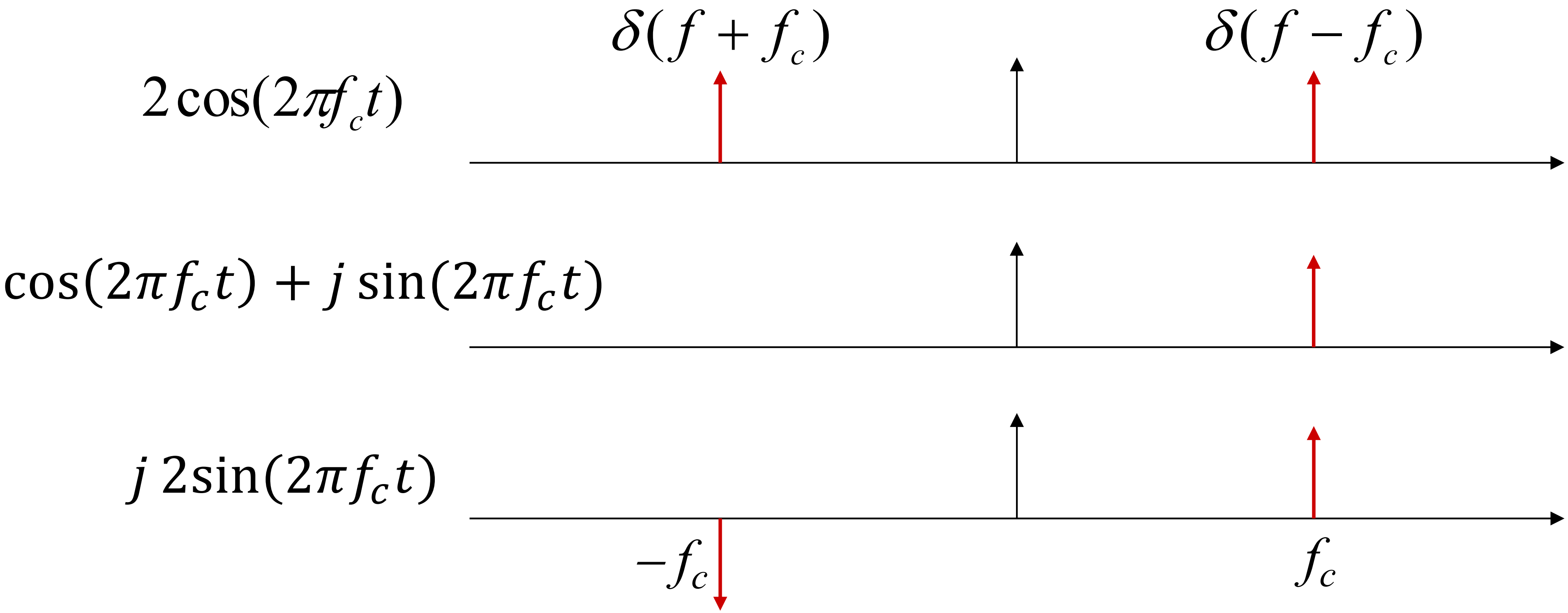
Negative Frequency

- So the introduction of *negative frequency part* is due to the need of *imaginary signal part*.
- Signal phase information is embedded in *imaginary signal part* of the signal.



Negative Frequency

- As a result, the following spectrums contain the same *frequency components* but *different phases*.



Negative Frequency

- Summary
  - *Complex-valued* baseband signal consists of information of amplitude and phase; while *real-valued* baseband signal only contains amplitude information.
  - One-sided spectrum only bears *amplitude information*, while two-sided spectrum (with negative frequency part) carries also *phase information*.

Virtues of Amplitude Modulation

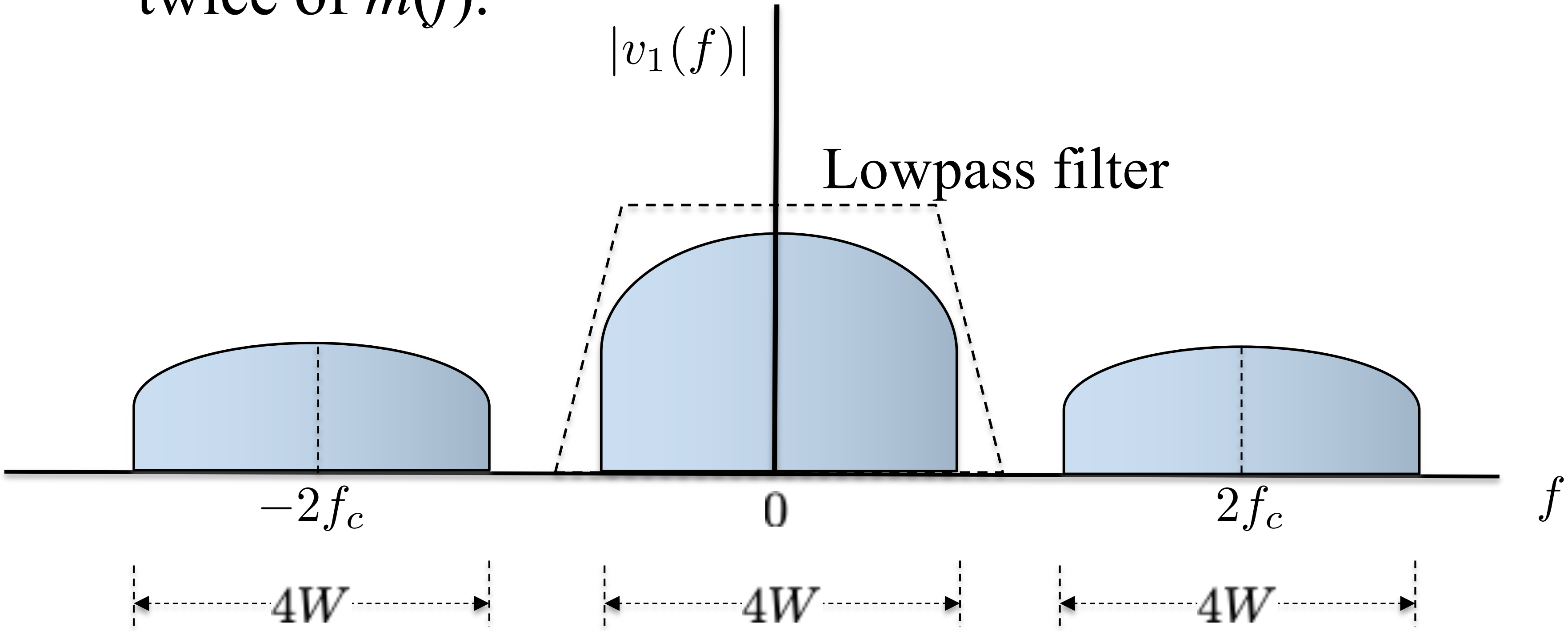
- AM receiver can be implemented in terms of *simple circuit* with *inexpensive electrical components*.
  - E.g., envelop detector

$$v_1(t) = s^2(t) = A_c^2[1 + k_a m(t)]^2 \cos^2(2\pi f_c t)$$

$$= \frac{A_c^2}{2}[1 + k_a m(t)]^2[1 + \cos(4\pi f_c t)]$$

Virtues of Amplitude Modulation

- The bandwidth of $m^2(t)$ is twice of $m(t)$. In other words, the bandwidth of $v_1(f) = m(f) \star m(f)$ is twice of $m(f)$.



Virtues of Amplitude Modulation

- So if $2f_c > 4W$,

$$\Rightarrow v_2(t) = \frac{A_c^2}{2}[1 + k_a m(t)]^2$$

$$\Rightarrow v_3(t) = \frac{A_c}{\sqrt{2}}[1 + k_a m(t)]$$

$$\text{if } m(t) \text{ is zero mean } \overset{\text{block DC}}{\Rightarrow} \frac{A_c k_a}{\sqrt{2}} m(t)$$

By means of a squarer, the receiver can recover the information-bearing signal **without a local carrier (or the knowledge of it)**.

Limitations of Amplitude Modulation (DSB-C)

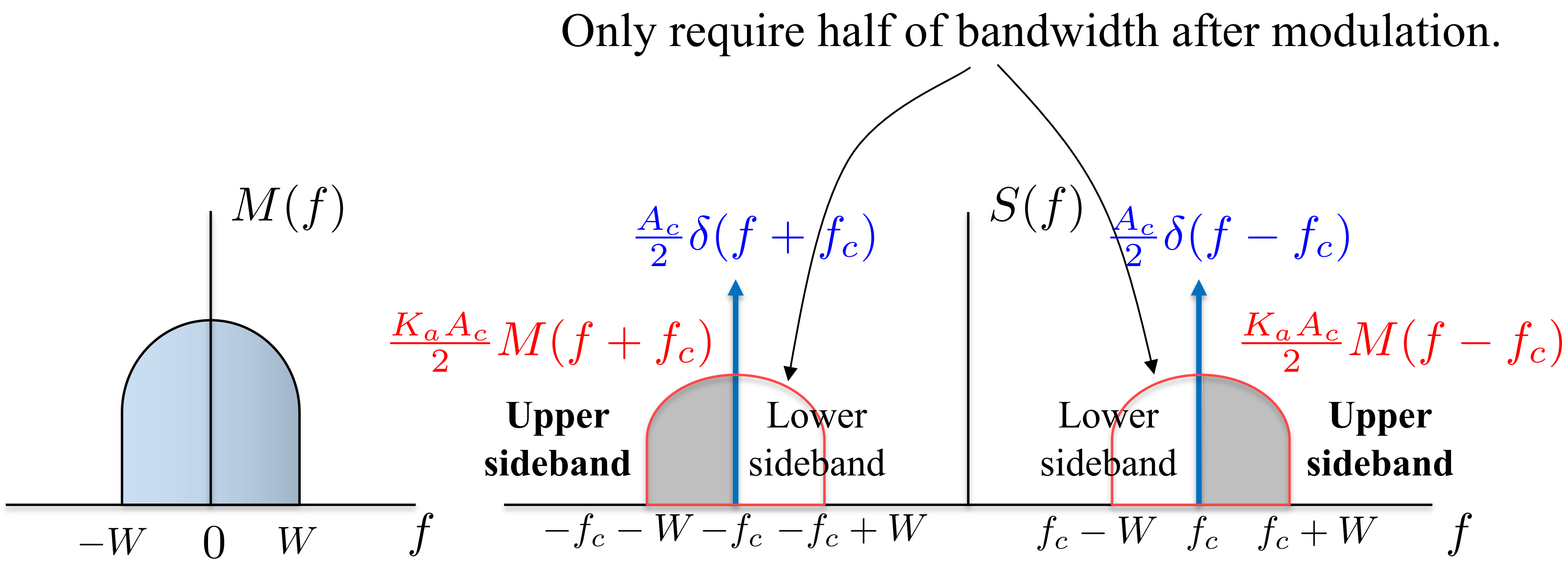
- Wasteful of power and bandwidth

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$
$$= \underbrace{A_c \cos(2\pi f_c t)}_{\text{with carrier}} + A_c k_a m(t)\cos(2\pi f_c t)$$

Waste of power in the information-less "with-carrier" part.

Limitations of Amplitude Modulation (DSB-C)

- Wasteful of power and bandwidth



Linear Modulation

- **Definition**
  - Both $s_I(t)$ and $s_Q(t)$ in

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

are *linear* function of $m(t)$.

Linear Modulation

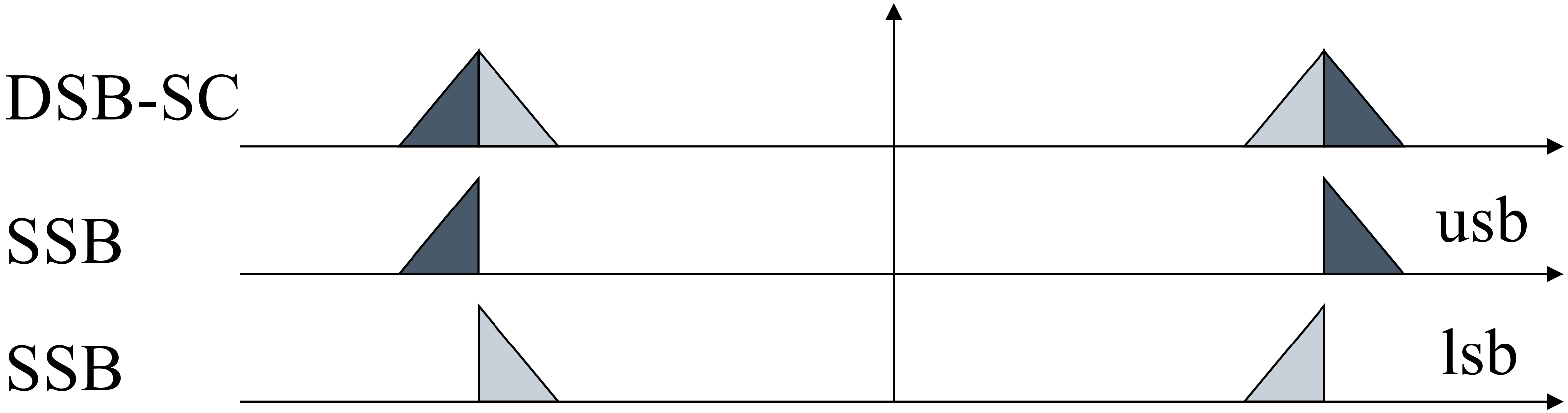
- For a single real-valued $m(t)$, three types of modulations can be identified according to how $s_Q(t)$ is *linearly* related to $m(t)$, at the case that $s_I(t)$ is exactly $m(t)$:
  - *Some modulation may have $s_I(t)$ and $s_Q(t)$ that respectively bear independent information.*

1. Double SideBand Suppressed Carrier (DSB-SC) modulation
2. Single SideBand (SSB) modulation
3. Vestigial SideBand (VSB) modulation

DSB-SC and SSB

Type of modulation	$s_I(t)$	$s_Q(t)$	
DSB-SC	$m(t)$	0	
SSB	$m(t)$	$\hat{m}(t)$	Upper side band transmission
SSB	$m(t)$	$-\hat{m}(t)$	Lower side band transmission

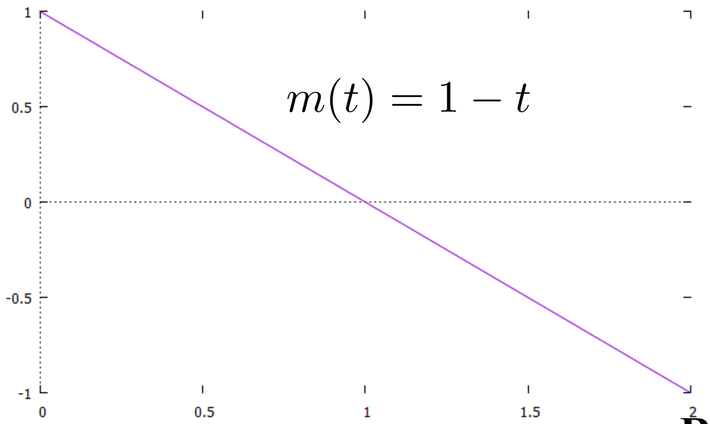
* $\hat{m}(t)$ = Hilbert transform of $m(t)$, which is used to completely "suppress" the other sideband.

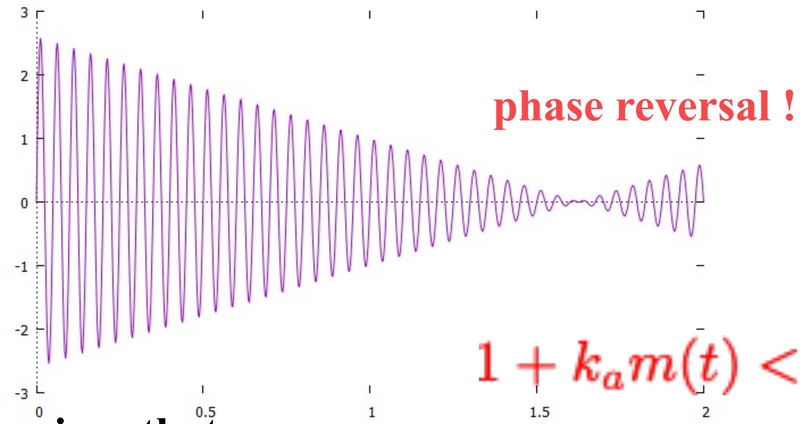


DSB-SC

- Different from DSB-C, $s(t)$ in DSB-SC undergoes a *phase reversal* whenever $m(t)$ crosses zero.

$$s(t) = m(t)\cos(2\pi f_c t) \qquad (1 + 1.6m(t))\cos(2\pi f_c t)$$





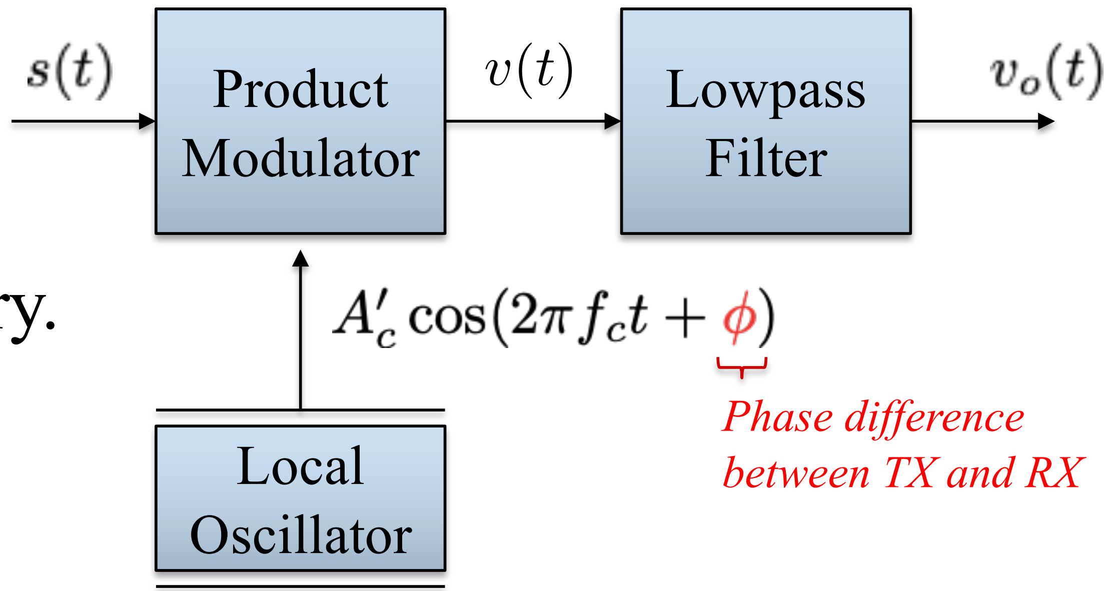
$1 + k_a m(t) < 0$

Require a receiver that can recognize the phase reversal !

overmodulaion

Coherent Detection for DSB-SC

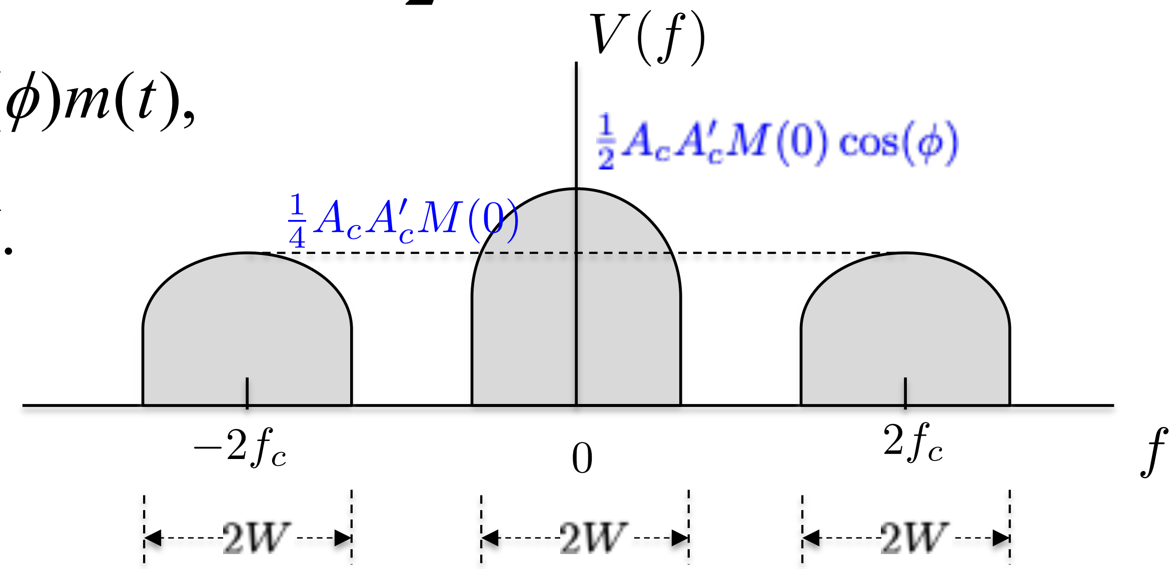
- For DSB-SC, we can no longer use the "envelope detector" (as used for DSB-C), in which no local carrier is required at the receiver.
- The *coherent detection* or *synchronous demodulation* becomes necessary.



Coherent Detection for DSB-SC

$$v(t) = A_c' \cos(2\pi f_c t + \phi) s(t)$$
$$= A_c A_c' \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t)$$
$$= \frac{1}{2} A_c A_c' \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A_c' \cos(\phi) m(t)$$
$$\overset{LowPass}{\rightarrow} \frac{1}{2} A_c A_c' \cos(\phi) m(t),$$

provided $f_c > W$.

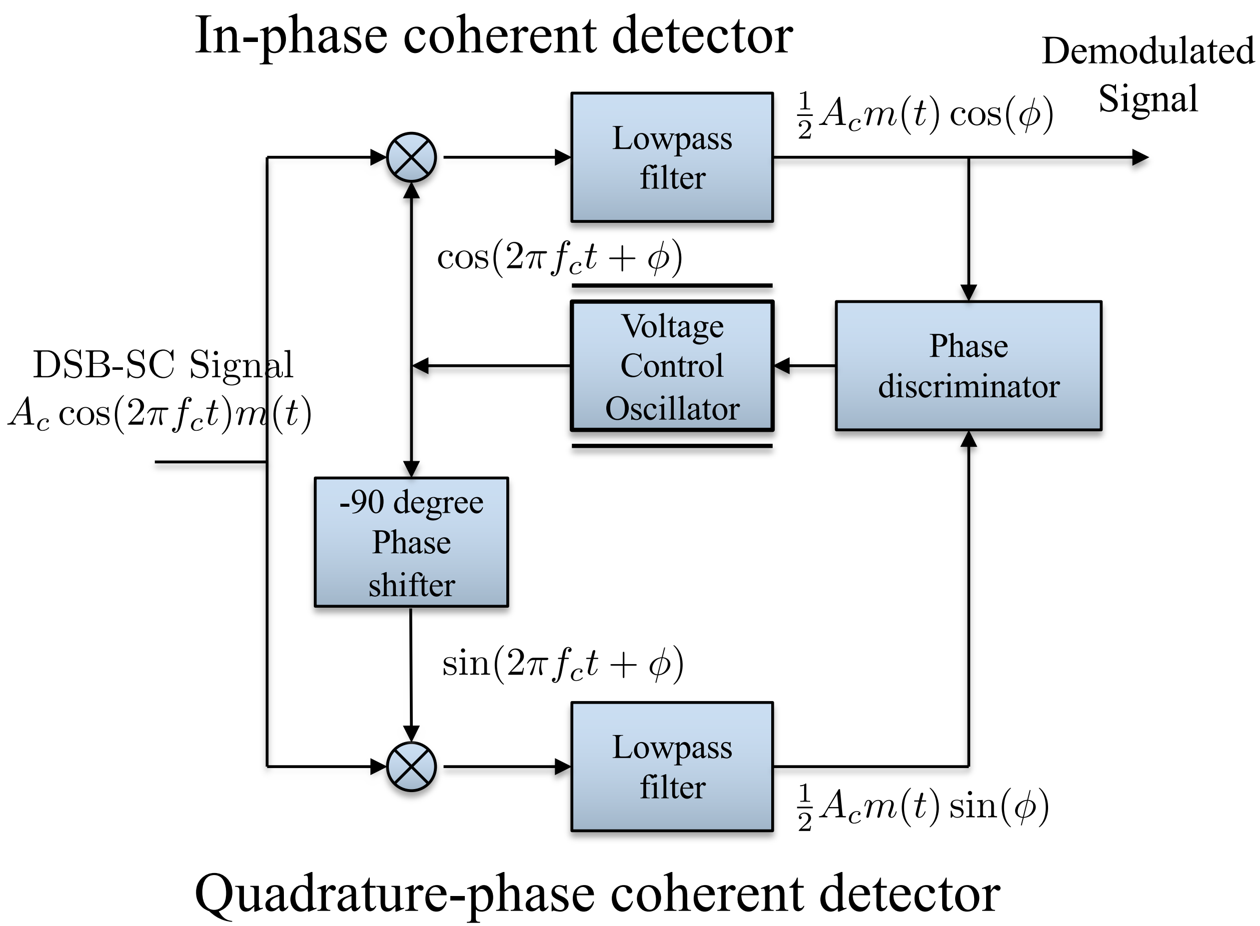


Coherent Detection for DSB-SC

- *Quadrature null effect* of the coherent detector
  - If $\phi = \pi/2$ or $-\pi/2$, the output of coherent detector for DSB-SC is *nullified*.
- If ϕ is not equal to either $\pi/2$ or $-\pi/2$, the output of coherent detector for DSB-SC is simply attenuated by a factor of $\cos(\phi)$, **if ϕ is a constant, independent of time.**
- However, in practice, ϕ often varies with time; therefore, it is necessary to have an additional mechanism to maintain the local carrier in the receiver in *perfect* synchronization with the local carrier in the transmitter.
- Such an additional mechanism adds the system complexity of the receiver.

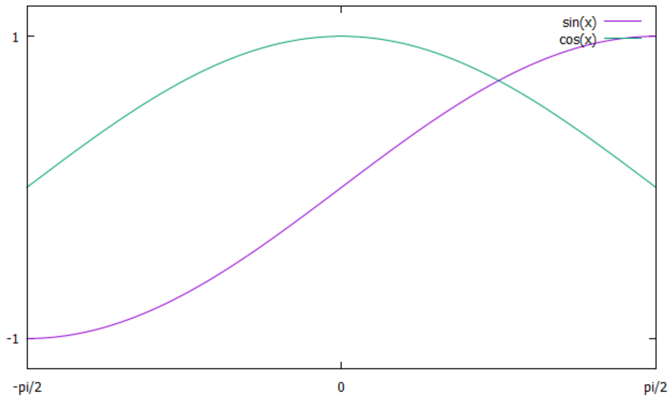
Costas Receiver for DSB-SC

- An exemplified design of synchronization mechanism is the Costas receiver, where two coherent detectors are used.



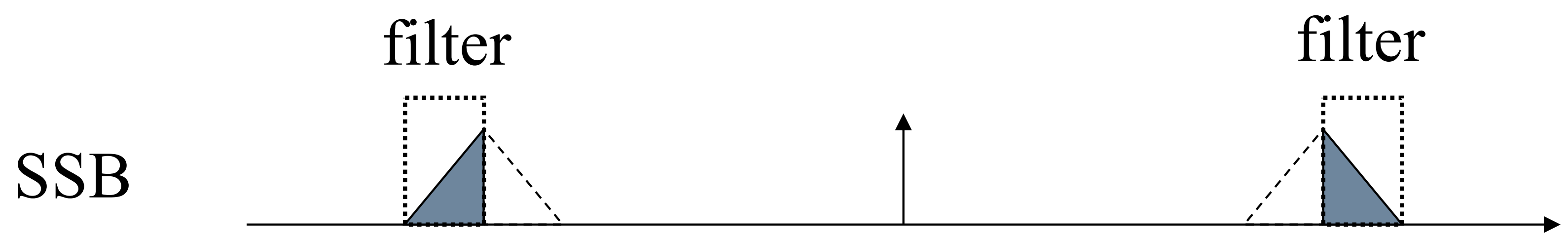
Costas Receiver for DSB-SC

- Conceptually, the Costas receiver adjusts the phase ϕ so that it is close to 0.
  - When ϕ drifts away from 0, the Q-channel output will have *the same polarity* as the I-channel output for *one direction* of phase drift, and *opposite polarity* for *another direction* of phase drift.
  - The phase discriminator then adjusts ϕ through the voltage controlled oscillator.



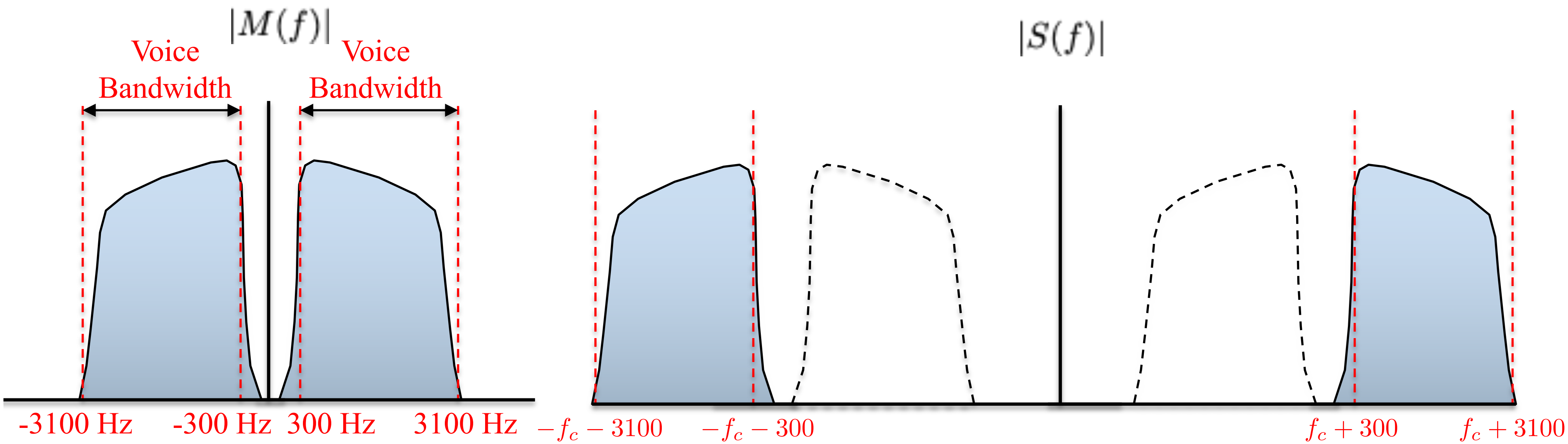
Single-Sideband (SSB) Modulation

- How to generate SSB signal?
  - 1. *Product modulator* to generate DSB-SC signal
  - 2. *Band-pass filter* to pass only one of the sideband and suppress the other.
- The above technique may not be applicable to a DSB-SC signal like below. Why?



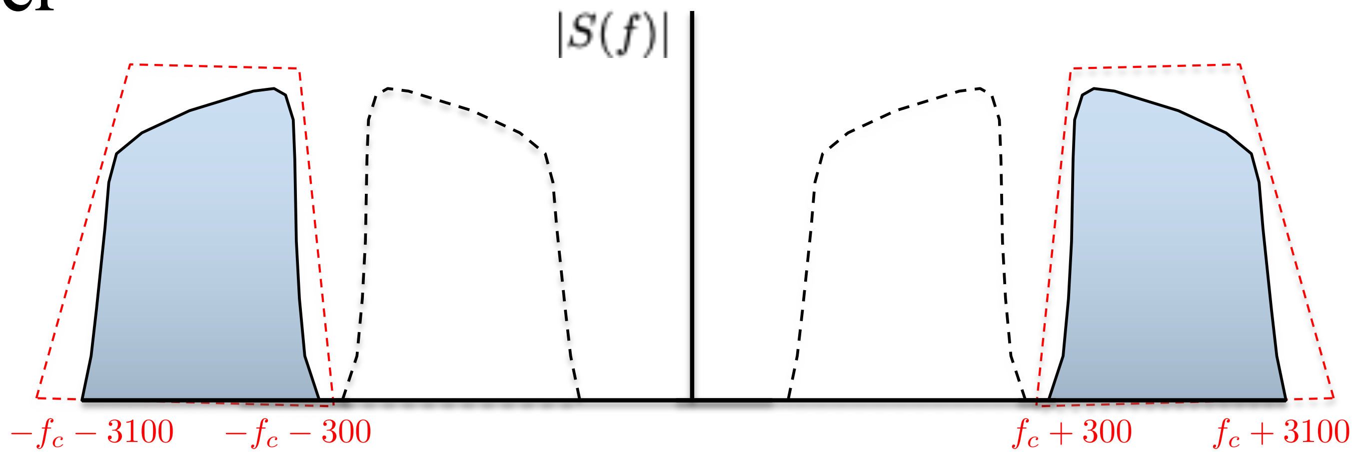
Single-Sideband (SSB) Modulation

- For the generation of a SSB modulated signal to be possible, the message spectrum must have an *energy gap* centered at the origin.



Single-Sideband (SSB) Modulation

- Example of a signal with –300 Hz ~ 300 Hz energy gap
  - Voice : A band of 300 Hz to 3100 Hz gives good articulation.
- Also required for SSB modulation is a highly selective filter

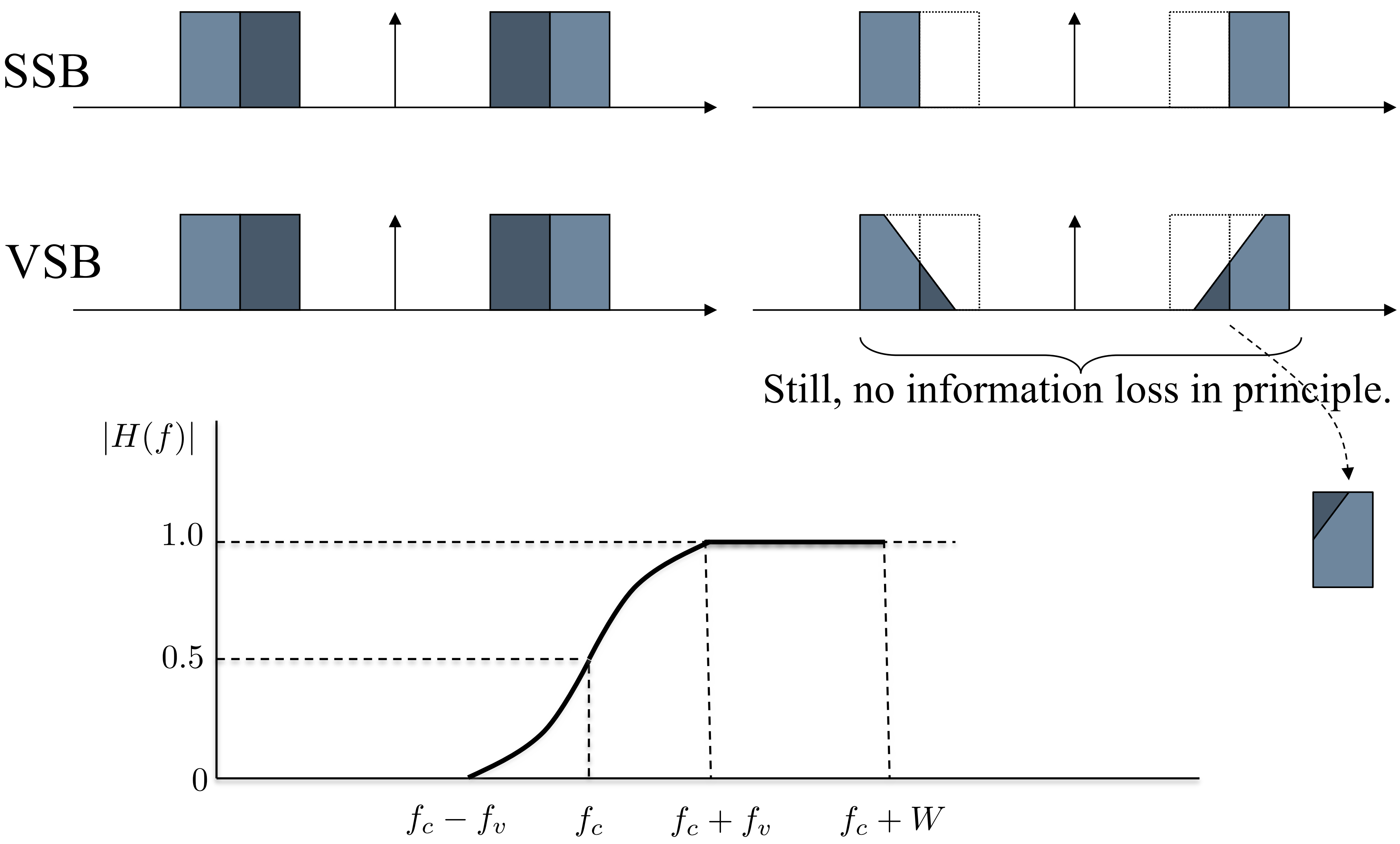


Single-Sideband (SSB) Modulation

- Phase synchronization is also an important issue for SSB demodulation. This can be achieved by:
  - Either a separate low-power pilot carrier
  - Or a highly stable local oscillator (for voice transmission)
    - Phase distortion that gives rise to a *Donald Duck* voice effect is relatively insensitive to human ear.

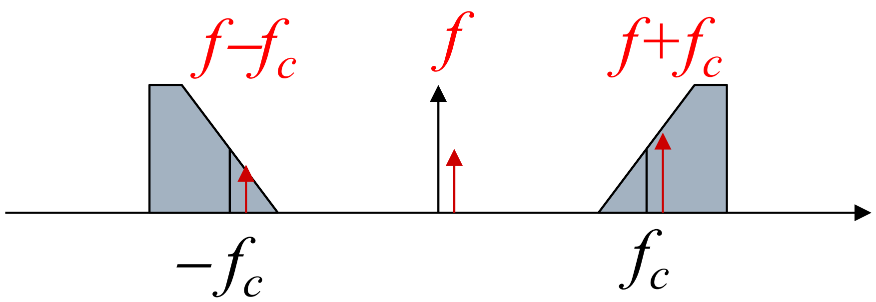
Vestigial Sideband (VSB) Modulation

- Instead of transmitting only one sideband as SSB, VSB modulation transmits a partially suppressed sideband and a vestige of the other sideband.


SSB
VSB
Still, no information loss in principle.
$|H(f)|$
1.0
0.5
0
$f_c - f_v$
f_c
$f_c + f_v$
$f_c + W$

Requirements for VSB Filter

1. The sum of values of the magnitude response $|H(f)|$ at any two frequencies, equally displaced above and below f_c , is **unity**. I.e., $|H(f_c - f)| + |H(f_c + f)| = 1$ for $-f_v < f < f_v$.
2. $H(f - f_c) + H(f + f_c) = 1$ for $-W < f < W$.



So the transmission band of VSB filter is $B_T = W + f_v$.

Generation of VSB Signal

- Analysis of VSB
  - Give a real baseband signal $m(t)$ of bandwidth W.
    - Then, $M(-f) = M^*(f)$ and $M(f) = 0$ for $|f| > W$.
  - Let $M_{VSB}(f) = M(f)[1 + H_Q(f)/j]/2$, where

$$H_Q(-f) = H_Q^*(f) \quad \text{and} \quad \frac{1}{j}H_Q(f) = \begin{cases} 1, & f \le -f_v \\ \in (0,1), & -f_v < f < 0 \\ 0, & f = 0 \end{cases}$$

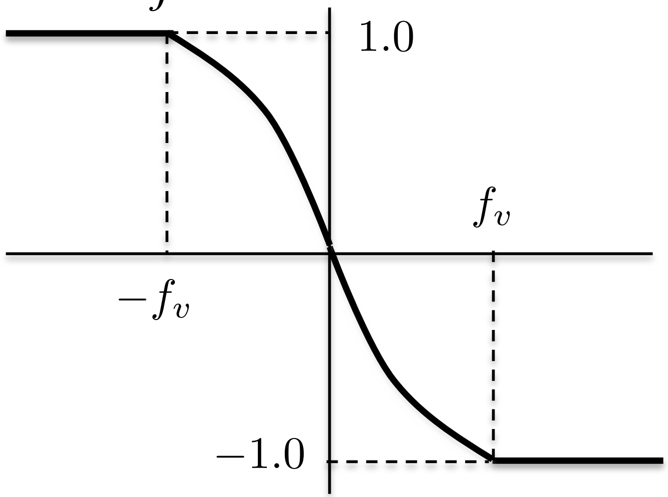
The filter is denoted by H_Q because it is used to generate $s_Q(t)$ (See Slide 4-23)

Generation of VSB Signal

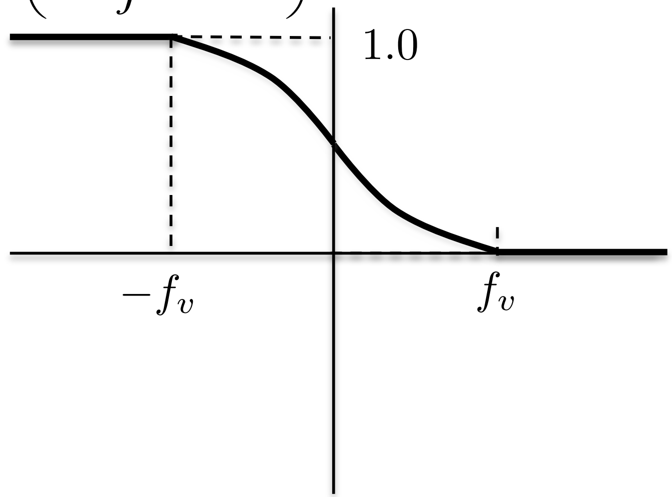
$$L_Q(f)=\frac{1}{j}H_Q(f)\text{ is real.}\Rightarrow L_Q(-f)=\frac{1}{j}H_Q(-f)=\frac{1}{j}H_Q^*(f)=\left[-\frac{1}{j}H_Q(f)\right]^*=-L_Q^*(f)$$

$$=-L_Q(f).$$

$$L_Q(f)=\frac{1}{j}H_Q(f)$$



$$[1+L_Q(f)]/2=\left(1+\frac{1}{j}H_Q(f)\right)/2$$



How to Recover from VSB Signal?

$$M_{VSB}(f) + M_{VSB}^*(-f)$$

$$= \frac{1}{2}\left(M(f)[1 + L_Q(f)] + M^*(-f)[1 + L_Q(-f)]^*\right)$$

$$= \frac{1}{2}\left(M(f)[1 + L_Q(f)] + M(f)[1 + L_Q(-f)]\right)$$

since $[1 + L_Q(-f)]$ is real, and $M^*(-f) = M(f)$

$$= \frac{1}{2}\left(M(f)[2 + L_Q(f) + L_Q(-f)]\right)$$

$= M(f)$, because $L_Q(-f) = -L_Q(f)$.

VSB Upper Sideband Transmission

Relation between VSB and DSB

$$S_{VSB}(f) = S_{DSB}(f)H(f)$$

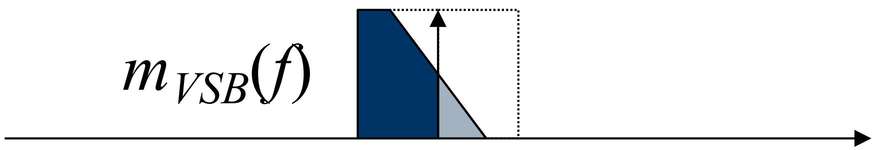
where $H(f) = 1 + \frac{1}{2}\left(L_Q(f + f_c) + L_Q(-f + f_c)\right)$.

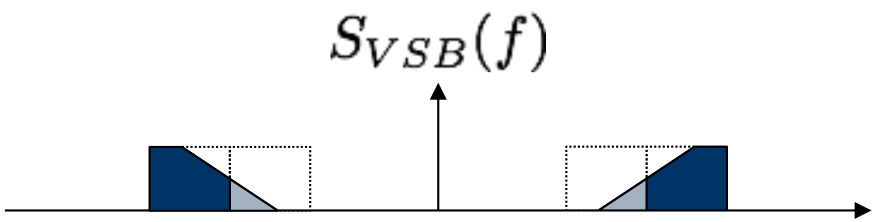
(See the derivation in the next two slides.)

VSB Upper Sideband Transmission

$$S_{DSB}(f) = \tfrac{1}{2}[M(f+f_c) + M(f-f_c)] = \tfrac{1}{2}[M(f+f_c) + M^*(-f+f_c)]$$

$$\begin{aligned}
S_{VSB}(f) &= \frac{1}{2}\left[M_{VSB}(f+f_c) + M^*_{VSB}(-f+f_c)\right] \\
&= \frac{1}{2}\left(M(f+f_c)\frac{[1+L_Q(f+f_c)]}{2} + M^*(-f+f_c)\frac{[1+L_Q(-f+f_c)]}{2}\right) \\
&= \frac{1}{2}\left(M(f+f_c)\,\mathbf{1}\{-f_c-W \le f \le -f_c+W\} \times \frac{[1+L_Q(f+f_c)]}{2}\,\mathbf{1}\{f \le -f_c+W\}\right. \\
&\quad \left. + M^*(-f+f_c)\,\mathbf{1}\{f_c-W \le f \le f_c+W\} \times \frac{[1+L_Q(-f+f_c)]}{2}\,\mathbf{1}\{f \ge f_c-W\}\right)
\end{aligned}$$

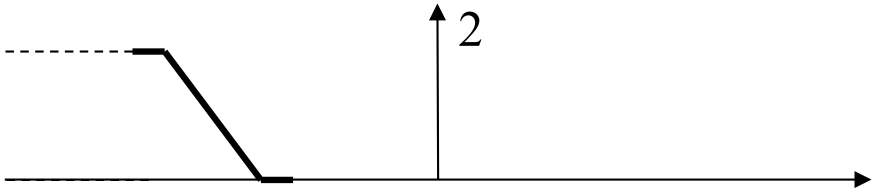




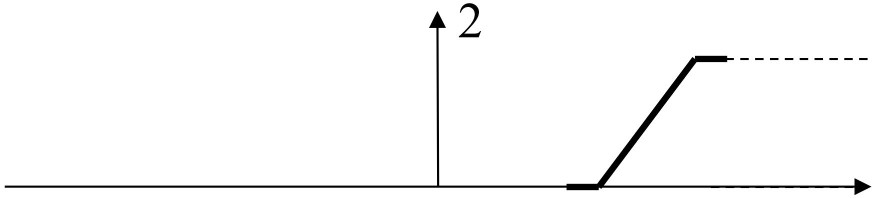
$$M_L(f)F_L(f)+M_R(f)F_R(f)=[M_L(f)+M_R(f)][F_L(f)+F_R(f)]$$
$$\text{if } M_L(f)F_R(f)=M_R(f)F_L(f)=0.$$

$$\overset{\text{cont.}}{=}\frac{1}{2}\left(M(f+f_c)\mathbf{1}\{-f_c-W\le f\le -f_c+W\}+M^*(-f+f_c)\mathbf{1}\{f_c-W\le f\le f_c+W\}\right)$$
$$\times\left(\frac{[1+L_Q(f+f_c)]}{2}\mathbf{1}\{f\le -f_c+W\}+\frac{[1+L_Q(-f+f_c)]}{2}\mathbf{1}\{f\ge f_c-W\}\right)$$
$$=\frac{1}{2}\left(M(f+f_c)+M^*(-f+f_c)\right)$$
$$\times\left(\frac{[1+L_Q(f+f_c)]}{2}\mathbf{1}\{f\le -f_c+W\}+\frac{[1+L_Q(-f+f_c)]}{2}\mathbf{1}\{f\ge f_c-W\}\right)$$
$$=s_{DSB}(f)\times\left(\frac{[1+L_Q(f+f_c)]}{2}\mathbf{1}\{f\le -f_c+W\}+\frac{[1+L_Q(-f+f_c)]}{2}\mathbf{1}\{f\ge f_c-W\}\right)$$
$$=s_{DSB}(f)\times\frac{1}{2}\left([1+L_Q(f+f_c)]\mathbf{1}\{f+f_c\le W\}+[1+L_Q(-f+f_c)]\mathbf{1}\{-f+f_c\le W\}\right)$$
$$\boxed{=s_{DSB}(f)\times\frac{1}{2}\left(2+L_Q(f+f_c)+L_Q(-f+f_c)\right)}$$

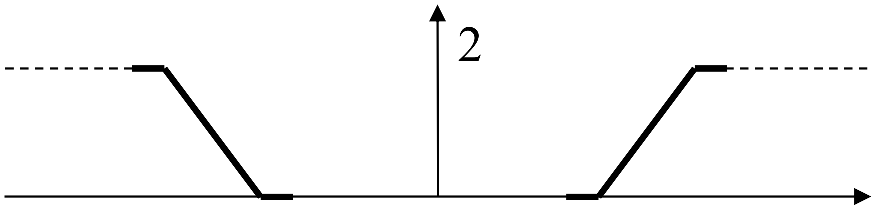
(See the next slide for detail.)



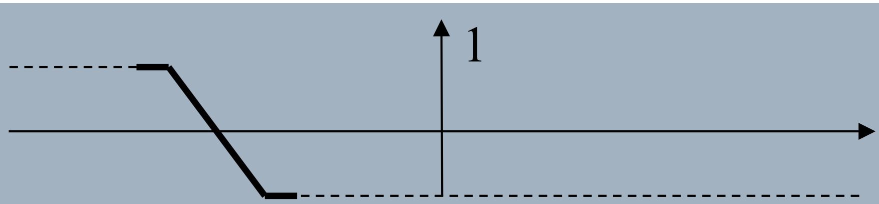
$$[1+L_Q(f+f_c)]\mathbf{1}\{f+f_c \le W\}$$



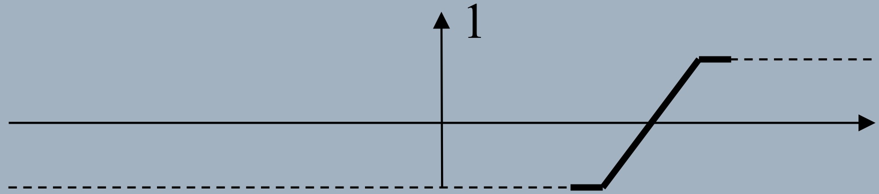
$$[1+L_Q(-f+f_c)]\mathbf{1}\{-f+f_c \le W\}$$



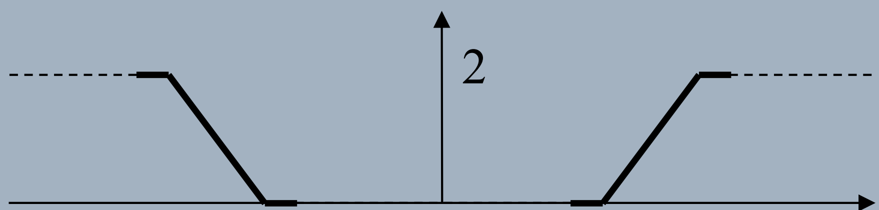
$$[1+L_Q(f+f_c)]\mathbf{1}\{f+f_c \le W\}$$
$$+[1+L_Q(-f+f_c)]\mathbf{1}\{-f+f_c \le W\}$$



$$L_Q(f+f_c)$$



$$L_Q(-f+f_c)$$



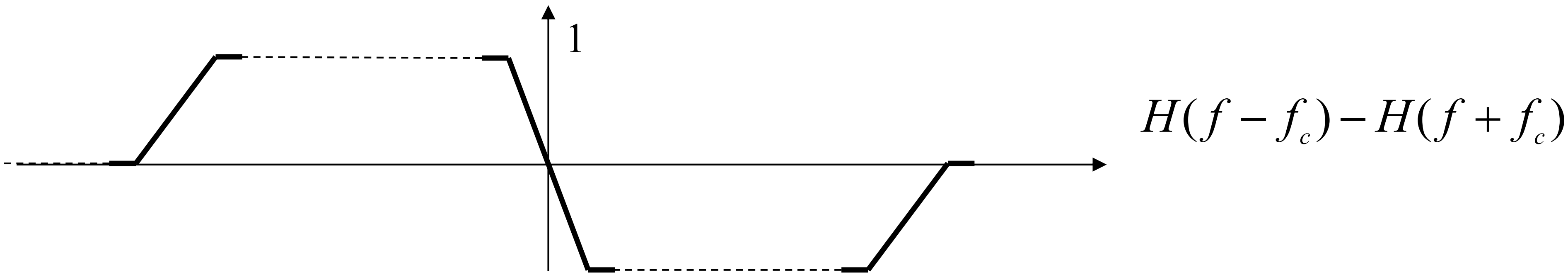
$$L_Q(f+f_c)+L_Q(-f+f_c)+2$$

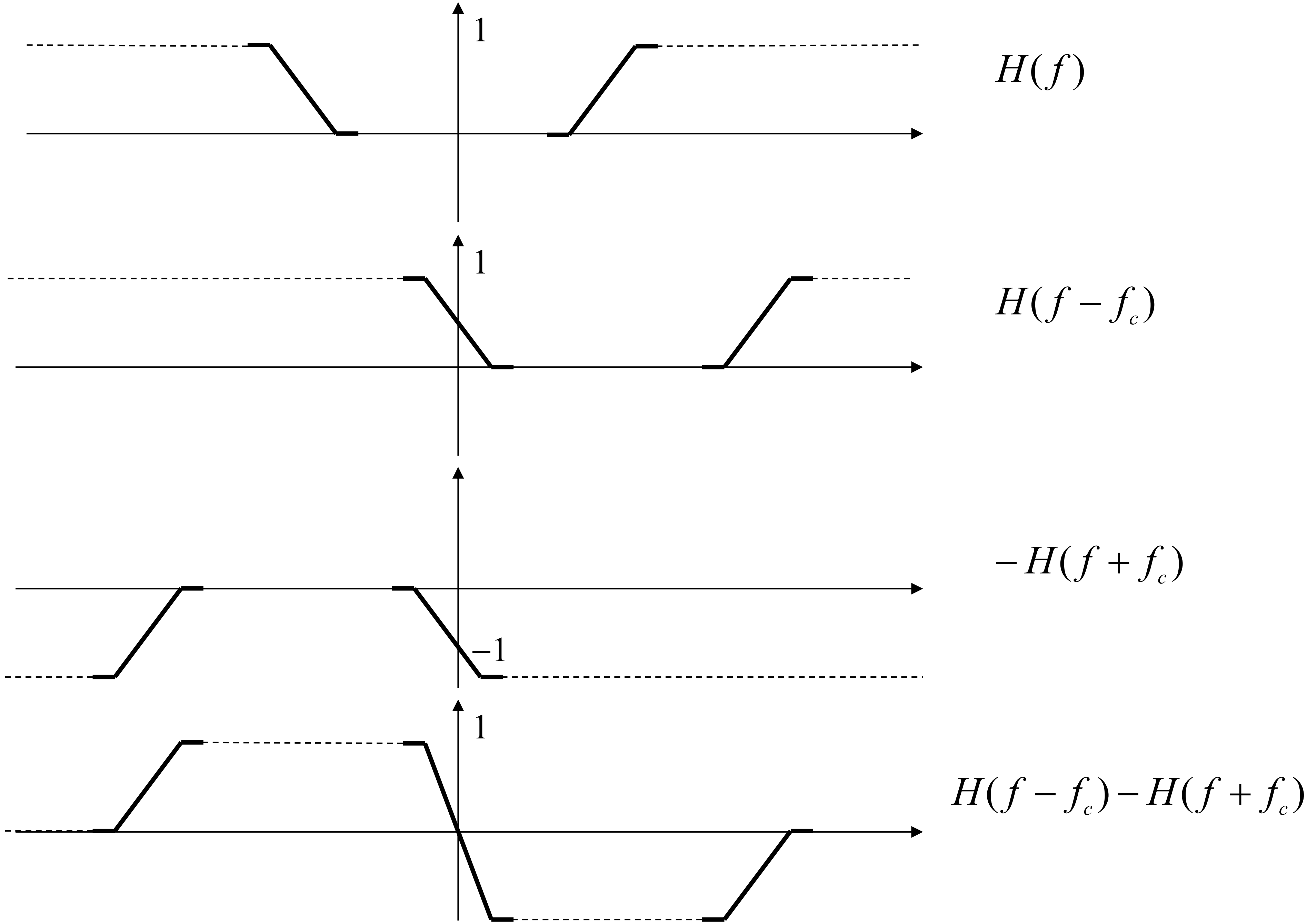
VSB Upper Sideband Transmission

Note $H(f) = 1 + \frac{1}{2}\left(L_Q(f + f_c) + L_Q(-f + f_c)\right)$

$$\Rightarrow L_Q(f) = H(f - f_c) - H(f + f_c) \text{ for } |f| \le W$$

$$\Rightarrow H_Q(f) = j[H(f - f_c) - H(f + f_c)] \text{ for } |f| \le W$$




1
H(f)
1
H(f − f_c)
−H(f + f_c)
−1
1
H(f − f_c) − H(f + f_c)

Mathematical Representation of VSB Signal

$$M_{VSB}(f) = M(f)[1 - jH_Q(f)]/2$$
$$= \frac{1}{2}M(f) - \frac{1}{2}jM(f)H_Q(f)$$
$$= \frac{1}{2}M(f) + \frac{1}{2}jM'(f)$$

where $M'(f) = -M(f)H_Q(f) = -jM(f)L_Q(f)$.

Notably, $m'(t)$ is real. This is an extension of Hilbert Transform.

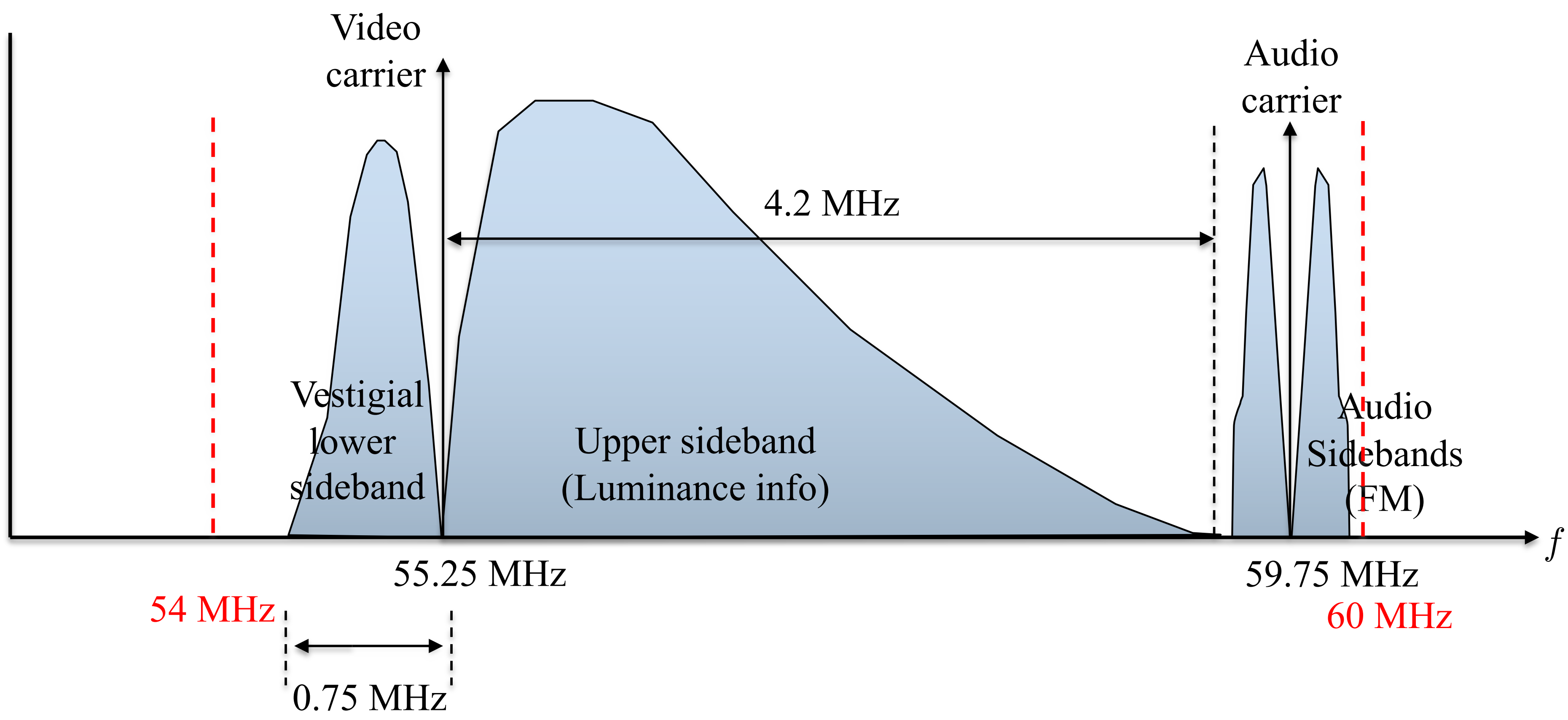
$$M'(-f) = -jM(-f)L_Q(-f) = jM^*(f)L_Q(f)$$
$$= (-j)^* M^*(f)L_Q^*(f) = [-jM(f)L_Q(f)]^* = [M'(f)]^*.$$

Application of VSB Modulation

- Television Signals
  1. The video signal exhibits a *large* bandwidth and *significant low-frequency* content.
     - Hence, no *energy gap* exists (SSB becomes impractical).
     - VSB modulation is adopted to save bandwidth.
     - Notably, since a rigid control of the transmission VSB filter at the very high-power transmitter is expensive, a “not-quite” VSB modulation is used instead (a little waste of bandwidth to save cost).

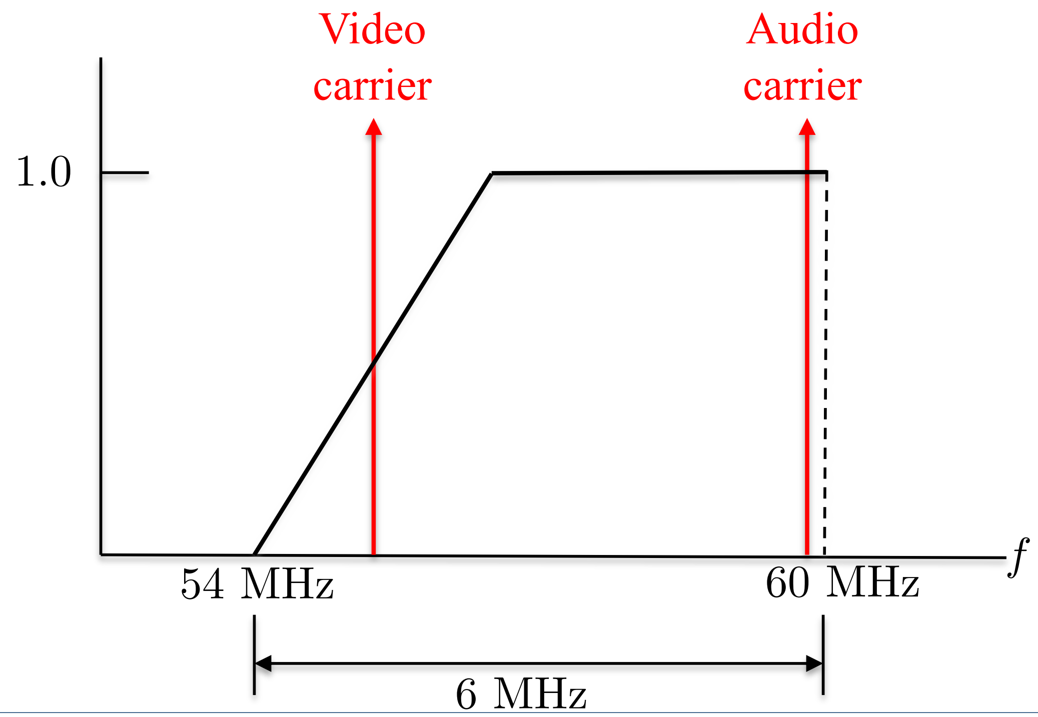
Application of VSB Modulation

The spectrum of a vestigial-sideband (VSB) TV transmission



Application of VSB Modulation

- As the transmission signal is not quite VSB modulated, the receiver needs to "re-shape" the received signal before feeding it to a VSB demodulator.



Application of VSB Modulation

2. In order to save the cost of the receiver (i.e., in order to use envelope detector at the receiver), an additional carrier is added.
   - Notably, additional carrier does not increase bandwidth, but just add transmission power.

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} A_c k_a \left(m(t)\cos(2\pi f_c t) - m'(t)\sin(2\pi f_c t)\right)$$

$$= A_c\left[1 + \frac{1}{2} k_a m(t)\right]\cos(2\pi f_c t) - \frac{1}{2} k_a A_c m'(t)\sin(2\pi f_c t)$$

Application of VSB Modulation

- Distortion of envelope detector

$$s^2(t) = A_c^2\left[1+\tfrac{1}{2}k_a m(t)\right]^2\cos^2(2\pi f_c t) + \tfrac{1}{4}k_a^2 A_c^2\left(m'(t)\right)^2\sin^2(2\pi f_c t)$$

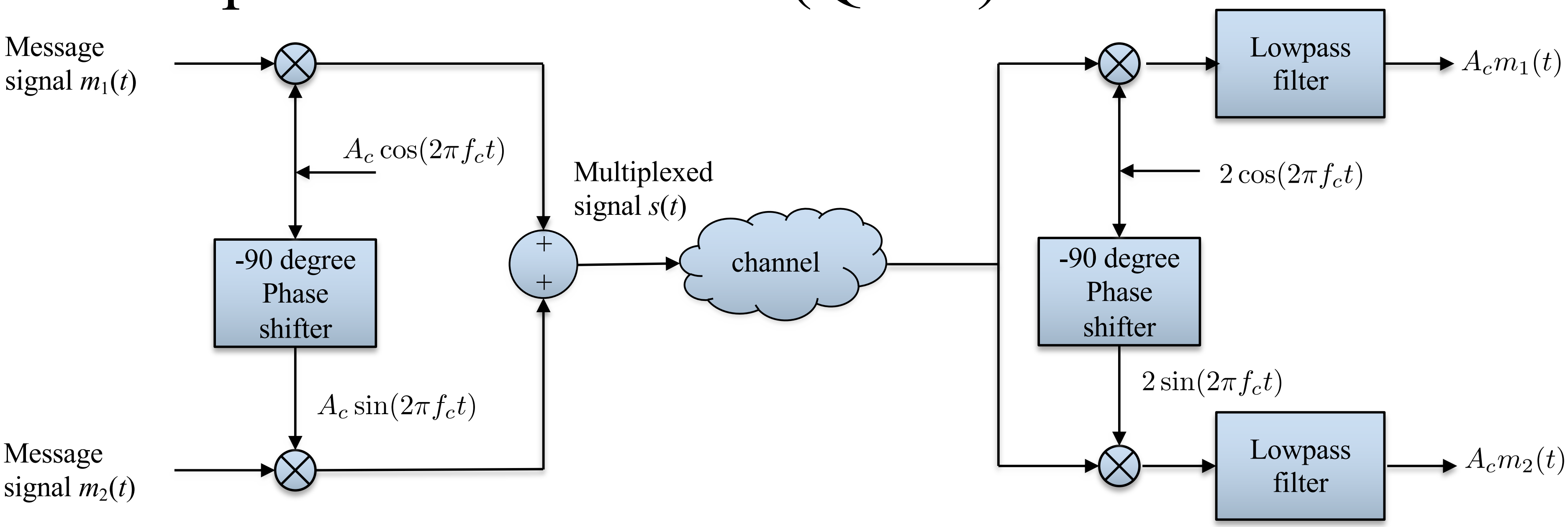
$$-k_a A_c^2 m'(t)\left[1+\tfrac{1}{2}k_a m(t)\right]\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$\xrightarrow{\text{Lowpass}} \frac{A_c^2}{2}\left(\left[1+\tfrac{1}{2}k_a m(t)\right]^2 + \underbrace{\tfrac{1}{4}k_a^2\left(m'(t)\right)^2}_{\text{distortion}}\right)$$

The distortion can be compensated by reducing the amplitude sensitivity k_a or increasing the width of the vestigial sideband. Both methods are used in the design of television broadcasting system.

Extension Usage of DSB-SC

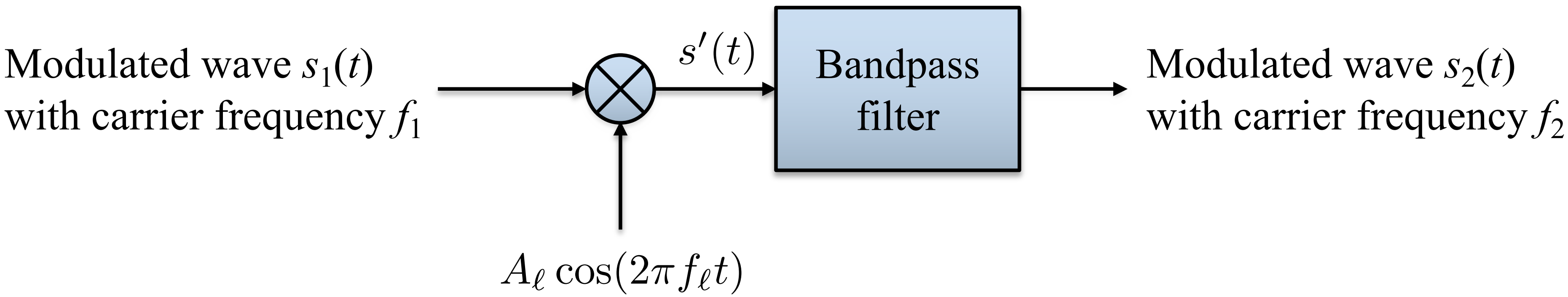
- Quadrature-Carrier Multiplexing or Quadrature Amplitude Modulation (QAM)



Synchronization is critical in QAM, which is often achieved by a separate low-power pilot tone outside the passband of the modulated signal.

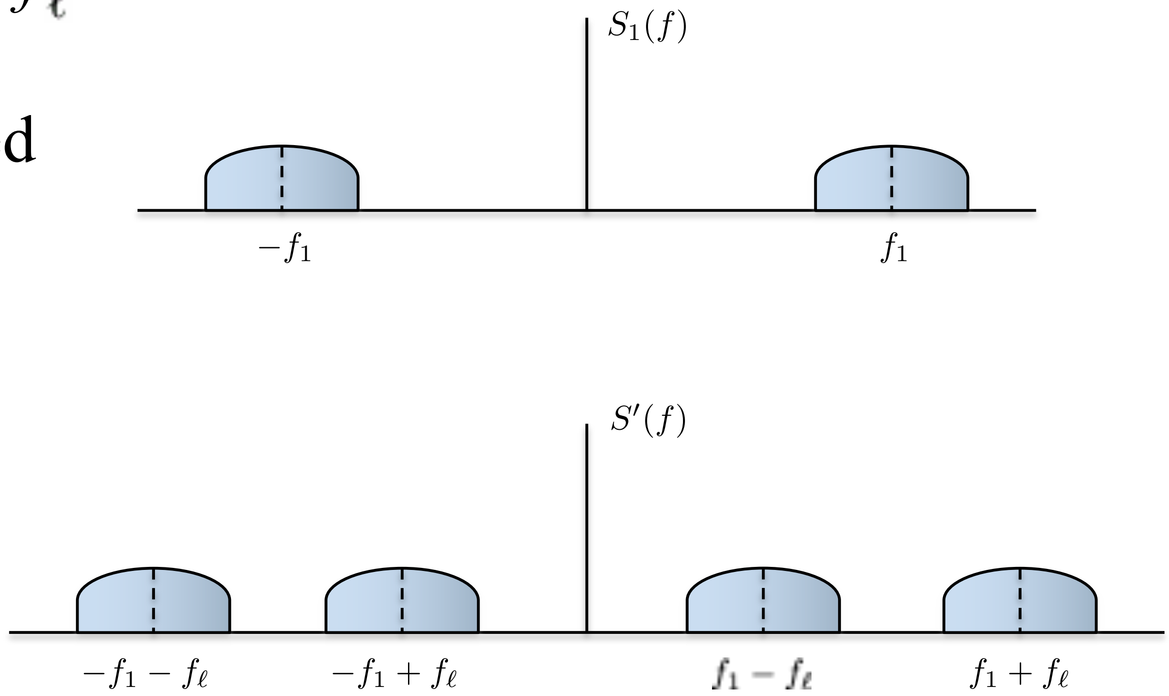
Translation of Frequency

- The basic operation of SSB modulation is simply a special case of *frequency translation*.
  - For this reason, SSB modulation is sometimes referred to as *frequency changing*, *mixing*, or *heterodyning*.
  - The mixer is a device that consists of a product modulator followed by a band-pass filter, which is exactly what SSB modulation does.



Translation of Frequency

- The process is named *upconversion*, if $f_1 + f_\ell$ is the wanted signal.
- The process is named *downconversion*, if $f_1 - f_\ell$ is the wanted signal.



Angle Modulation

- Angle modulation
  - The angle of the carrier is varied in accordance with the baseband signal.
- Angle modulation provides us with a practical means of exchanging *channel bandwidth* for improved *noise performance*.
  - Angle modulation can provide better discrimination against noise and interference than the amplitude modulation, at the expense of increased transmission bandwidth.

Angle Modulation

- Commonly used angle modulation
  - Phase modulation (PM)

    $s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$, where k_p is phase sensitivity.
  - Frequency modulation (FM)

$$s(t) = A_c \cos\left[2\pi \int_0^t (f_c + k_f m(\tau)) d\tau\right]$$
$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right],$$

where k_f is frequency sensitivity.

Angle Modulation

- Main differences between Amplitude Modulation and Angle Modulation
  1. Zero crossing spacing of angle modulation no longer has a perfect regularity as amplitude modulation does.
  2. Angle modulated signal has constant envelope; yet, the envelope of amplitude modulated signal is dependent on the message signal.

Angle Modulation

- Similarity between PM and FM
  - PM is simply an FM with $\int_0^t m(\tau)d\tau$ in place of $m(t)$.

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

  - Hence, only FM will be introduced.

Frequency Modulation (FM)

- $s(t)$ of FM is a **nonlinear** function of $m(t)$.

$$s(t) = A_c \cos\left[2\pi \int_0^t f_i(\tau) d\tau\right] = A_c \cos\left[2\pi \int_0^t (f_c + k_f m(\tau)) d\tau\right]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau\right]$$

- So, its general analysis is hard.
- To simplify the analysis, we may assume a single-tone transmission, where

$$m(t) = A_m \cos(2\pi f_m t)$$

- From the formula in the previous slide,

$$
\begin{aligned}
f_i(t) &= f_c + k_f m(t) \\
&= f_c + k_f A_m \cos(2\pi f_m t) \\
&= f_c + \Delta f \cdot \cos(2\pi f_m t)
\end{aligned}
$$

where $\Delta f = k_f A_m$ is the frequency deviation.

$$
\begin{aligned}
\Rightarrow s(t) &= A_c \cos\left[2\pi \int_0^t f_i(\tau) d\tau\right] \\
&= A_c \cos\left[2\pi \int_0^t [f_c + \Delta f \cdot \cos(2\pi f_m \tau)] d\tau\right] \\
&= A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)\right]
\end{aligned}
$$

where $\beta = \Delta f / f_m$ is often called the modulation index of FM signal.

- *Modulation index* β is the largest deviation from $2\pi f_c t$ in an FM system.

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$

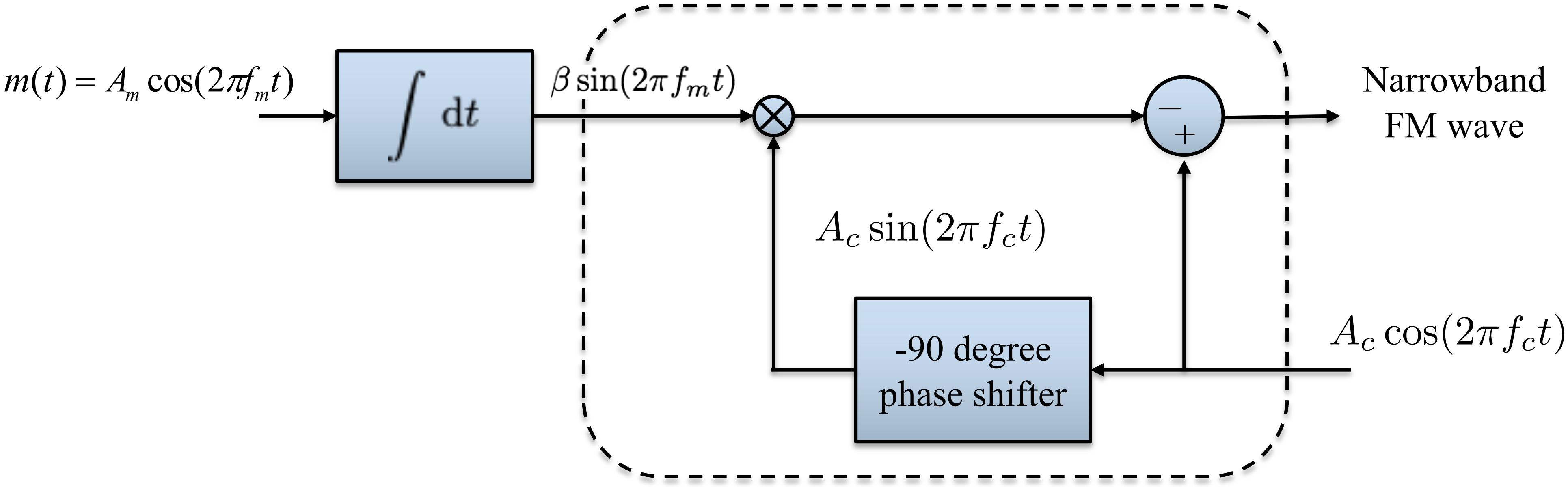
- We then obtain

$$f_c - \beta f_m = f_c - \Delta f \le f_i(t) = f_c + \Delta f \cdot \cos(2\pi f_m t) \le f_c + \Delta f = f_c + \beta f_m$$

$$\Delta f = \beta f_m$$

1. A *small* β corresponds to a *narrowband* FM.
2. A *large* β corresponds to a *wideband* FM.

Narrowband Frequency Modulation



$$
\begin{aligned}
s(t) &= A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \\
&= A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)] \\
&\approx A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)\beta \sin(2\pi f_m t) \quad \text{(Often, } \beta < 0.3\text{)}
\end{aligned}
$$

Narrowband Frequency Modulation

- Comparison between approximate narrowband FM and DSB-C modulation

$$s_{FM}(t) \approx A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)\beta \sin(2\pi f_m t)$$

$$= A_c \cos(2\pi f_c t) + \frac{\beta A_c}{2}\cos(2\pi(f_c + f_m)t) - \frac{\beta A_c}{2}\cos(2\pi(f_c - f_m)t)$$

$$s_{AM}(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

$$= A_c[1 + k_a A_m \cos(2\pi f_m t)]\cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \frac{k_a A_c A_m}{2}\cos(2\pi(f_c + f_m)t) + \frac{k_a A_c A_m}{2}\cos(2\pi(f_c - f_m)t)$$

Narrowband Frequency Modulation

- Represent them in terms of their lowpass isomorphism.

$$\tilde{s}_{FM}(t) = (A_c + j0) + \frac{\beta A_c}{2}[\cos(2\pi f_m t) + j\sin(2\pi f_m t)]$$

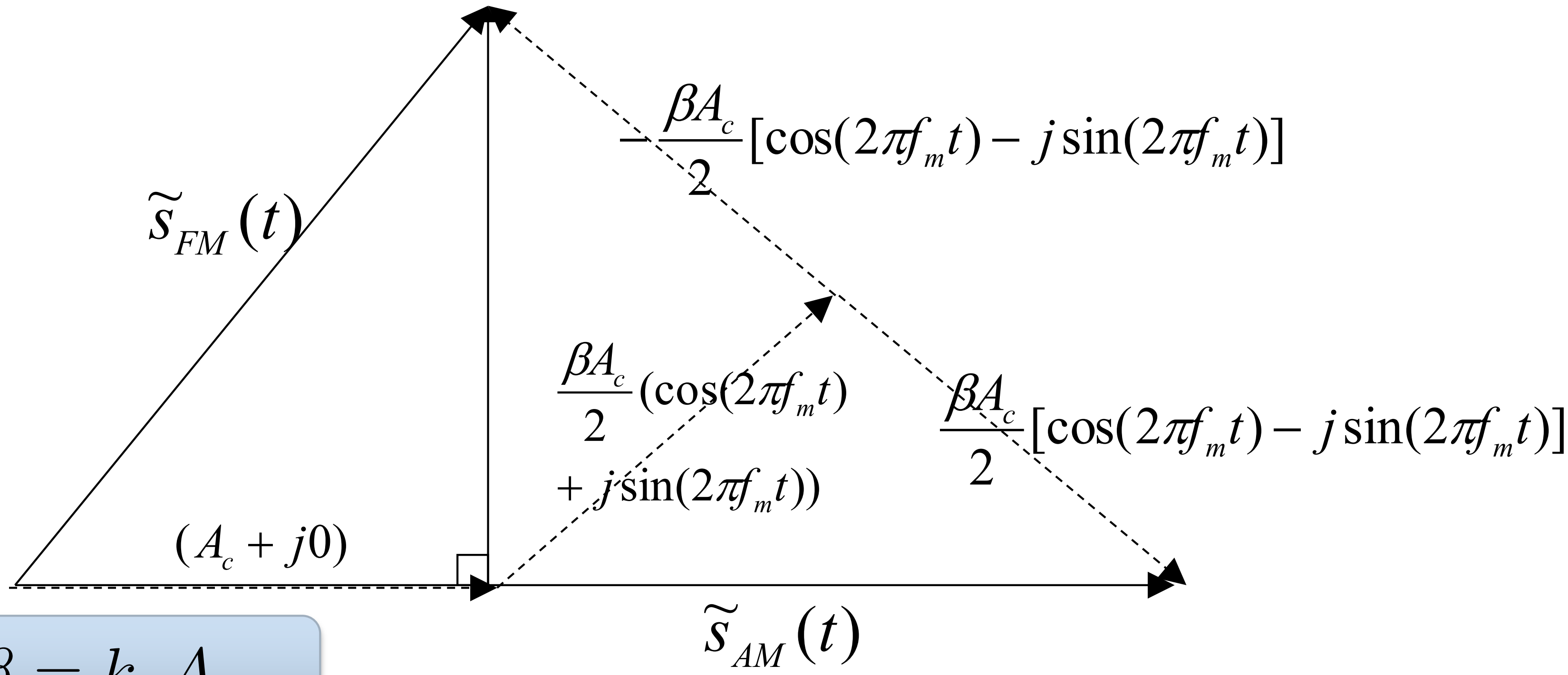
$$-\frac{\beta A_c}{2}[\cos(2\pi f_m t) - j\sin(2\pi f_m t)]$$

$$\tilde{s}_{AM}(t) = (A_c + j0) + \frac{k_a A_c A_m}{2}[\cos(2\pi f_m t) + j\sin(2\pi f_m t)]$$

$$+\frac{k_a A_c A_m}{2}[\cos(2\pi f_m t) - j\sin(2\pi f_m t)]$$

Narrowband Frequency Modulation

- Phaser diagram



Let $\beta = k_a A_m$.

Spectrum of Single-Tone FM

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$
$$= \text{Re}\{A_c \exp(j[2\pi f_c t + \beta \sin(2\pi f_m t)])\}$$
$$= \text{Re}\{\tilde{s}(t)\exp(j2\pi f_c t)\}$$
$$\Rightarrow \tilde{s}(t) = A_c \exp(j[\beta \sin(2\pi f_m t)])$$

(See Slide 3-53)

$$\Rightarrow \tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m n t}$$

$$\sum_{n=-\infty}^{\infty} J_n(x) e^{jn\phi} = e^{jx\sin(\phi)}$$

where $J_n(\cdot)$ is the nth order Bessel function of the first kind.

Spectrum of Single-Tone FM

$$\Rightarrow \quad \tilde{S}(f) = \int_{-\infty}^{\infty} \tilde{s}(t) e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} \left(A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m nt} \right) e^{-j2\pi ft} dt$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \int_{-\infty}^{\infty} e^{-j2\pi (f - nf_m)t} dt$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \delta(f - nf_m)$$

Spectrum of Single-Tone FM

Consequently,

$$S(f) = \frac{1}{2}\left[\tilde{S}(f - f_c) + \tilde{S}^*(-f - f_c)\right]$$

$$= \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(-f - f_c - nf_m)\right]$$

$$= \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$

Spectrum of Single-Tone FM

- The time-average PSD of a deterministic signal $s(t)$ is given by (See Slide 2-30)

$$\overline{PSD}(f) = \lim_{T\to\infty} \frac{1}{2T} S(f) S_{2T}^*(f)$$

where $S_{2T}(f)$ is the Fourier transform of $s(t)\cdot\mathbf{1}\{|t|\le T\}$.

  - From

$$s_{2T}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + nf_m)t)\cdot\mathbf{1}\{|t|\le T\}$$

we obtain:

$$S_{2T}(f) = A_c T \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\operatorname{sinc}(2T(f - f_c - nf_m)) + \operatorname{sinc}(2T(f + f_c + nf_m))\right]$$

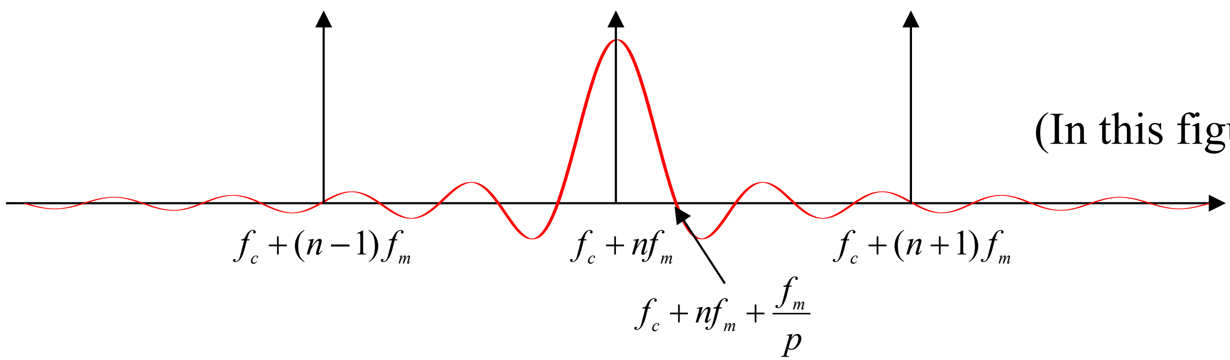
For simplicity, assume that $2T$ *increases along the multiple of* $1/f_m$., i.e., $2T = p/f_m$, where p is an integer. Also assume that f_c is a multiple of f_m, i.e., $f_c = qf_m$., where q is an integer. Then

$$\overline{PSD}(f)$$

$$= \lim_{T\to\infty} \frac{1}{2T} S(f) S_{2T}^*(f)$$

$$= \lim_{p\to\infty} \frac{A_c^2}{4} \sum_{k=-\infty}^{\infty} J_k(\beta)\left[\delta(f - f_c - kf_m) + \delta(f + f_c + kf_m)\right]$$

$$\times \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\text{sinc}(p(f - f_c - nf_m)/f_m) + \text{sinc}(p(f + f_c + nf_m)/f_m)\right]$$



(In this figure, $p = 5$.)

$$
\begin{aligned}
\overline{PSD}(f) = \frac{A_c^2}{4}\lim_{p\to\infty}\Bigg\{ & \sum_{n=-\infty}^{\infty} J_n(\beta)\operatorname{sinc}(p(f-f_c-nf_m)/f_m)\sum_{k=-\infty}^{\infty} J_k(\beta)\delta(f-f_c-kf_m) \\
& + \sum_{n=-\infty}^{\infty} J_n(\beta)\operatorname{sinc}(p(f-f_c-nf_m)/f_m)\sum_{k=-\infty}^{\infty} J_k(\beta)\delta(f+f_c+kf_m) \\
& + \sum_{n=-\infty}^{\infty} J_n(\beta)\operatorname{sinc}(p(f+f_c+nf_m)/f_m)\sum_{k=-\infty}^{\infty} J_k(\beta)\delta(f-f_c-kf_m) \\
& + \sum_{n=-\infty}^{\infty} J_n(\beta)\operatorname{sinc}(p(f+f_c+nf_m)/f_m)\sum_{k=-\infty}^{\infty} J_k(\beta)\delta(f+f_c+kf_m) \Bigg\} \\
= \frac{A_c^2}{4}\Bigg\{ & \sum_{n=-\infty}^{\infty} J_n^2(\beta)\delta(f-f_c-nf_m) + \sum_{n=-\infty}^{\infty} J_n(\beta)J_{-n-2q}(\beta)\delta(f-f_c-nf_m) \\
& + \sum_{n=-\infty}^{\infty} J_n(\beta)J_{-n-2q}(\beta)\delta(f+f_c+nf_m) + \sum_{n=-\infty}^{\infty} J_n^2(\beta)\delta(f+f_c+nf_m) \Bigg\} \\
\approx \frac{A_c^2}{4}\Bigg\{ & \sum_{n=-\infty}^{\infty} J_n^2(\beta)[\delta(f-f_c-nf_m)+\delta(f+f_c+nf_m)] \Bigg\}
\end{aligned}
$$

Average Power of Single-Tone FM Signal

- Hence, the power of a single-tone FM signal is given by:

$$\int_{-\infty}^{\infty} \overline{PSD}(f)df = \frac{A_c^2}{2}\left(\sum_{n=-\infty}^{\infty} J_n^2(\beta) + \sum_{n=-\infty}^{\infty} J_n(\beta) J_{-n-2q}(\beta)\right)$$

$$= \frac{A_c^2}{2}\left(1 + \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta) J_{n+2q}(\beta)\right)$$

$$\approx \frac{A_c^2}{2}$$

- **Question**: Can we use $2\Delta f$ to be the bandwidth of a single-tone FM signal?

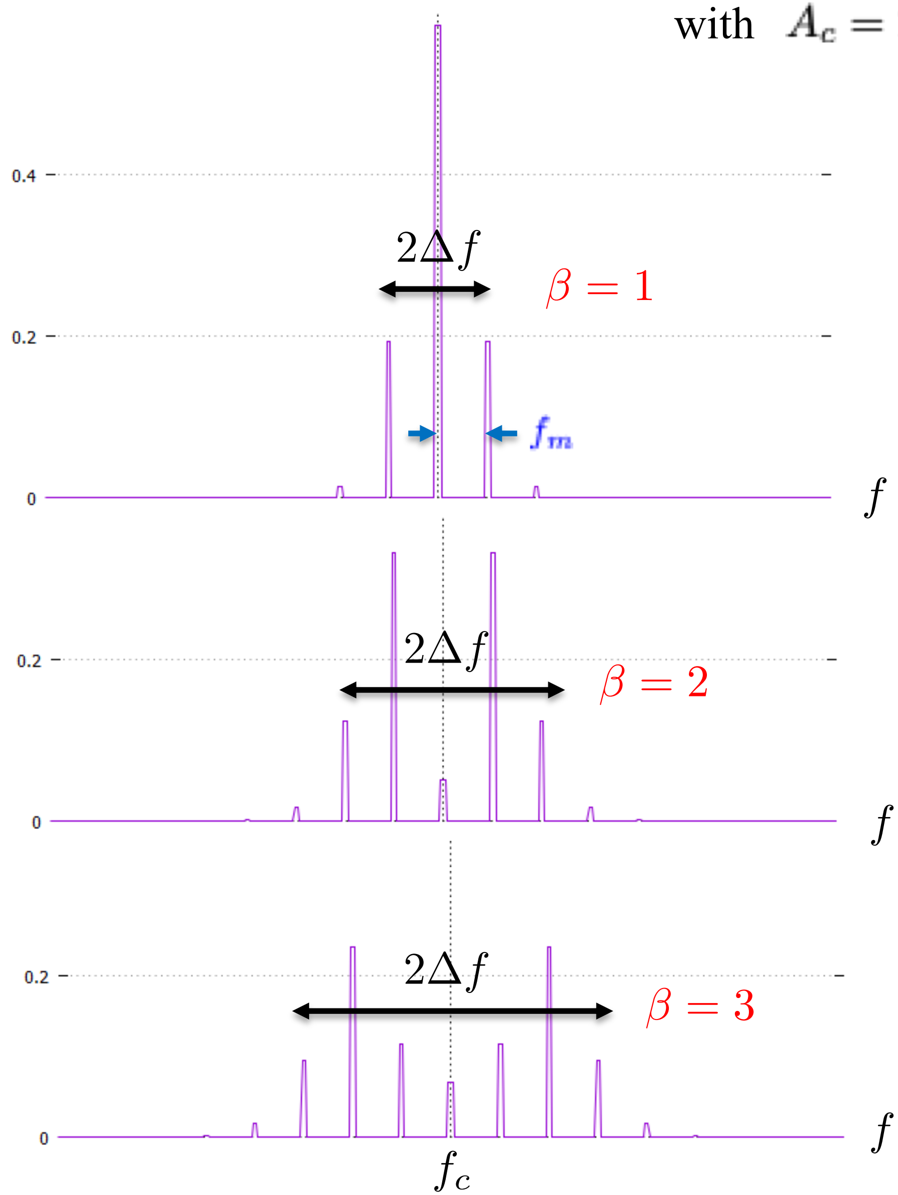
$$\overline{PSD}(f) \approx \sum_{n=-\infty}^{\infty} J_n^2(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

with $A_c = 2$

Illustration

- Fix f_m and k_f, but vary $\beta = \Delta f / f_m = k_f A_m / f_m$.

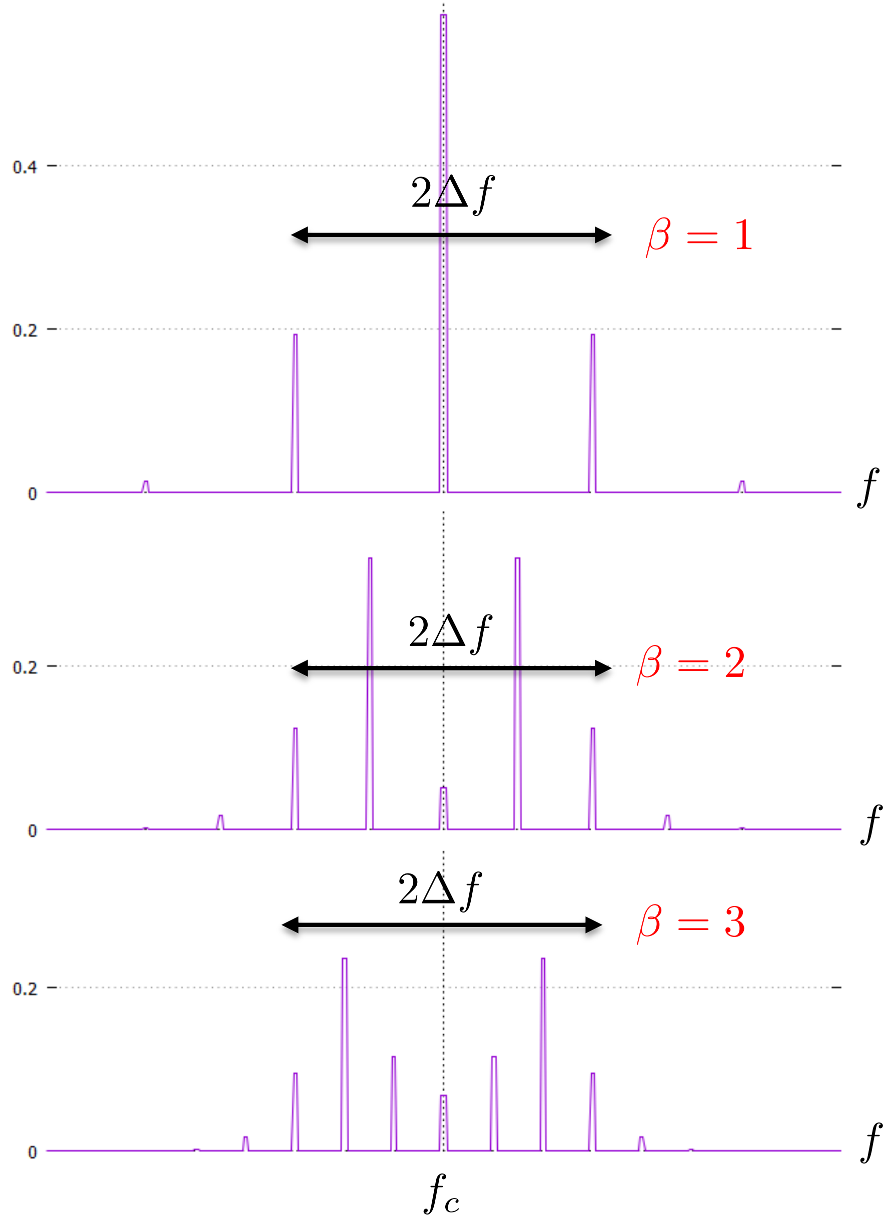
$$\Delta f = \beta f_m = k_f A_m$$



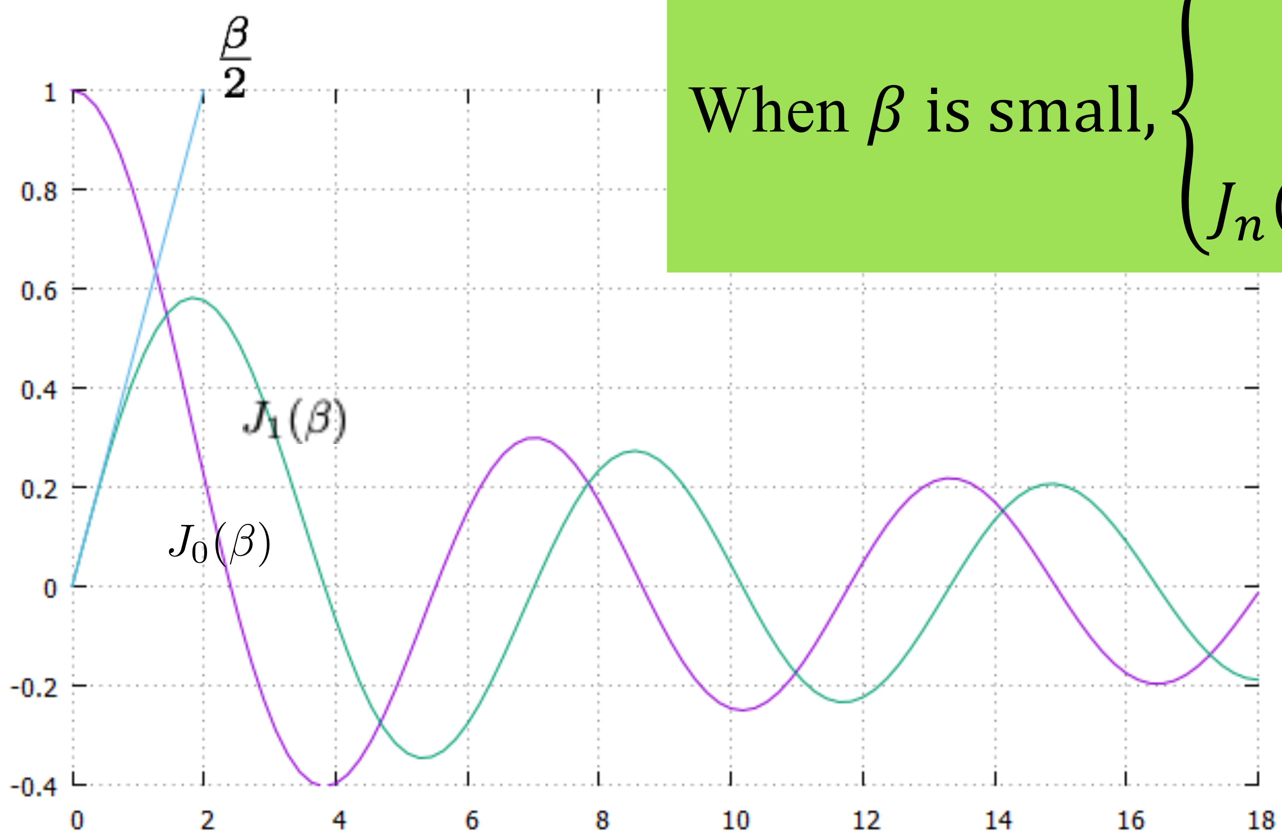
Illustration

- Fix A_m and k_f, but vary $\beta = \Delta f / f_m = k_f A_m / f_m$.

$$\Delta f = \beta f_m = k_f A_m$$



Spectrum of Single-Tone FM



When β is small, $\begin{cases} J_0(\beta) \approx 1 \\ J_1(\beta) \approx \frac{\beta}{2} \\ J_n(\beta) \approx 0 \text{ for } n \geq 2 \end{cases}$

Spectrum of Single-Tone FM

- This results in an approximate spectrum for narrowband single-tone FM signal spectrum as

$$\overline{PSD}(f) \approx \frac{A_c^2}{4} \sum_{n=-\infty}^{\infty} J_n^2(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$

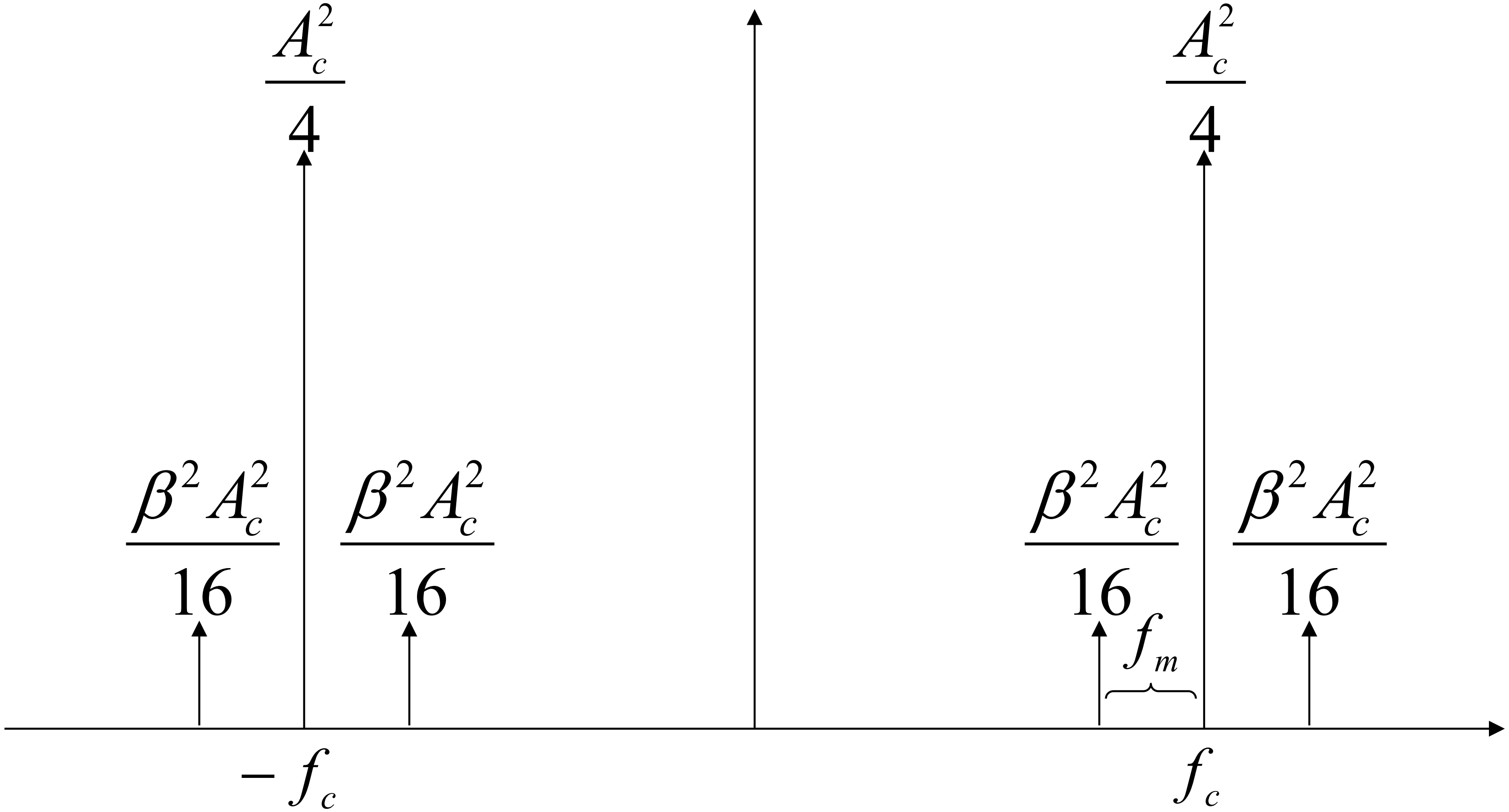
$$\approx \frac{A_c^2}{4} J_{-1}^2(\beta)\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right]$$

$$+ \frac{A_c^2}{4} J_0^2(\beta)\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$+ \frac{A_c^2}{4} J_1^2(\beta)\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right]$$

$$J_n(\beta) = (-1)^n J_{-n}(\beta) \Rightarrow J_n^2(\beta) = J_{-n}^2(\beta)$$

$$
\begin{aligned}
&= \frac{A_c^2}{4} J_1^2(\beta)\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right] \\
&\quad + \frac{A_c^2}{4} J_0^2(\beta)\left[\delta(f - f_c) + \delta(f + f_c)\right] \\
&\quad + \frac{A_c^2}{4} J_1^2(\beta)\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right] \\
&\approx \frac{\beta^2 A_c^2}{16}\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right] \\
&\quad + \frac{A_c^2}{4}\left[\delta(f - f_c) + \delta(f + f_c)\right] \\
&\quad + \frac{\beta^2 A_c^2}{16}\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right]
\end{aligned}
$$


$\frac{A_c^2}{4}$
$\frac{A_c^2}{4}$
$\frac{\beta^2 A_c^2}{16}$
$\frac{\beta^2 A_c^2}{16}$
$\frac{\beta^2 A_c^2}{16}$
$\frac{\beta^2 A_c^2}{16}$
f_m
$-f_c$
f_c

Transmission Bandwidth of FM Signals

- Carson's rule – An empirical bandwidth
  - An empirical rule for transmission bandwidth of FM signals
    - For large β, the bandwidth is essentially $2\Delta f$.
    - For small β, the bandwidth is effectively $2f_m$.
    - So, Carson proposed (in 1922) that:

$$B_T \approx 2\Delta f + 2f_m = 2\Delta f\left(1+\frac{1}{\beta}\right)$$

Transmission Bandwidth of FM Signals

- "Universal-Curve" transmission bandwidth
  - The transmission bandwidth of an FM wave is the *minimum* separation between two frequencies beyond which none of the side frequencies is greater than 1% of the carrier amplitude, obtained when the modulation is removed.

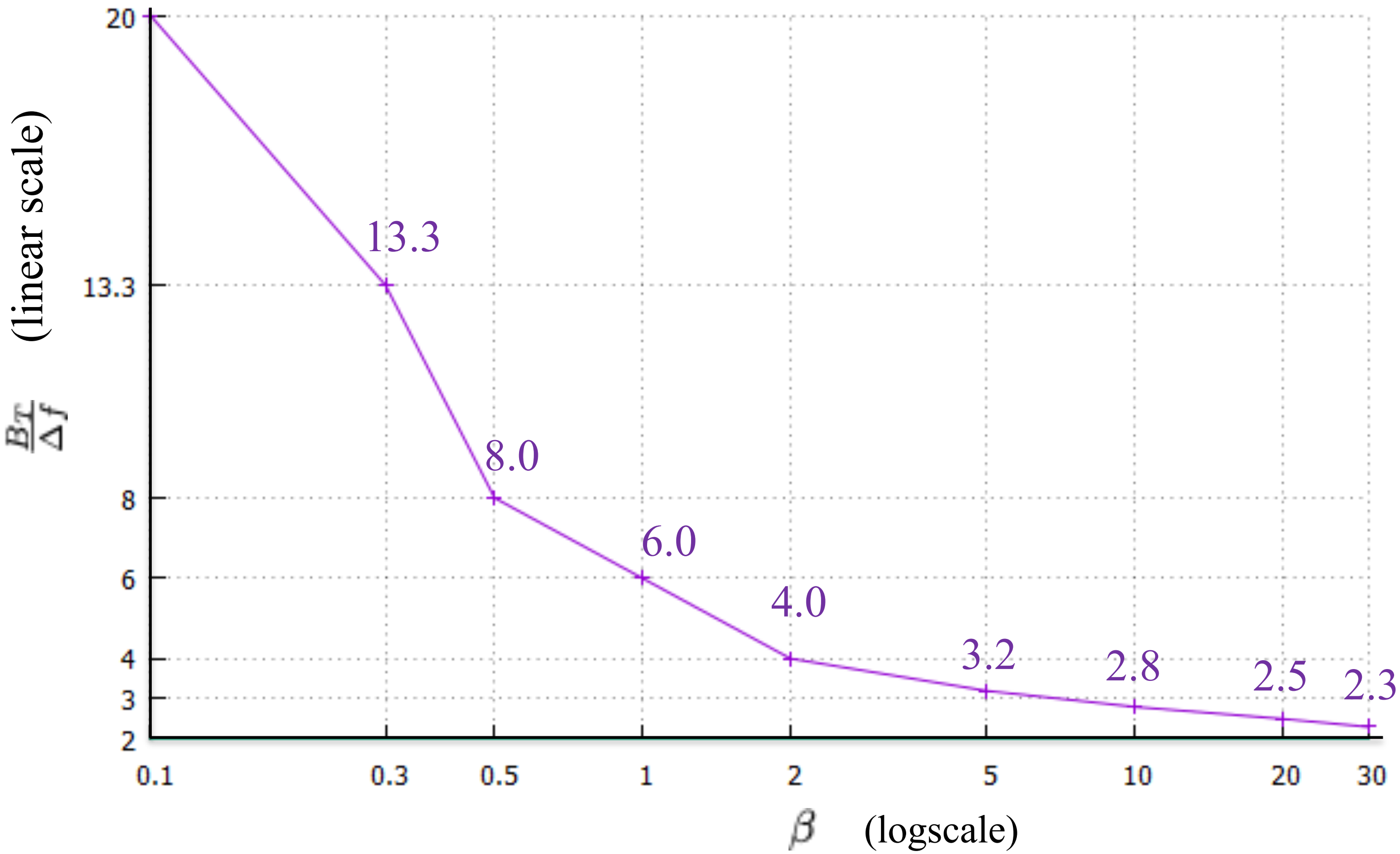
$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$

$$A_c \cos(2\pi f_c t) \rightarrow \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$\Rightarrow B_T = 2n_{\max} f_m, \text{ where } n_{\max} = \max\left\{n : \frac{A_c}{2}|J_n(\beta)| > 0.01\frac{A_c}{2}\right\}.$$

$$\frac{B_T}{\Delta f} = \frac{2n_{\max} f_m}{\beta f_m} = \frac{2n_{\max}}{\beta}$$

$\Rightarrow$ For fixed Δf, a smaller β causes a larger B_T.



Bandwidth of a General FM Wave

- Now suppose $m(t)$ is no longer a single tone but a general message signal of bandwidth W.
  - Hence, the "worst-case" tone is $f_m = W$.
    - For non-sinusoidal modulation, the *deviation ratio* $D = \Delta f / W$ is used instead of the *modulation index* β.
    - The *derivation ratio* D plays the same role for non-sinusoidal modulation as the *modulation index* β for the case of sinusoidal modulation.
  - We can then use Carson's rule or "Universal Curve" to determine the transmission bandwidth B_T.

Bandwidth of a General FM Wave

- Final notes
  - Carson's rule usually underestimates the transmission bandwidth.
  - Universal curve is too conservative in bandwidth estimation.
  - So, a choice of a transmission bandwidth in-between is acceptable for most practical purposes.

Bandwidth of FM radio in North America

- FM radio in North America requires the maximum frequency derivation $\Delta f = 75$ kHz.
  - If some message signal has bandwidth $W = 15$ kHz, then the deviation ratio $D = \Delta f / W = 75/15 = 5$.
  - Then

$$B_{T,\text{Carson}} = 2\Delta f\left(1+\frac{1}{D}\right) = 2\times 75\left(1+\frac{1}{D}\right) = 180\,\text{kHz}$$

$$B_{T,\text{Universal Curve}} = 75 \times 3.2 = 240 \text{ kHz}$$

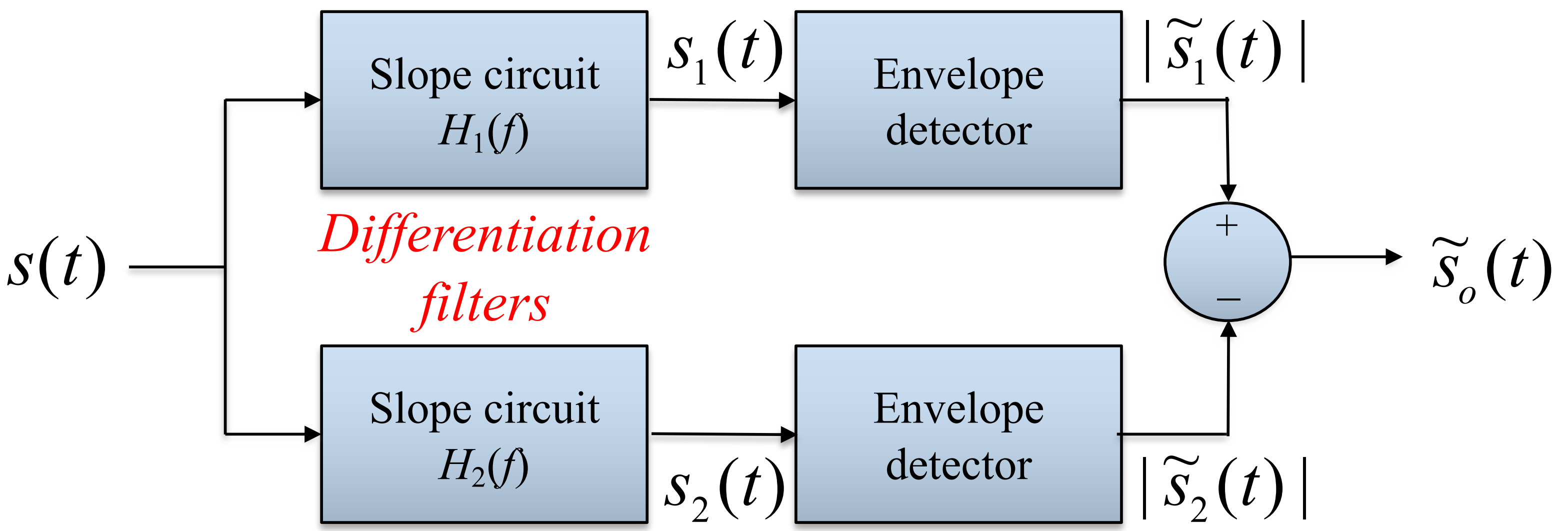
(See Slide 4-81)

Bandwidth of FM radio in North America

- In practice, a bandwidth of 200 kHz is allocated to each FM transmitter.
- This supports what has been claimed: Carson's rule underestimates B_T, while "Universal Curve" overestimates B_T.

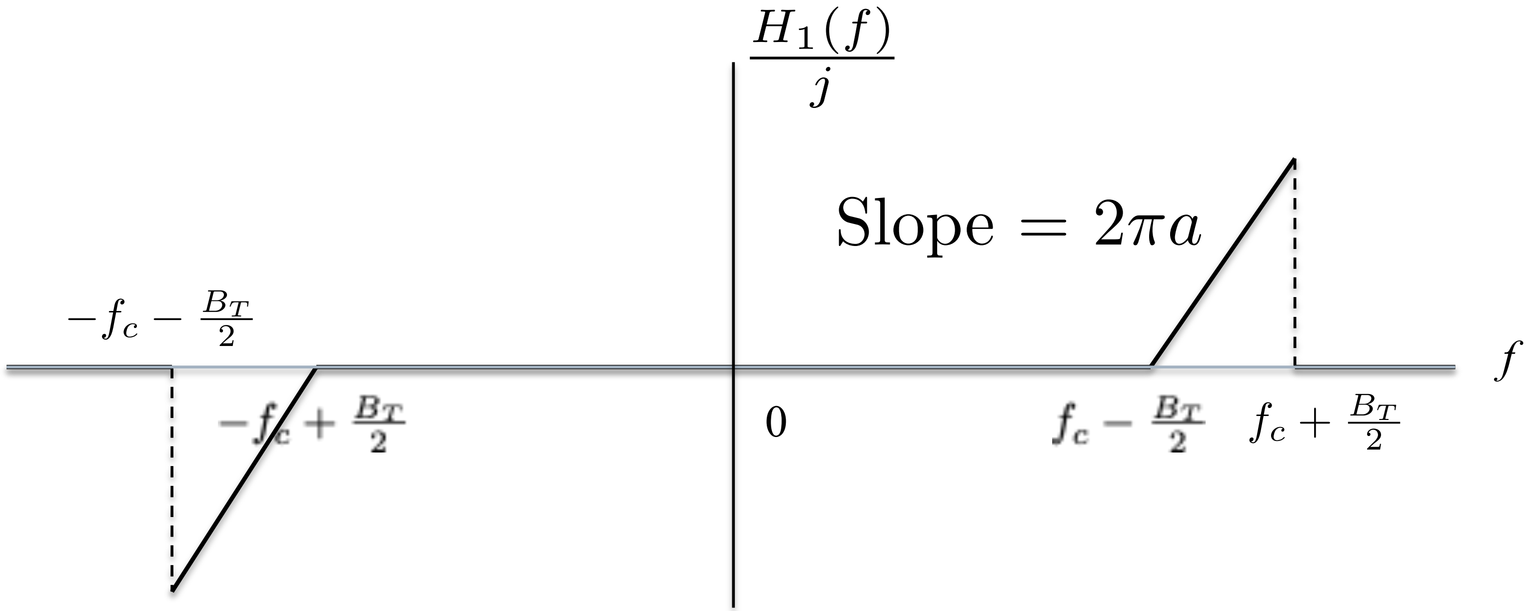
Demodulation of FM Signals

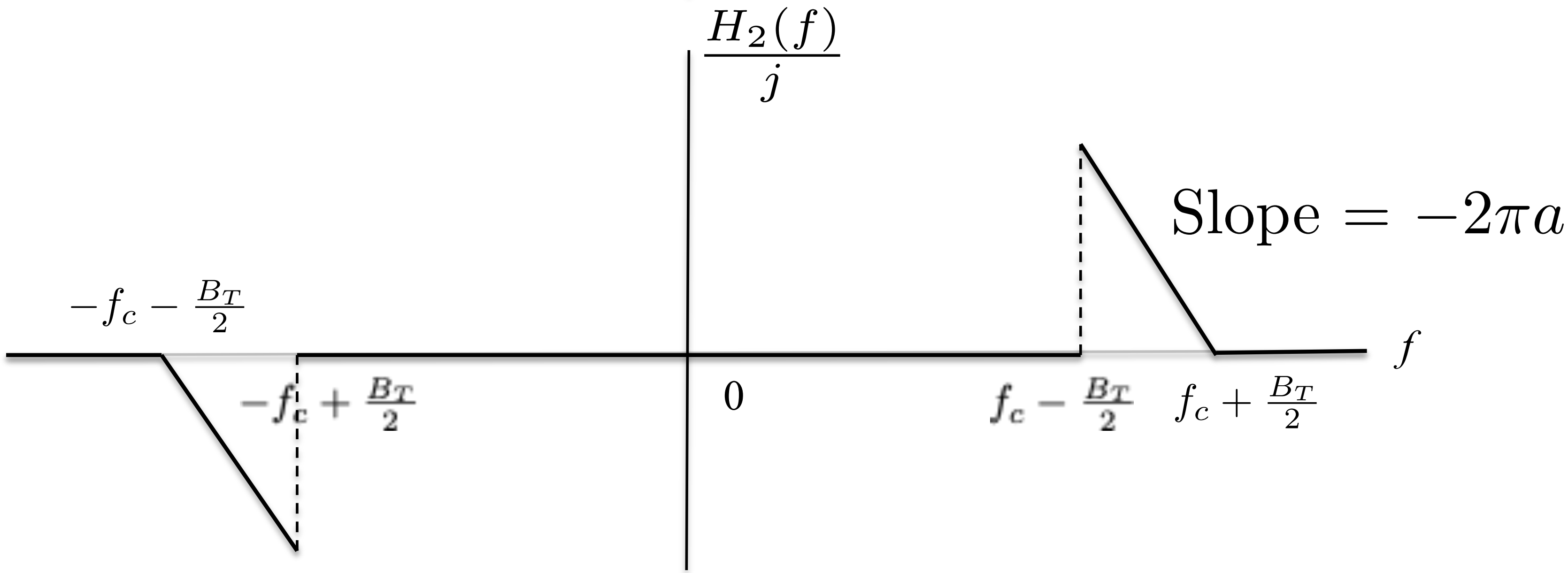
- Indirect Demodulation – Phase-locked loop
- Direct Demodulation
  - Balanced frequency discriminator



$$H_1(f) = \begin{cases} j2\pi a\left(f - f_c + \dfrac{B_T}{2}\right), & |f - f_c| \le \dfrac{B_T}{2} \\ j2\pi a\left(f + f_c - \dfrac{B_T}{2}\right), & |f + f_c| \le \dfrac{B_T}{2} \\ 0, & \text{elsewhere} \end{cases}$$

$$H_2(f) = \begin{cases} -j2\pi a\left(f + f_c + \dfrac{B_T}{2}\right), & |f + f_c| \le \dfrac{B_T}{2} \\ -j2\pi a\left(f - f_c - \dfrac{B_T}{2}\right), & |f - f_c| \le \dfrac{B_T}{2} \\ 0, & \text{elsewhere} \end{cases}$$

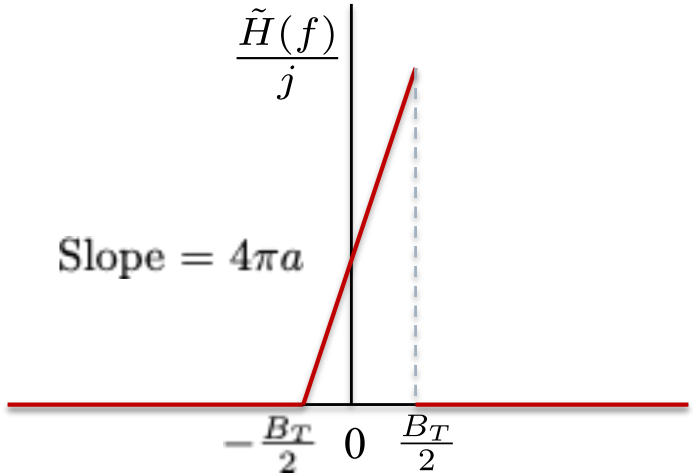

$\frac{H_1(f)}{j}$
Slope $= 2\pi a$
$-f_c - \frac{B_T}{2}$
$-f_c + \frac{B_T}{2}$
0
$f_c - \frac{B_T}{2}$
$f_c + \frac{B_T}{2}$
f


$\frac{H_2(f)}{j}$
Slope $= -2\pi a$
$-f_c - \frac{B_T}{2}$
$-f_c + \frac{B_T}{2}$
0
$f_c - \frac{B_T}{2}$
$f_c + \frac{B_T}{2}$
f

Analysis of Direct Demodulation in terms of Lowpass Equivalences

$$\tilde{H}_1(f) = \begin{cases} 2H_1(f+f_c), & |f| \leq \dfrac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} j4\pi a\left(f + \dfrac{B_T}{2}\right), & |f| \leq \dfrac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$



$$\Rightarrow \tilde{S}_1(f) = \frac{1}{2}\tilde{H}_1(f)\tilde{S}(f) = \begin{cases} j2\pi a\left(f + \dfrac{B_T}{2}\right)\tilde{S}(f), & |f| \leq \dfrac{B_T}{2} \\ 0, & \text{elsewhere} \end{cases}$$

$$\Rightarrow \tilde{s}_1(t) = a\left[\frac{d\tilde{s}(t)}{dt} + j\pi B_T \tilde{s}(t)\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

$$\Rightarrow \tilde{s}(t) = A_c \exp\left[j2\pi k_f \int_0^t m(\tau)d\tau\right]$$

$$\Rightarrow \tilde{s}_1(t) = a\left[\frac{d\tilde{s}(t)}{dt} + j\pi B_T \tilde{s}(t)\right]$$

$$= a\left[\left(jA_c 2\pi k_f m(t)\exp\left[j2\pi k_f \int_0^t m(\boldsymbol{\tau})d\boldsymbol{\tau}\right]\right)\right.$$

$$\left. + j\pi B_T \left(A_c \exp\left[j2\pi k_f \int_0^t m(\boldsymbol{\tau})d\boldsymbol{\tau}\right]\right)\right]$$

$$= j\pi B_T a A_c \left[\frac{2k_f}{B_T} m(t) + 1\right]\exp\left[j2\pi k_f \int_0^t m(\boldsymbol{\tau})d\boldsymbol{\tau}\right]$$

$$\Rightarrow s_1(t) = \mathrm{Re}\{\tilde{s}_1(t)\exp(j2\pi f_c t)\}$$

$$= \mathrm{Re}\left\{ j\pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right] \exp\left[j2\pi k_f \int_0^t m(\tau)d\tau \right] \exp(j2\pi f_c t)\right\}$$

$$= -\pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right] \sin\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right)$$

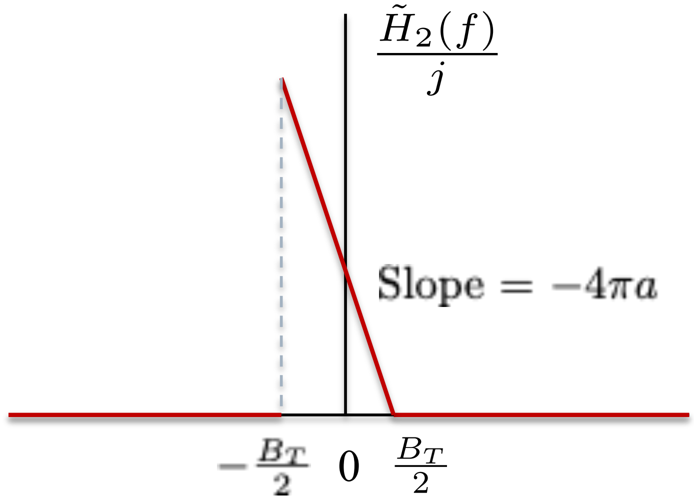
$$= \pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right] \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau + \frac{\pi}{2}\right)$$

$\Rightarrow$ If $\left|\frac{2k_f}{B_T} m(t)\right| < 1$ and $f_c >> W$, then envelope detector can be used to obtain the amplitude of the lowpass equivalent message.

$$|\tilde{s}_1(t)| = \pi B_T a A_c \left[1 + \frac{2k_f}{B_T} m(t)\right]$$

Similarly,

$$\tilde{H}_2(f) = \begin{cases} -j4\pi a\left(f - \frac{B_T}{2}\right), & |f| \le \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$



$$\Rightarrow \tilde{S}_2(f) = \frac{1}{2}\tilde{H}_2(f)\tilde{S}(f) = \begin{cases} -j2\pi a\left(f - \frac{B_T}{2}\right)\tilde{S}(f), & |f| \le \frac{B_T}{2} \\ 0, & \text{elsewhere} \end{cases}$$

$$\Rightarrow \tilde{s}_2(t) = -a\left[\frac{d\tilde{s}(t)}{dt} - j\pi B_T \tilde{s}(t)\right]$$

$$= j\pi B_T a A_c\left[1 - \frac{2k_f}{B_T}m(t)\right]\exp\left[j2\pi k_f \int_0^t m(\tau)d\tau\right]$$

$$\Rightarrow s_2(t) = \mathrm{Re}\{\tilde{s}_2(t)\exp(j2\pi f_c t)\}$$
$$= \pi B_T a A_c \left[1 - \frac{2k_f}{B_T} m(t)\right] \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau + \frac{\pi}{2}\right)$$

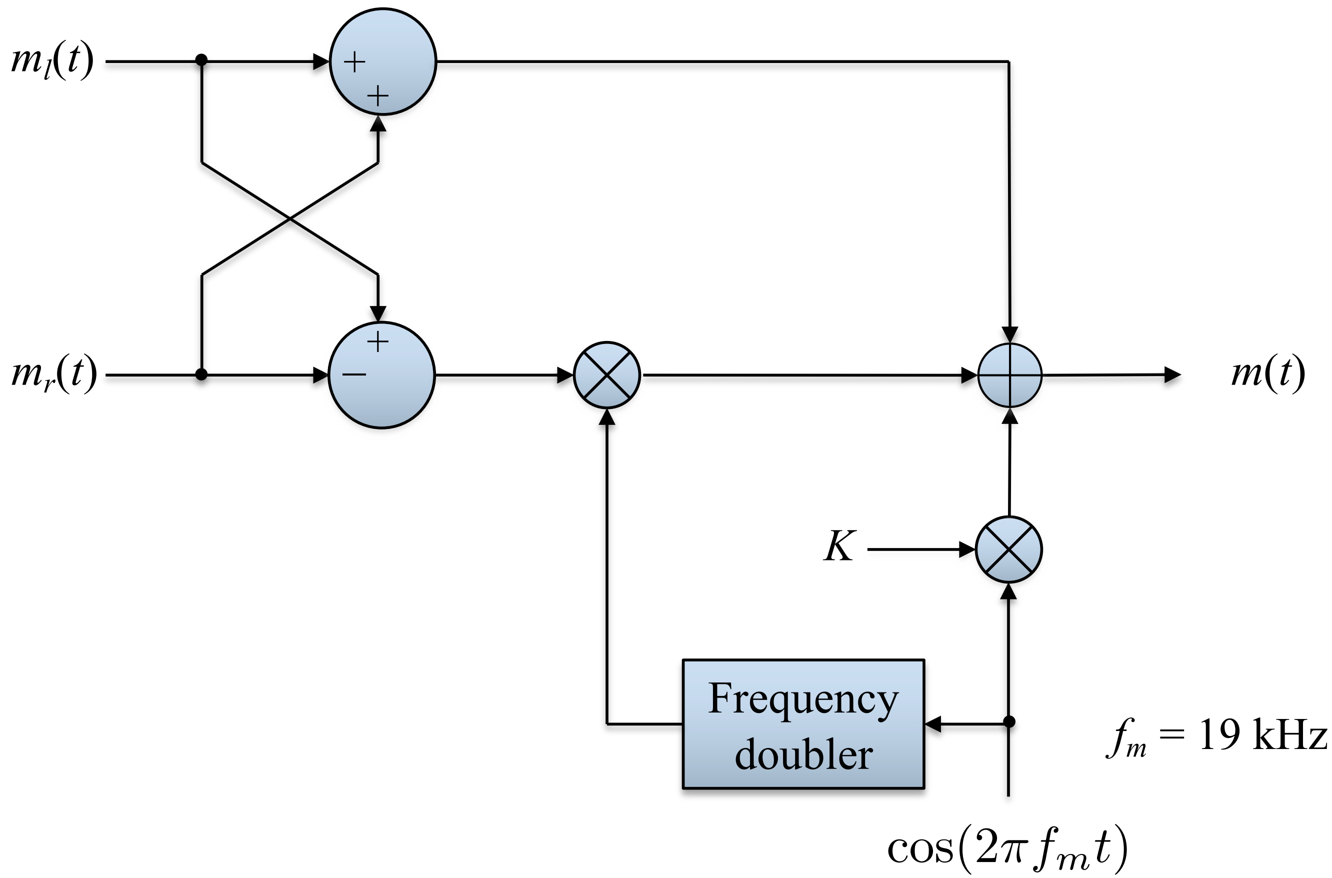
$$\Rightarrow |\tilde{s}_2(t)| = \pi B_T a A_c \left[1 - \frac{2k_f}{B_T} m(t)\right]$$

$$\Rightarrow \tilde{s}_o(t) = |\tilde{s}_1(t)| - |\tilde{s}_2(t)| = 4\pi k_f a A_c m(t)$$

Final Note: a is a parameter of the two filters, which can be used to adjust the amplitude of the resultant output.

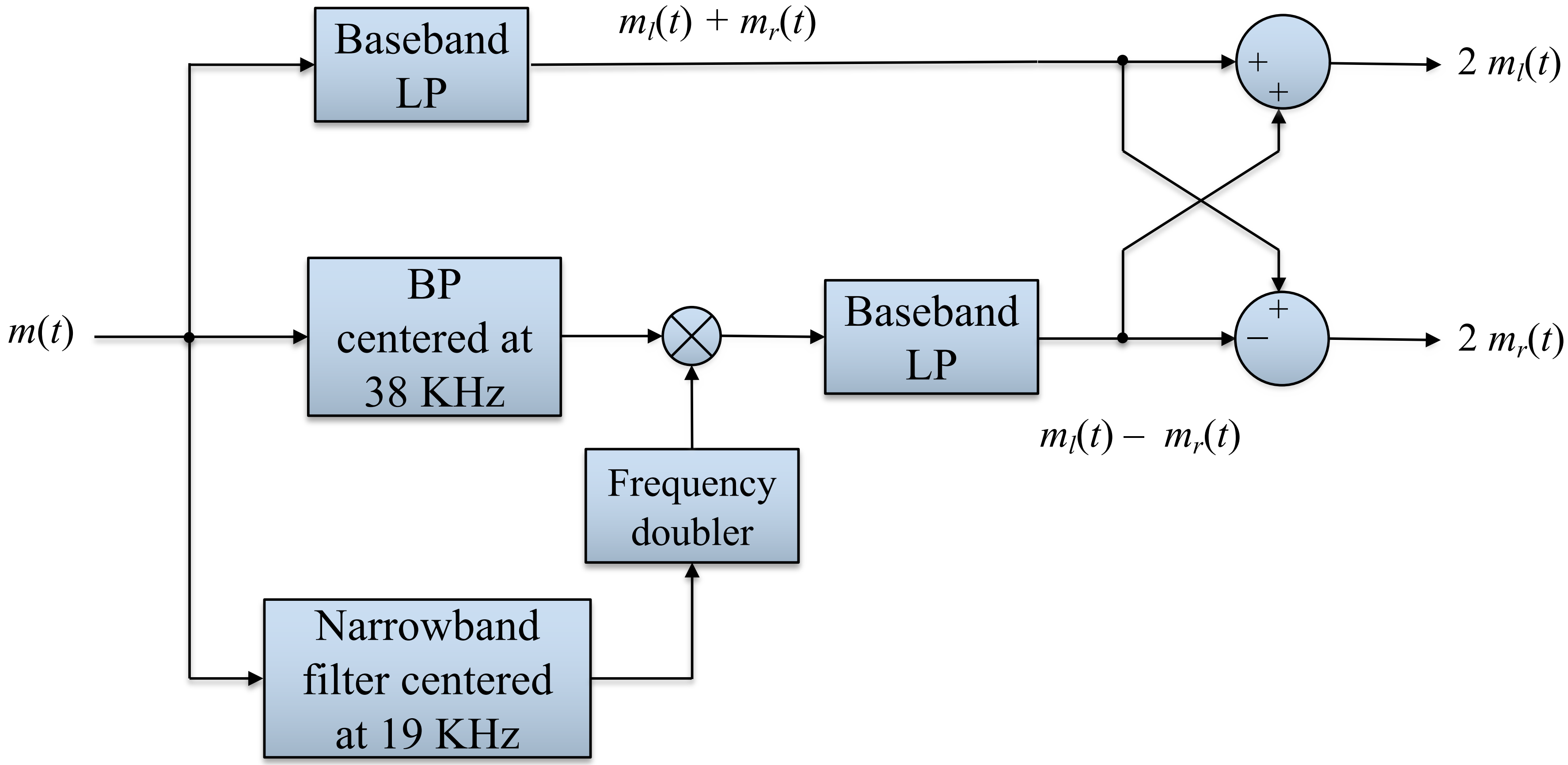
FM Stereo Multiplexing

- How to do *Stereo Transmission* in FM radio?
  - Two requirements:
    - Backward compatible with monophonic radio receivers
    - Operate within the allocated FM broadcast channels
  - To fulfill these requirements, the baseband message signal has to be re-made.



$$m(t) = \underbrace{[m_l(t) + m_r(t)]}_{\text{For monophonic reception}} + [m_l(t) - m_r(t)]\cos(4\pi f_m t) + \underbrace{K\cos(2\pi f_m t)}_{\text{For coherent detection}}$$

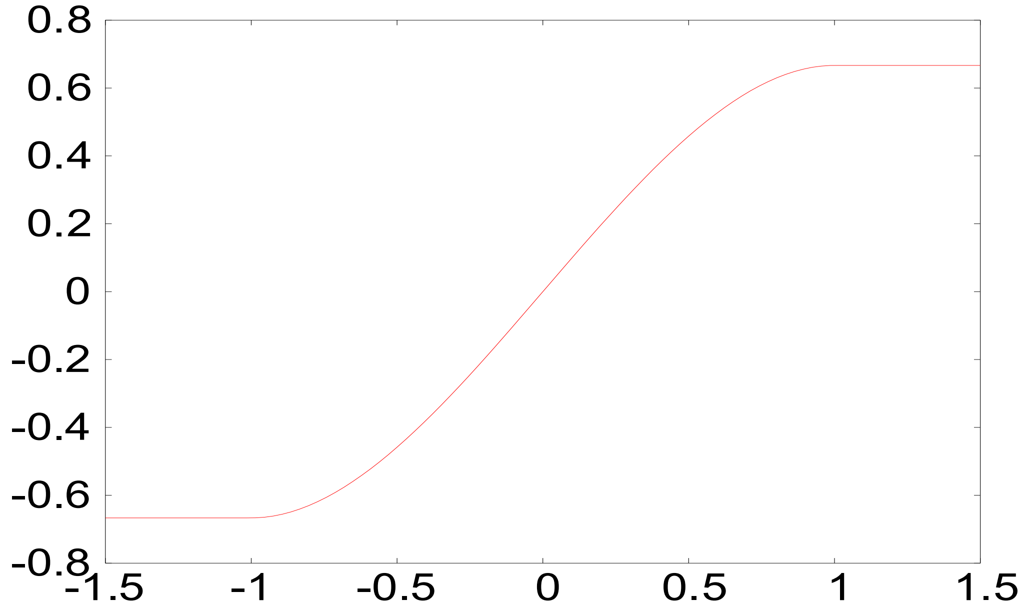
Demultiplexer in receiver of FM stereo.



Impact of Nonlinearity in FM Systems

- The channel (including background noise, interference and circuit imperfection) may introduce nonlinear effects on the transmission signals.
  - For example, nonlinearity due to amplifiers.

$$v_o(t) = \begin{cases} v_i(t) - \frac{1}{3} v_i^3(t), & |v_i(t)| \le 1 \\ \frac{2}{3}, & v_i(t) > 1 \\ -\frac{2}{3}, & v_i(t) < 1 \end{cases}$$



Impact of Nonlinearity in FM Systems

- Suppose

$$\begin{cases} v_o(t) = a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) \\ v_i(t) = A_c \cos\left[2\pi f_c t + \varphi(t)\right] \\ \varphi(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau \end{cases}$$

Then

$$\begin{aligned} v_o(t) &= a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) \\ &= a_1 A_c \cos[2\pi f_c t + \varphi(t)] + a_1 A_c^2 \cos^2[2\pi f_c t + \varphi(t)] \\ &\quad + a_1 A_c^3 \cos^3[2\pi f_c t + \varphi(t)] \end{aligned}$$

$$v_o(t) = a_1 A_c \cos[2\pi f_c t + \varphi(t)] + \frac{1}{2} a_2 A_c^2 \left(1 + \cos[4\pi f_c t + 2\varphi(t)]\right)$$

$$+ \frac{1}{4} a_3 A_c^3 \left(3\cos[2\pi f_c t + \varphi(t)] + \cos[6\pi f_c t + 3\varphi(t)]\right)$$

$$= \frac{1}{2} a_2 A_c^2 + \underbrace{\left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right)\cos[2\pi f_c t + \varphi(t)]}_{B_{T,\text{Carson}} = 2\Delta f + 2W}$$

$$+ \underbrace{\frac{1}{2} a_2 A_c^2 \cos[4\pi f_c t + 2\varphi(t)]}_{B_{T,\text{Carson}} = 4\Delta f + 2W} + \underbrace{\frac{1}{4} a_1 A_c^3 \cos[6\pi f_c t + 3\varphi(t)]}_{B_{T,\text{Carson}} = 6\Delta f + 2W}$$

Thus, in order to recover $s(t)$ from $v_o(t)$ using bandpass filter (i.e., to remove $2f_c$ and $3f_c$ terms from $v_o(t)$), it requires:

$$2f_c - (4\Delta f + 2W)/2 > f_c + (2\Delta f + 2W)/2$$

or equivalently, $f_c > 3\Delta f + 2W$.

The filtered output is therefore:

$$v_{o,\text{filtered}}(t) = \left(a_1 A_c + \frac{3}{4} a_3 A_c^3\right)\cos[2\pi f_c t + \varphi(t)]$$

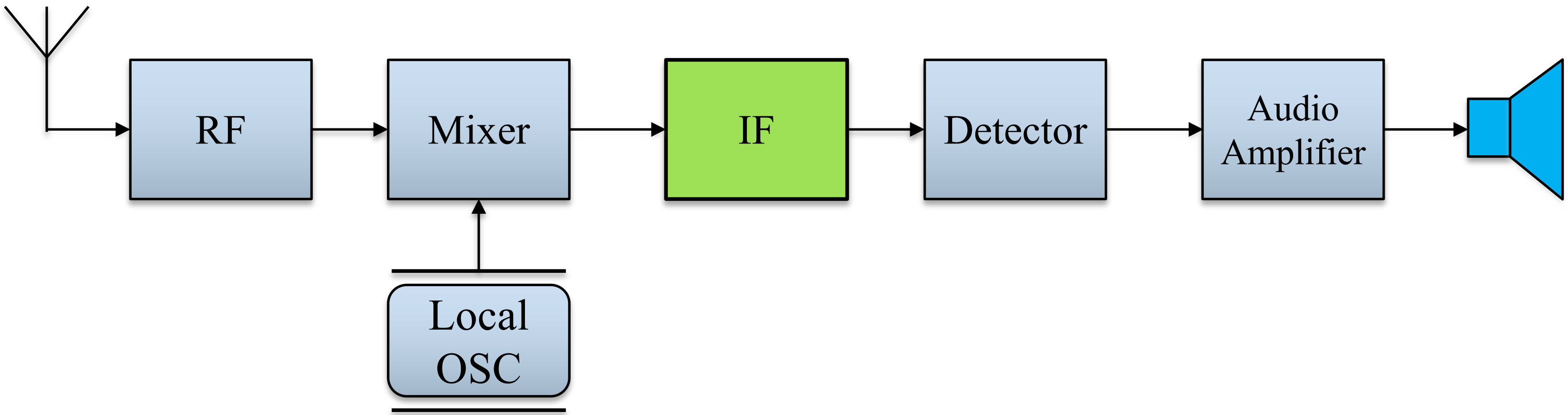
- **Observations**
  - Unlike AM, FM is not affected by distortion produced by transmission through a channel with *amplitude nonlinearities*.
  - So, FM allows the usage of highly nonlinear amplifiers and power transmitters.

Intermediate Frequency (IF) Session

- In a broadcasting system, the receiver not only has the task of demodulation but also requires to perform some other system functions, such as:
  - Carrier-frequency tuning, to select the desired signals
  - Filtering, to separate the desired signal from other unwanted signals
  - Amplifying, to compensate for the loss of signal power incurred in the course of transmission

Intermediate Frequency (IF) Session

- A *superheterodyne* receiver or *superhet* is designed to facilitate the fulfillment of these functions, especially the first two.
  - It overcomes the difficulty of having to build a *tunable highly selective and variable* filter (rather a fixed filter is applied on IF section).



Intermediate Frequency (IF) Session

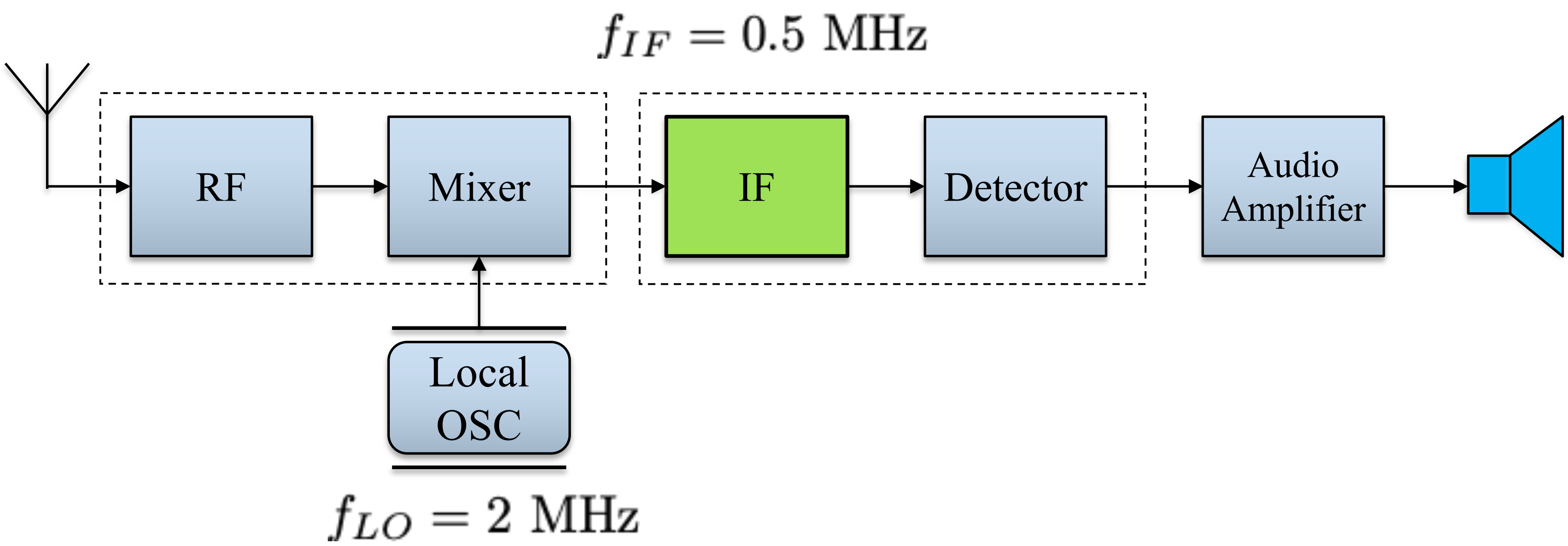


Image Interference

- Fix f_{IF} and f_{LO} at the receiver end. What is the f_{RF} that will survive at the IF section output?
  - Answer: $f_{RF} = | f_{LO} \pm f_{IF} |$
  - Example. Suppose the receiver uses 2 MHz local oscillator, and receives two RF signals respectively centered at 2.5 MHz and 1.5 MHz.

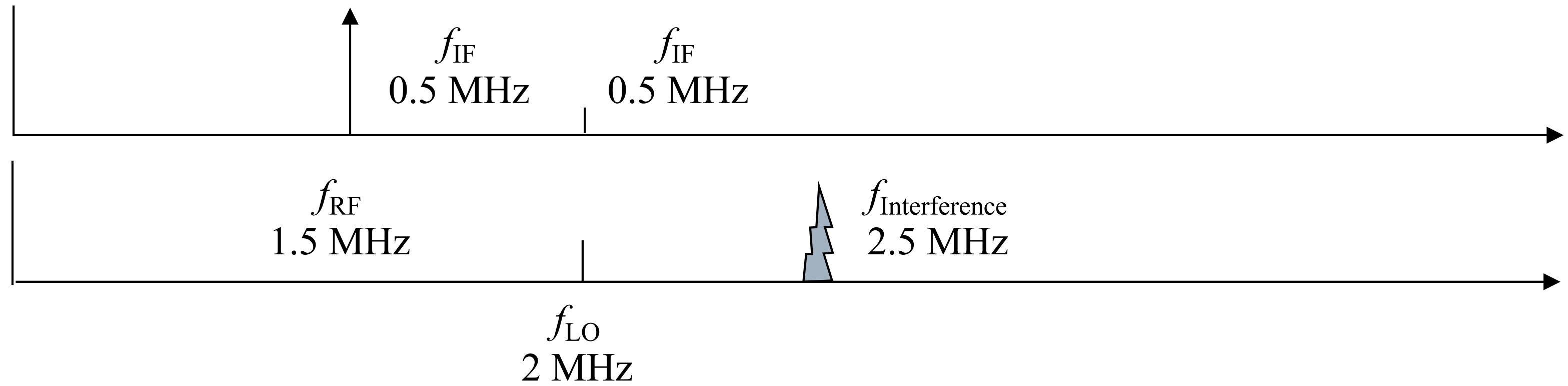


Image Interference

- A cure of image interference is to employ a highly selective stages in the RF session in order to favor the desired signal (at f_{RF}) and discriminate the undesired signal.

Advantage of Constant Envelope for FM

- Observations
  - For FM, any variation in amplitude is caused by noise or interference.
  - For FM, the information is resided on the variations of the instantaneous frequency.
  - So, we can use an *amplitude limiter* to remove the amplitude variation, but to retain the frequency variation after the IF section.

Advantage of Constant Envelope for FM

- Amplitude limiter
  - Clipping the modulated wave at the IF section output almost to the zero axis to result in a near-rectangular wave.
  - Pass the rectangular wave through a bandpass filter centered at f_{IF} to suppress harmonics due to clipping.
  - Then, the filter output *retains the frequency variation with constant amplitude*.

Summary

- Four types of AM modulations are introduced
  - (expensive) DSB-C transmitter + (inexpensive) envelope detector, which is good for applications like radio broadcasting.
  - (less expensive) DSB-SC transmitter + (more complex) coherent detector, which is good for applications like limited-transmitter-power point-to-point communication.
  - (less bandwidth) VSB transmitter + coherent detector, which is good for applications like television signals and high speed data.
  - (minimum transmission power/bandwidth) SSB transmitter + coherent detector, which is perhaps only good for applications whose message signals have an energy gap on zero frequency.

Summary

- FM modulation, a representative of Angle Modulation
  - A nonlinear modulation process
  - Carson's rule and universal curve on transmission bandwidth