11. Maximum Power Point Tracking of a Solar Module

Nowadays, in order to deal with the problems of global energy crisis and environmental protection, solar photovoltaic (PV) systems constructed by a solar panel (Figure 11-1) or a solar module (Figure 11-2) have been developed for a great diversity of applications and, especially, can be found in remote areas where no public grid is available. Here, we will only focus on the maximum power point tracking problem of a solar system, containing a solar PV module, a DC/DC converter and a maximum power controller.

A solar panel with 24 solar modules

Figure 11-1

A solar module with 4 solar cells

Figure 11-2

In general, a solar power system inevitably encounters low efficiency problem caused by the variation of irradiance and temperature. As a consequence, a solar power system can not be operated at maximum power unless it is under power control. Recently, a lot of investigators have proposed their control technologies for the PV panels to generate maximum power, which are named as the maximum power point tracking, or MPPT in brief.

The developed MPPT technologies can be classified into six categories, which are listed as below:

(1) the perturbation and observation method (P&O),

(2) the incremental conductance method,

(3) the gradient method,

(4) the approximate straight line method,

(5) the voltage feedback method,

(6) the power feedback method.

In this topic, we will focus on the most common one, the P&O method which is designed under the strategy of perturbing the load so as to drive the operating point toward the maximum power point (MPP). However, the main disadvantage of the P&O happens when the solar system is finally operated in the vicinity of MPP wherein the operating point oscillates and causes a power loss to degrade the solar energy conversion efficiency.

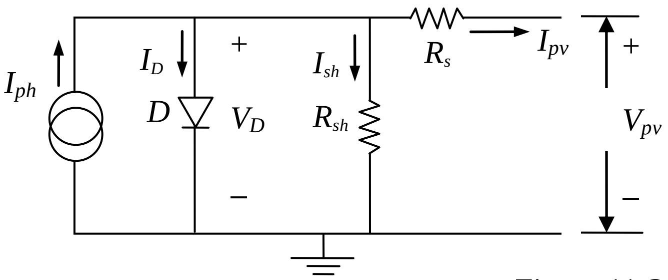

Figure 11-3

An equivalent circuit of a solar cell is shown in Figure 11-3, which is modeled as below:

$$I_{pv} = I_{ph} - I_D - I_{sh} \tag{11-1}$$

where I_{pv} is the output current through the series resistor R_s, I_{ph} is the photogenerated current, I_D is the diode current and I_{sh} is the current through the shunt resistor R_{sh}. It is easy to check that

$$I_{sh} = \frac{V_{pv} + R_s I_{pv}}{R_{sh}} \tag{11-2}$$

where V_{pv} is the output voltage. In addition, from Shockley diode equation, the diode current can be represented as

$$I_D = I_o\left(e^{\frac{qV_D}{\eta KT}} - 1\right) = I_o\left(e^{\frac{q(V_{pv}+R_s I_{pv})}{\eta KT}} - 1\right) \tag{11-3}$$

where I_0 is the reverse saturation current, q=1.6×10^{-19} C is the elementary charge, K=1.38×10^{-23} J/K is the Boltzmann's constant, T is the absolute temperature, and η is the diode ideality fctor. Note that η=1~2 and η=1 for an ideal diode. As a result, we have

$$I_{ph} = I_{pv} + I_o\left(e^{\frac{q(V_{pv}+R_s I_{pv})}{\eta KT}} - 1\right) + \frac{V_{pv} + R_s I_{pv}}{R_{sh}} \tag{11-4}$$

which is a nonlinear static system.

In practice, the shunt resistance R_{sh} is large enough and the series resistance R_s is very small. For simplicity, we rewrite (11-4) as the following approximate model:

$$I_{pv} = I_{ph} - I_o\left(e^{\frac{qV_{pv}}{\eta KT}} - 1\right) \tag{11-5}$$

where the voltage R_sI_{pv} across the series resistor and the current I_{sh} through the shunt resistor are neglected. From (11-5), it can be also obtained that

$$V_{pv} = \frac{\eta KT}{q} ln \frac{I_o + I_{ph} - I_{pv}}{I_o} \tag{11-6}$$

Therefore, the power generated by a solar cell can be expressed as

$$P_{pv} = I_{pv}V_{pv} = \left[I_{ph} - I_o\left(e^{\frac{qV_{pv}}{\eta KT}} - 1\right)\right] \cdot V_{pv} \tag{11-7}$$

and Figure 11-4 shows the resulted PV characteristics by P_{pv}-V_{pv}. From the curve, we know that the solar cell generates a maximum power P_{max}.

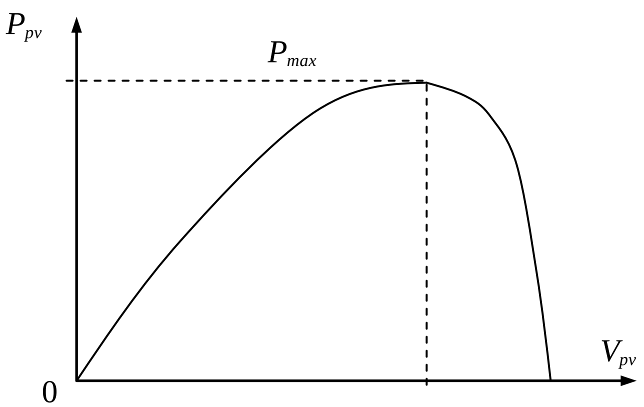

Figure 11-4

If we further consider the solar module constructed by $M \times N$ solar cells, M rows and N columns, then the voltage and current provided by the solar module are $V_{SM}=MV_{pv}$ and $I_{SM}=NI_{pv}$, respectively. From (11-6), we have

$$V_{SM} = M\frac{\eta KT}{q} ln \frac{I_o + I_{ph} - \frac{I_{SM}}{N}}{I_o} \tag{11-8}$$

The equivalent circuit of a solar module is depicted in Figure 11-5.

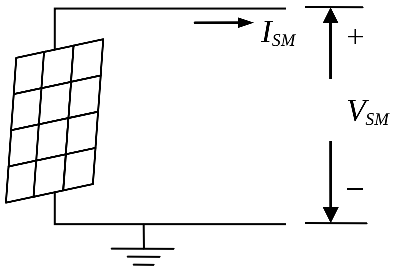

Figure 11-5

To efficiently retrieve the maximum power from the solar module, a switching step-down DC-DC converter called buck converter is used as shown in Figure 11-5. The power transistor Q is switched on and off at a frequency f_s higher than the frequency bandwidth of the lowpass filter. Let the period be $T_s=t_{on}+ t_{off}$, where $T_s=1/f_s$ is constant and t_{on} and t_{off} are respectively the time intervals of Q on and Q off. The buck converter is a device mainly to change an input voltage into a lower output voltage, and is often applied to the power supply and DC motor speed control.

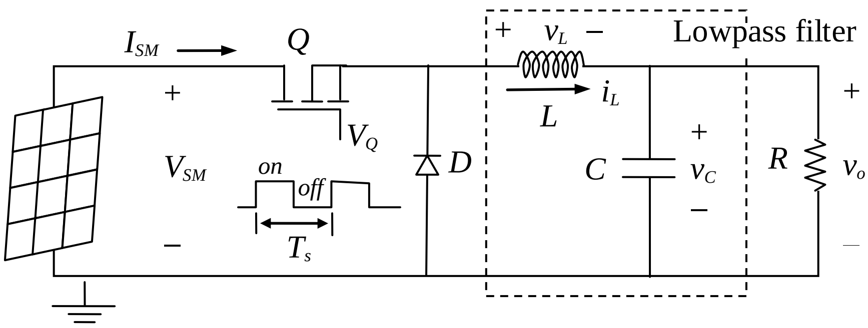

Figure 11-6

Let the output voltage of the buck converter be $v_o(t)$ and its average be denoted as V_o. It is known that V_o is lower than the input voltage V_{SM}, i.e., $V_o<V_{SM}$. In general, the operation of a buck converter can be divided into the continuous mode and the discontinuous mode, depending on the switching actions of the switching transistor Q. Figure 11-7 shows the characteristic of the inductor with a larger value of L during one period T_s.

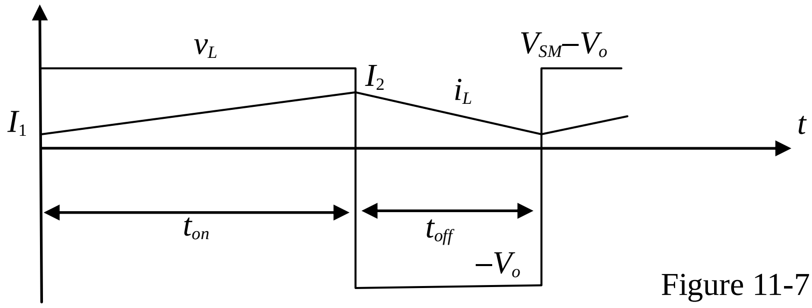

Figure 11-7

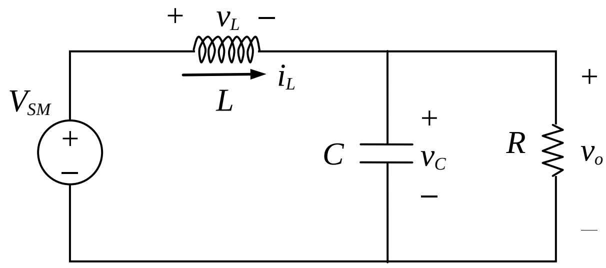

Figure 11-8

During the continuous mode, Q is switched on and the equivalent circuit is shown in Figure 11-8. Since the input voltage V_{SM} is greater than the average output voltage V_o, i.e., $v_L(t)= V_s–V_o >0$, the inductor current $i_L(t)$ is increased due to the fact of $\frac{di_L(t)}{dt}=\frac{1}{L}v_L(t)$. For a larger L, $i_L(t)$ will rises linearly from I_1 to I_2 during the time t_{on} that Q is on, as shown in Figure 11-7. Therefore,

$$V_{SM}-V_o=L\frac{I_2-I_1}{t_{on}}=L\frac{\Delta I}{t_{on}} \tag{11-9}$$

which leads to

$$t_{on}=\frac{L\Delta I}{V_s-V_o} \tag{11-10}$$

Note that the energy is stored in the magnetic field of the inductor when Q is on.

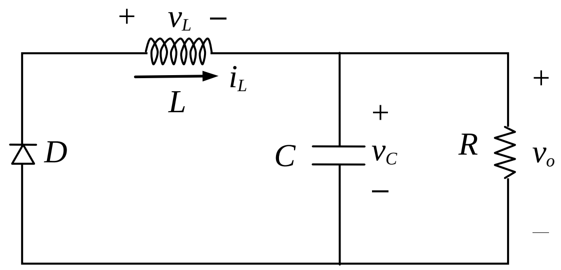

Figure 11-9

During the discontinuous mode, Q is switched off and the equivalent circuit is shown in Figure 11-9. It is known that the current $i_L(t)$ and the voltage $v_C(t)$ are continuous and thus can not be changed abruptly. As a result, the voltage of the inductor $v_L(t)$ is suddenly changed from V_{SM}–V_o to $-V_o$ and its current is also linearly decreased from I_2 to I_1. Similarly, we have

$$-V_o = L\frac{I_1 - I_2}{t_{off}} = -L\frac{\Delta I}{t_{off}} \tag{11-11}$$

which leads to

$$t_{off} = \frac{L\Delta I}{V_o} \tag{11-12}$$

From (11-10) and (11-12), we have

$$V_o = V_s\frac{t_{on}}{T} = V_s D \tag{11-13}$$

where $D = \frac{t_{on}}{T}$ is called the duty ratio. Note that (11-13) implies the output voltage can be controlled by the dytu ratio D.

The dynamic equation of Figure 11-6, the solar module combined with the buck converter, can be described in continuous mode and discontinuous mode. For the continuos mode, from Figure 11-8 we have

$$\begin{cases} \dfrac{di_L(t)}{dt} = \dfrac{V_{SM}}{L} - \dfrac{v_o(t)}{L} \\ \\ \dfrac{dv_o(t)}{dt} = \dfrac{i_L(t)}{C} - \dfrac{v_o(t)}{RC} \end{cases} \tag{11-14}$$

where the term V_{SM} is given in (11-8). Since $I_{SM}=i_L(t)$, V_{SM} is a function of $i_L(t)$ and expressed as

$$V_{SM} \equiv V_{SM}(i_L(t)) = M\frac{\eta KT}{q} ln\frac{I_o + I_{ph} - \frac{i_L(t)}{N}}{I_o} \tag{11-15}$$

For the discontinuos mode, from Figure 11-9 we have

$$\begin{cases} \dfrac{di_L(t)}{dt} = -\dfrac{v_o(t)}{L} \\ \dfrac{dv_o(t)}{dt} = \dfrac{i_L(t)}{C} - \dfrac{v_o(t)}{RC} \end{cases} \tag{11-16}$$

where V_{SM} is not included.

Based on the high frequency switching and the use of lowpass filter, the equivalent dynamic equation for (11-4) and (11-6) can be witten into

$$\frac{di_L(t)}{dt} = \frac{V_{SM}(i_L(t))}{L}D - \frac{v_o(t)}{L} \tag{11-17}$$

$$\frac{dv_o(t)}{dt} = \frac{i_L(t)}{C} - \frac{v_o(t)}{RC} \tag{11-18}$$

where $V_{SM}(i_L(t))$ is given in (11-15). Next the maximum power point tracking (MPPT) control will be designed for the duty ratio D.

The basic concept of MPPT is to set a suitable D such that the solar module can generate and provide the maximum power to the payload R. The power generated by the solar module is expressed as

$$P_{SM} = V_{SM} I_{SM} = V_{SM}(i_L(t)) \cdot i_L(t) \geq 0 \tag{11-19}$$

and from Figure 11-4 the power is always nonnegative and has a maximum. It is clear that if the derivative of P_{SM} with respect to time is positive then P_{SM} will be finally reach its maximum value. Taking the derivative of P_{SM} with respect to time yields

$$\begin{aligned}\dot{P}_{SM} &= \frac{\partial P_{SM}}{\partial i_L}\frac{di_L(t)}{dt} \\ &= \left(V_{SM}(i_L(t)) + i_L(t)\frac{\partial V_{SM}(i_L(t))}{\partial i_L}\right)\left(\frac{V_{SM}(i_L(t))}{L}D - \frac{v_o(t)}{L}\right)\end{aligned} \tag{11-20}$$

In the sense of discretized, let $t=kT$ where T is the sampling time used in the controller design. Then, (11-20) is approximated as

$$\dot{P}_{SM} \approx \frac{S}{L\Delta i_L(kT)}\left(V_{SM}(i_L(kT))D - v_o(kT)\right) \tag{11-21}$$

where

$$Q_{SM} = V_{SM}(i_L(kT))\Delta i_L(kT) + i_L(kT)\Delta V_{SM}(i_L(kT)) \tag{11-22}$$

$$\Delta i_L(kT) = i_L(kT) - i_L((k-1)T) \tag{11-23}$$

$$\Delta V_{SM}(i_L(kT)) = V_{SM}(i_L(kT)) - V_{SM}(i_L((k-1)T)) \tag{11-24}$$

Let the duty ratio be chosen as

$$D = \frac{v_o(kT) + \varepsilon \cdot sign(\Delta i_L(kT)Q_{SM})}{V_{SM}(i_L(kT))} \tag{11-25}$$

where $\varepsilon>0$, then from (11-21) we have

$$\dot{P}_{SM} \approx \frac{|Q_{SM}|}{L|\Delta i_L(kT)|}\varepsilon \geq 0 \tag{11-26}$$

Obviously, the use of (11-25) can guarantee that the power provided by the solar module will finally reach its maximum point and the larger ε makes the power to reach its maximumpoint faster. This completes the MPPT control.