Supplement: Instantaneous frequency

Instantaneous Frequency

Treat $\cos(\underbrace{2\pi f_c t})$

Angle (degree)

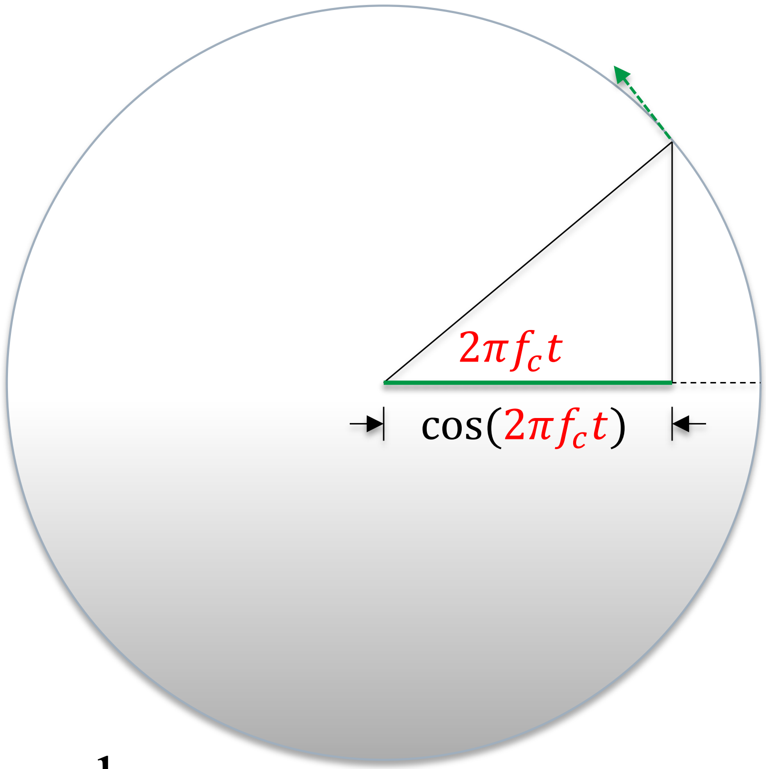
Phase (ratio)

as a projection of $e^{j2\pi f_c t}$ onto the x-axis.

$$f_i(t) = \frac{1}{2\pi}\frac{\mathrm{d}(2\pi f_c t)}{\mathrm{d}t} = f_c$$

Example. $f_c = 10$ Hz

10 circulations per second



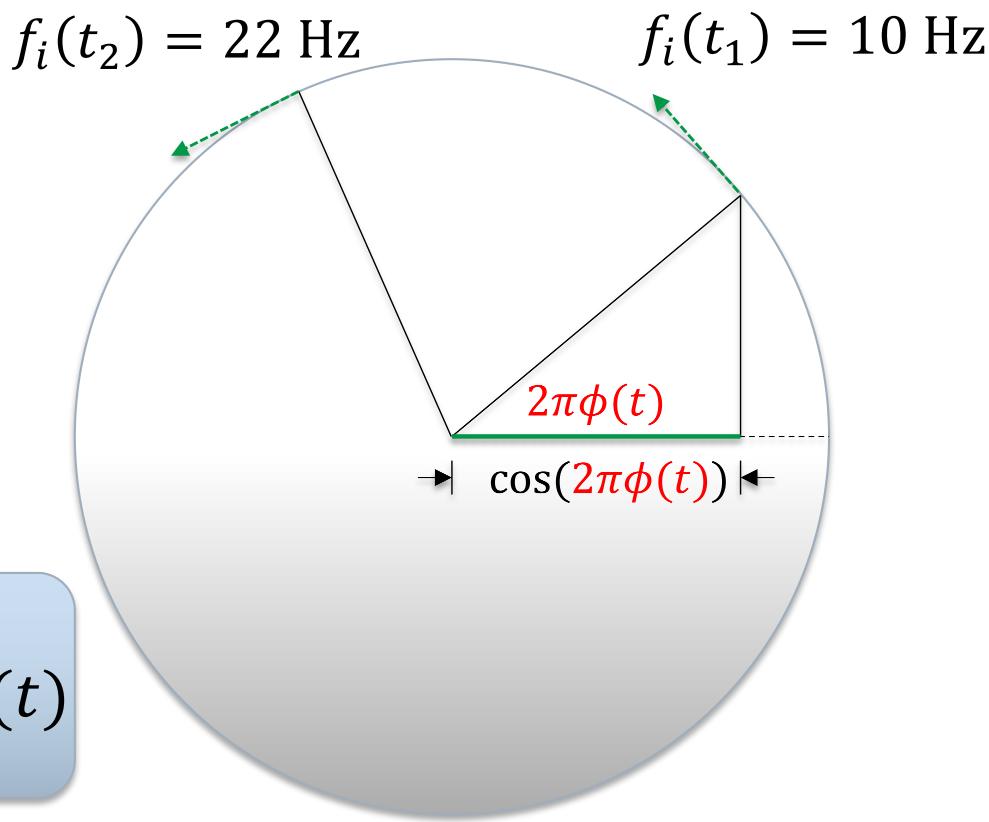
Instantaneous Frequency

Treat $\cos(\underbrace{2\pi\phi(t)})$

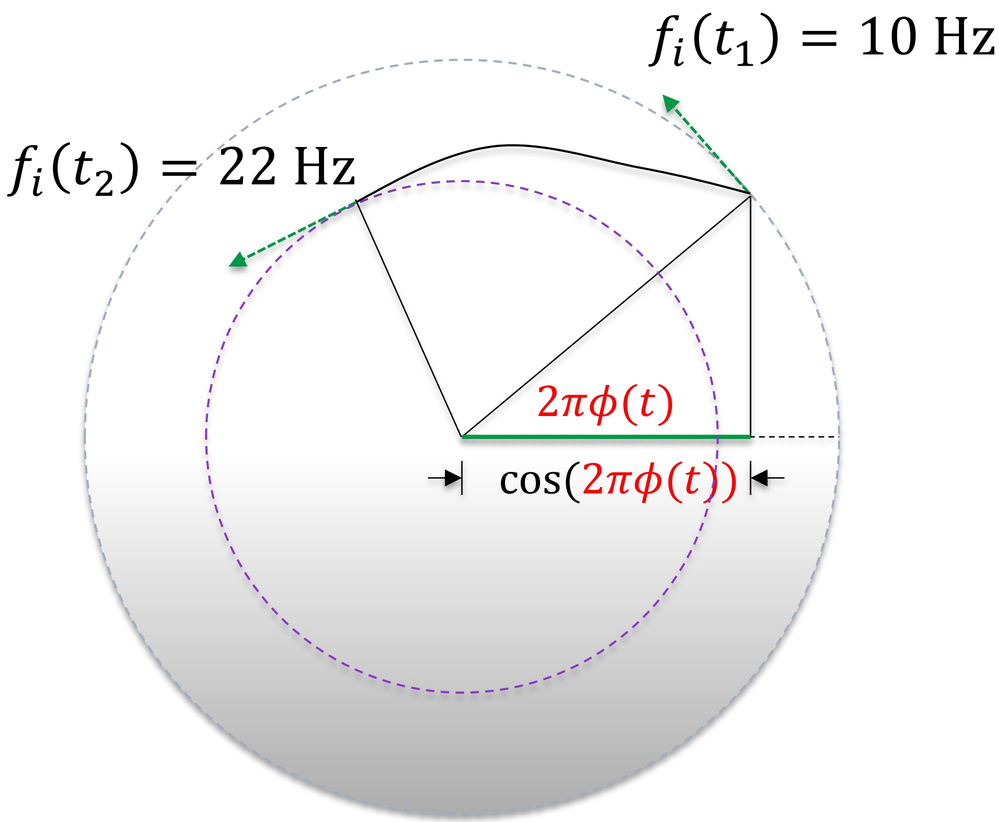
Phase (ratio)

as a projection of $e^{j2\pi\phi(t)}$ onto the x-axis

$$f_i(t) = \frac{1}{2\pi}\frac{\mathrm{d}(2\pi\phi(t))}{\mathrm{d}t} = \phi'(t)$$

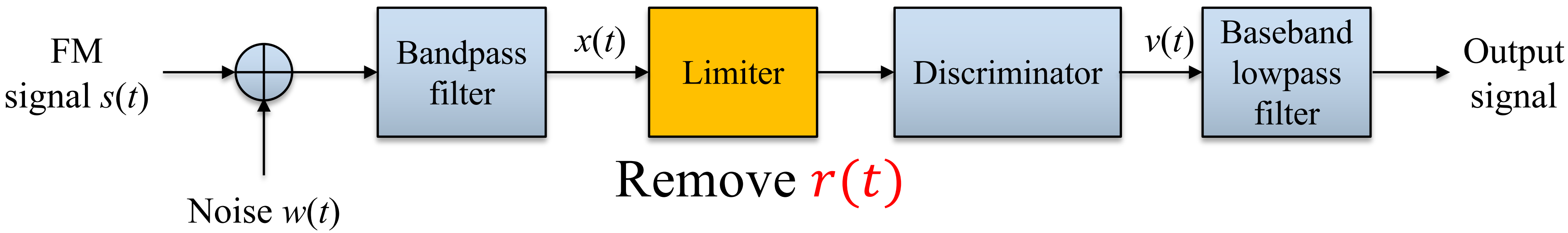


By this interpretation, $\cos(2\pi\phi(t))$ and $r(t)\cos(2\pi\phi(t))$ should have the same instantaneous frequency.



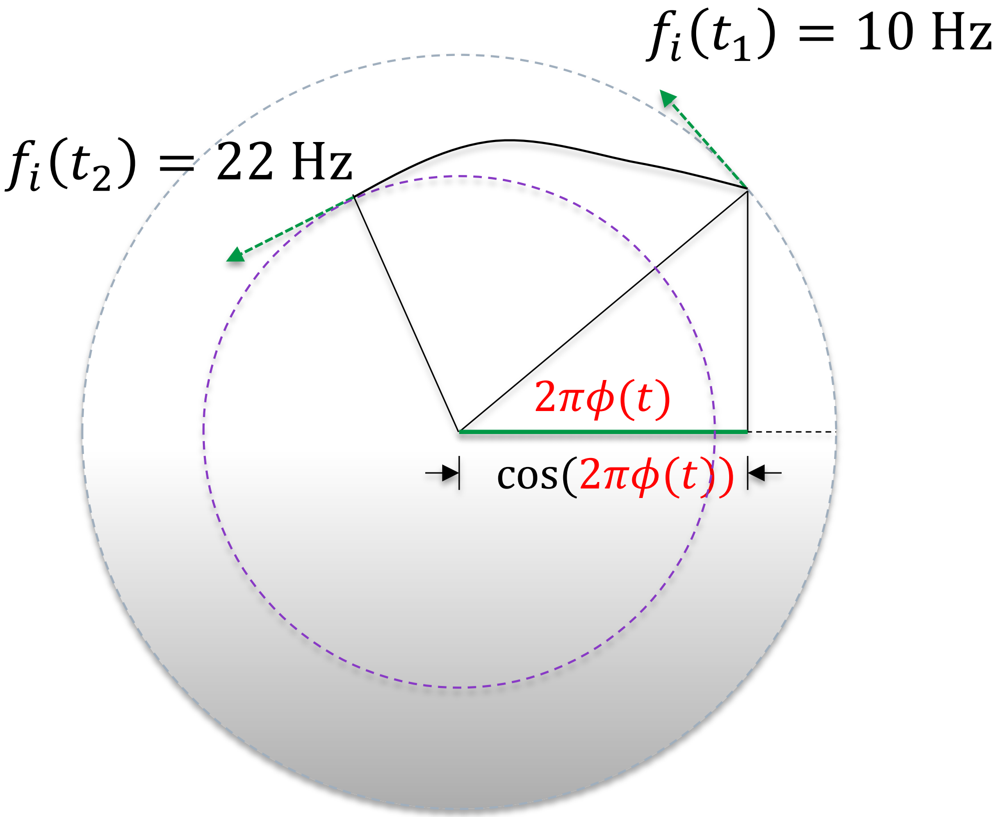
$$f_i(t) = \frac{1}{2\pi}\frac{\mathrm{d}(2\pi\phi(t))}{\mathrm{d}t} = \phi'(t)$$

This explains why we add a limiter in the FM demodulation process.



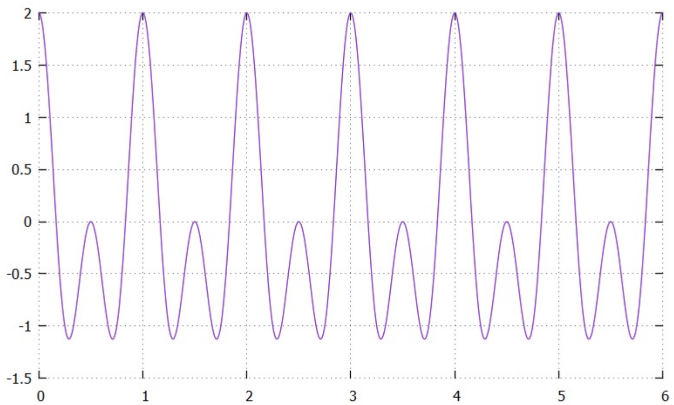
By this interpretation, $\cos(2\pi\phi(t))$ and $e^{j2\pi\phi(t)}$ should have the same instantaneous frequency.

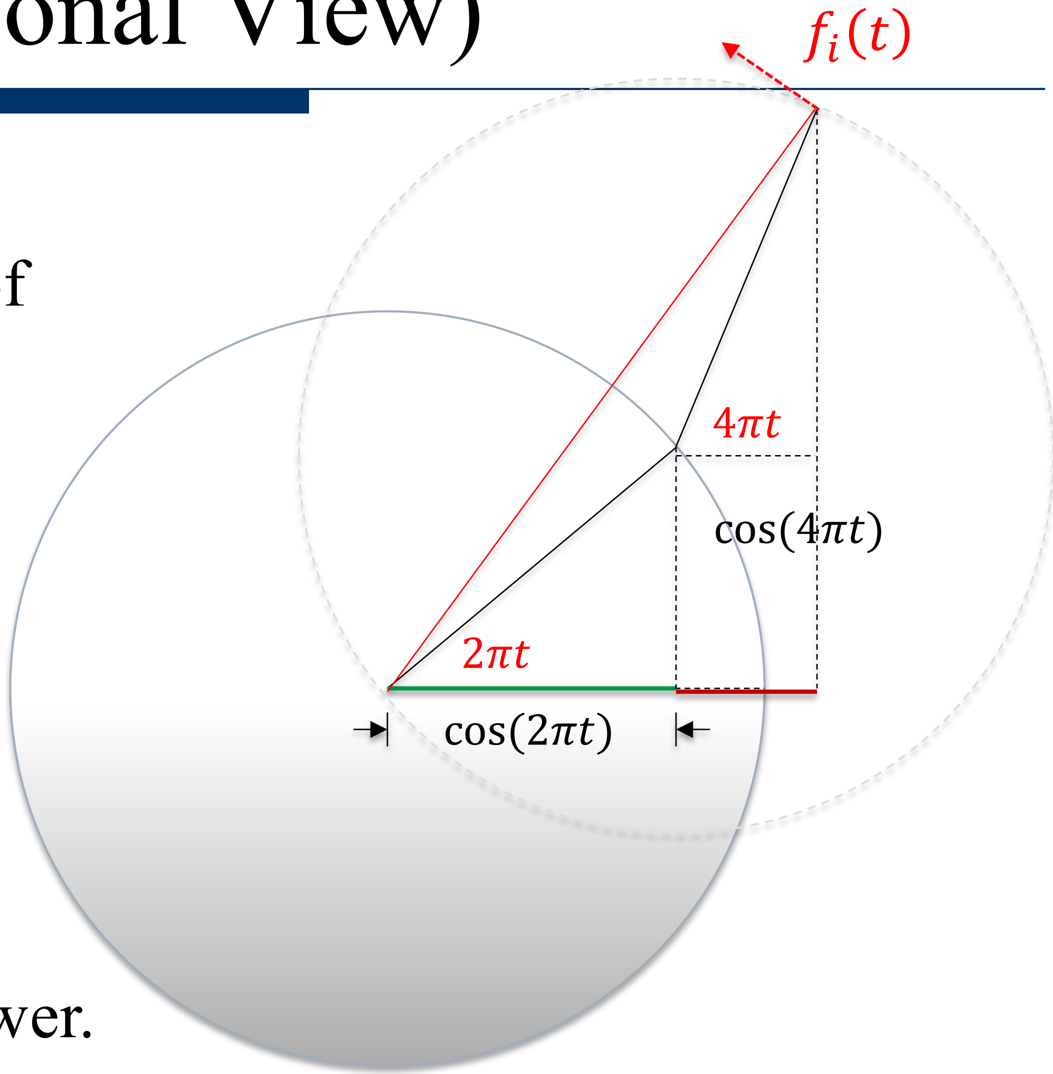
$$f_i(t) = \frac{1}{2\pi}\frac{\mathrm{d}(2\pi\phi(t))}{\mathrm{d}t} = \phi'(t)$$



Discussion (Personal View)

Question: What is the instantaneous frequency of $\cos(2\pi t) + \cos(4\pi t)$?

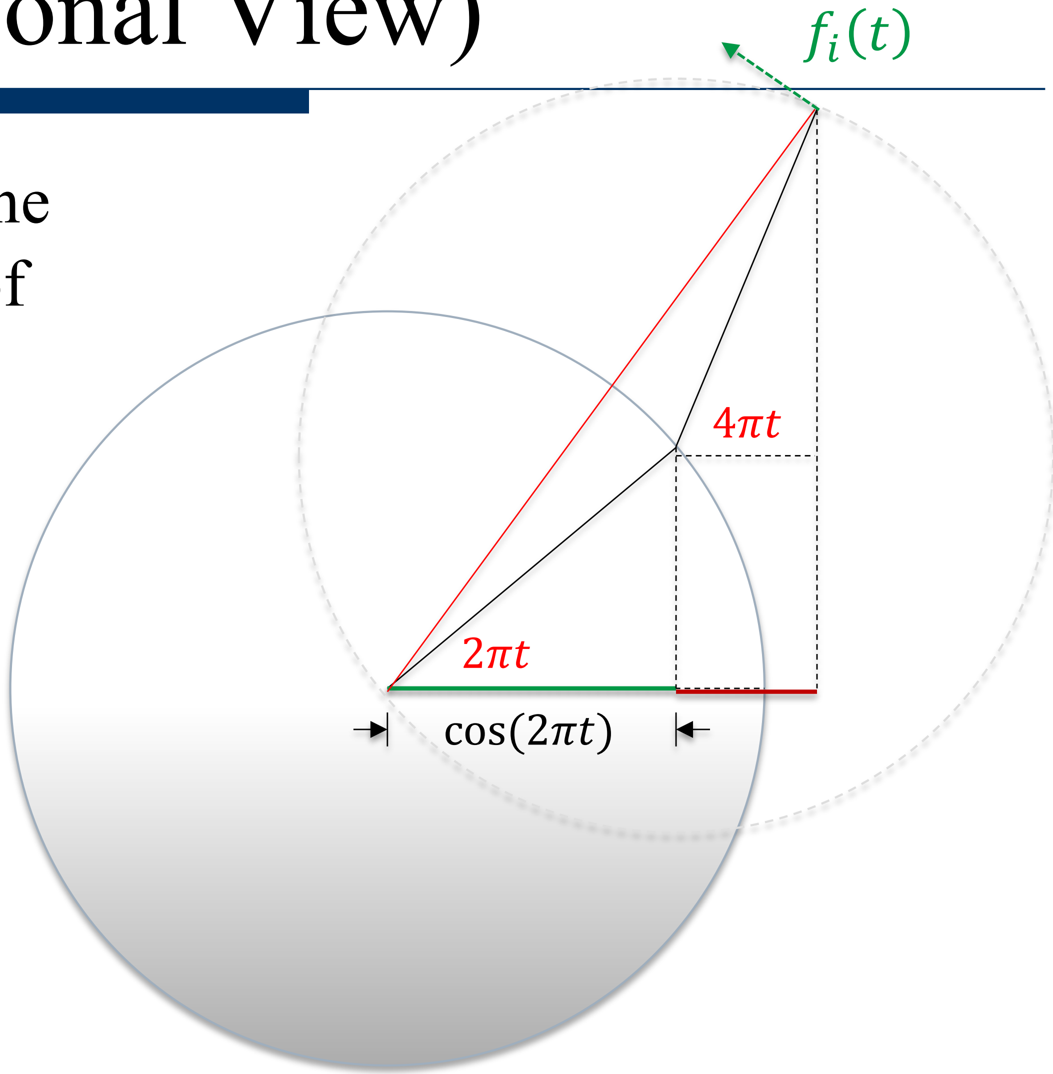




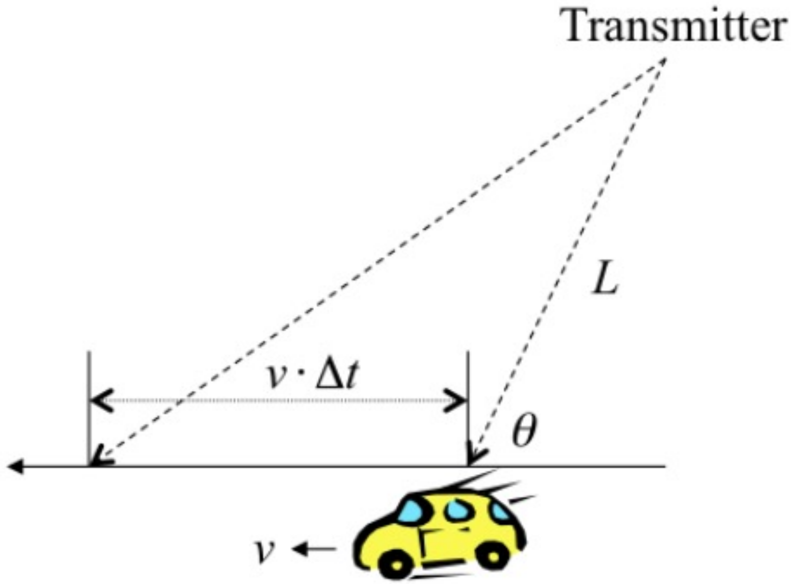
No universally good answer.

Discussion (Personal View)

Same Question: What is the instantaneous frequency of $e^{j2\pi t} + e^{j4\pi t}$?



Derivation of Doppler Shift



- Difference in path length

$$\begin{aligned}\Delta L &= \sqrt{(L\sin(\theta))^2 + (L\cos(\theta) + v \cdot \Delta t)^2} - L \\ &= \sqrt{L^2 + v^2(\Delta t)^2 + 2L \cdot v \cdot \Delta t \cdot \cos(\theta)} - L\end{aligned}$$

- Phase change $\Delta\phi = 2\pi \frac{\Delta L}{(c/f_c)} \quad \left(= 2\pi \frac{\Delta L}{\text{wavelength}} \right)$

- Estimated Doppler shift

$$\begin{aligned}\lambda_m &= \lim_{\Delta t \to 0} \frac{1}{2\pi} \frac{\Delta\phi}{\Delta t} \\ &= \frac{1}{c/f_c} \lim_{\Delta t \to 0} \frac{\sqrt{L^2 + v^2(\Delta t)^2 + 2L \cdot v \cdot \Delta t \cdot \cos(\theta)} - L}{\Delta t} \\ &= \frac{vf_c}{c}\cos(\theta) = f_m \cos(\theta)\end{aligned}$$

Example. $v = 108$ km/hour, $f_c = 5$ GHz and $c = 1.08 \times 10^9$ km/hour.

$$\Longrightarrow \lambda_m = 500\cos(\theta) \text{ Hz.}$$