3. Simulation of DC Motor

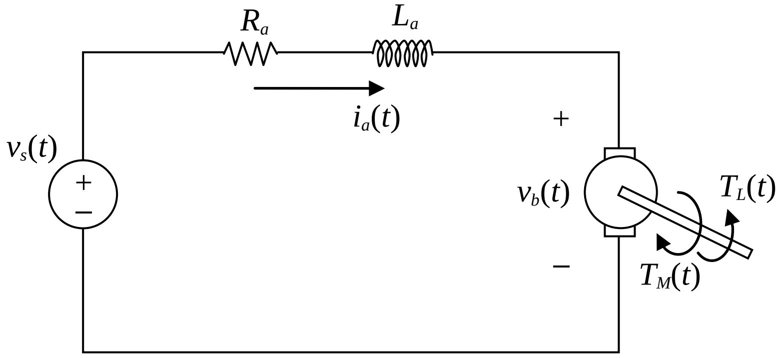

Figure3-1

The dynamic model of a DC motor, depicted in Figure3-1, has been derived to contain the electrical part:

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + k\omega(t) = v_s(t) \tag{3-1}$$

and the mechanical part:

$$J_M \dot{\omega}(t) + B_M \omega(t) - k i_a(t) = -T_L(t) \tag{3-2}$$

$$\dot{\theta}(t) = \omega(t) \tag{3-3}$$

where $i_a(t)$ is the armature current, $v_s(t)$ is the voltage source, $\omega(t)$ and $\theta(t)$ is the angular velocity and position of the rotor, and $T_L(t)$ the external torque from payload. In practice, the motor is connected to a transmitting device and then a payload. For example, the dynamic model of the payload can be a rotary structure described as

$$J_L \dot{\omega}_L(t) + B_L \omega_L(t) = T_L'(t) \tag{3-4}$$

$$\dot{\theta}_L(t) = \omega_L(t) \tag{3-5}$$

where the torque $T_L'(t)$ comes from the shaft of a transmittig device and is assumed satisfying

$$T_L(t) - T'(t) = K_s(\theta(t) - \theta_L(t)) + B_s(\omega(t) - \omega_L(t)) \tag{3-6}$$

For simplicity, let the payload strictly stick to the motor, i.e., $\theta_L(t)=\theta(t)$ and $\omega_L(t)=\omega(t)$. Then, $T_L(t) = T'(t)$ and from (3-2) and (3-4) we have

$$J\dot{\omega}(t)+B\omega(t)-ki_a(t)=0 \tag{3-7}$$

where $J=J_M+J_L$ and $B=B_M+B_L$. Next, we will use (3-1), (3-7) and (3-3) as the dynamic model of the DC motor in simulation.

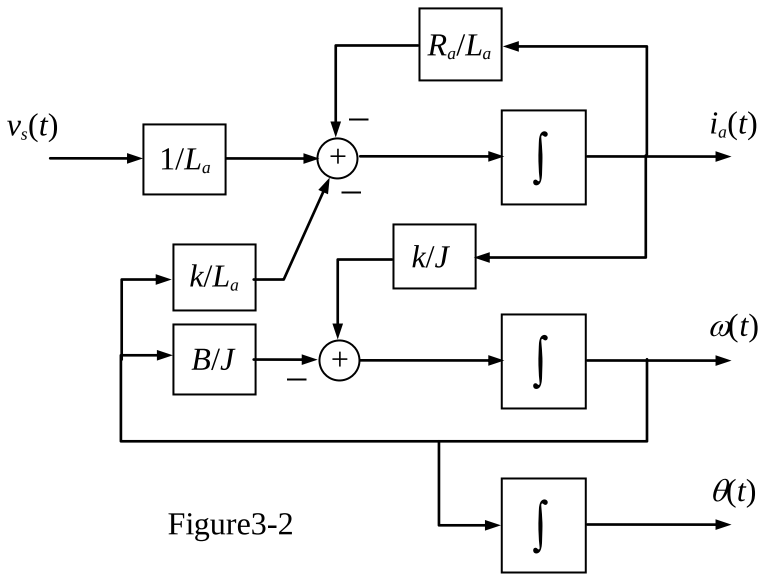

Figure3-2

To simulate the dynamic behavior of the DC motor, we have to rewrite (3-1), (3-7) and (3-3) as below:

$$\frac{di_a(t)}{dt}=-\frac{R_a}{L_a}i_a(t)-\frac{k}{L_a}\omega(t)+\frac{1}{L_a}v_s(t) \tag{3-8}$$

$$\dot{\omega}(t)=-\frac{B}{J}\omega(t)+\frac{k}{J}i_a(t) \tag{3-9}$$

$$\dot{\theta}(t)=\omega(t) \tag{3-10}$$

and Figure3-2 shows the block diagram correspondingly. For demonstration, let's consider the following case: R_a=0.5Ω, L_a=1.6×10^{-3} H, J=4×10^{-4} N-m/(rad/s^2), k=0.05 V/(rad/s) and B=1.5×10^{-4} N-m/(rad/s), and simulate the DC motor in MATLAB Simulink with the block diagram in Figure3-3. The simulation results of $i_a(t)$, $\omega(t)$ and $\theta(t)$ for initially idled DC motor with input $v_s(t)=V_s$=1 V are given in Figure3-4, where $i_a(t)$ and $\omega(t)$ finally approach I_a=0.0608 A and Ω=19.393 rad/s. Based on these results, is that possible to retrieve the motor parameters R_a, L_a, J, k and B?

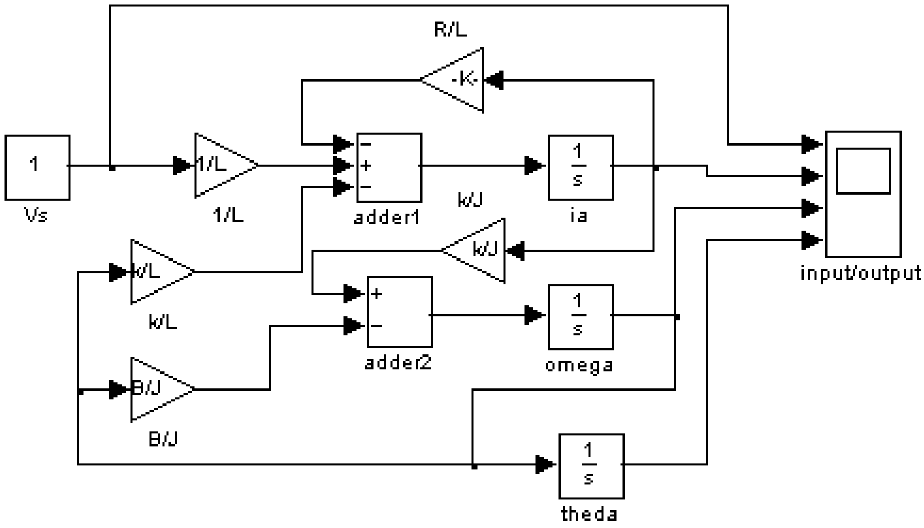

Figure3-3

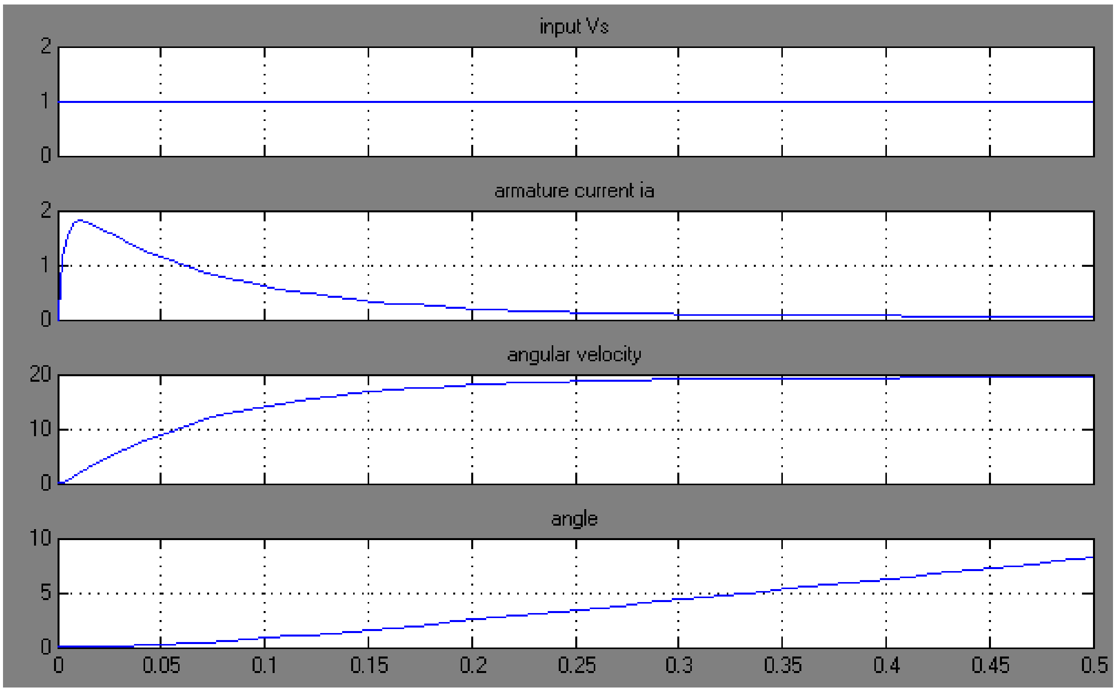

Figure3-4

From (3-8) and (3-9), we knows that once the DC motor is in steady state, i.e., the motor rotates in a constant velocity $\omega(t)=\Omega$ and the armature current reaches a constant value $i_a(t)=I_a$. That implies the variations of $i_a(t)$ and $\omega(t)$ almost vanish and (3-8) and (3-9) become

$$0=-\frac{R_a}{L_a}i_a(t)-\frac{k}{L_a}\omega(t)+\frac{1}{L_a}v_s(t) \tag{3-11}$$

$$0=-\frac{B}{J}\omega(t)+\frac{k}{J}i_a(t) \tag{3-12}$$

which result in $R_a I_a + k\Omega = V_s$ and $B\Omega - kI_a = 0$. Since V_s=1 V, I_a=0.0608 A and Ω=19.393 rad/s, we have $\frac{k}{B}=\frac{\Omega}{I_a}=318.96$ and $k=\frac{V_s-R_aI_a}{\Omega}=0.0516-0.0031R_a$. Clearly, the parameters can not be obtained just from the simulation results.

In practice, we can simply measure the armature resistance R_a=0.5Ω, and then obtain $k=0.0516-0.0031R_a=0.05005$ and $B=\frac{k}{318.96}=1.57\times10^{-4}$, which are approximated to the values in simulation. As for the values of J, we have to simplify the model by neglecting the small inductanc L_a, i.e., (3-8) is changed into

$$0=-R_ai_a(t)-k\omega(t)+v_s(t) \tag{3-13}$$

and (3-9) is rewritten as

$$\dot{\omega}(t)=-\left(\frac{B}{J}+\frac{k^2}{JR_a}\right)\omega(t)+\frac{k}{JR_a}v_s(t) \tag{3-14}$$

That means the time constant of $\omega(t)$ is $\left(\frac{B}{J}+\frac{k^2}{JR_a}\right)^{-1}$. From the simulation result of $\omega(t)$, we can find the time constant is 0.078 sec, i.e., $\frac{B}{J}+\frac{k^2}{JR_a}=\frac{1}{0.078}$ which leads to $J=0.078\left(B+k^2/R_a\right)=4.02\times10^{-4}$, near to the value in siimulation as expected. The only term left to be determined is the small inductance L_a. Actually, in the spped control of DC motor, we do not care about the exact model and often employ the simplied mode (3-14), which is given as

$$\dot{\omega}(t)=-12.82\,\omega(t)+249\,v_s(t) \tag{3-15}$$

It is easy to check that if $v_s(t)$=1 V then $\omega(t)$ reaches 19.422 rad/sec as t increases, which is near to the simulation result 19.393 rad/sec.