CV3 Complex Numbers: Regions in the Complex Plane

Here, we will explain some regions often used in the complex number system:

(1) ε neighborhood of z_0

If we want to denote a neighbor region with distance ε to a specified point z_0, we can write it as $|z-z_0|<\varepsilon$ which is defined as the ε neighborhood of z_0.

(2) deleted ε neighborhood of z_0

While the point z_0 is not included in the neighbor region, we can write it as $0<|z-z_0|<\varepsilon$ which is defined as the deleted ε neighborhood of z_0.

(3) interior point z_i of a set S

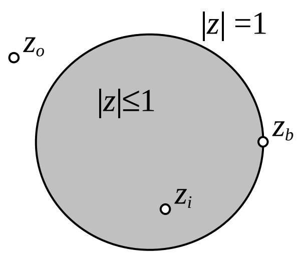
If there exists a neighborhood region of z_i that contains only points of S, then z_i is an interior point of S.

(4) exterior point z_o of a set S

If there exists a neighborhood region of z_o that contains no points of S, then z_o is an an exterior point of S.

(5) boundary point z_b of a set S

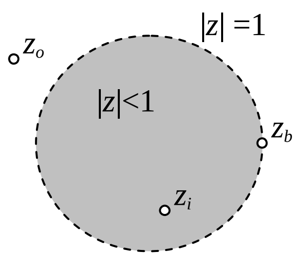
If there exists a neighborhood region of z_b that contains points in S and not in S, then z_b is an a boundary point of S.



(6) boundary of a set S

All the boundary points of S form the boundary of S.

Example $|z|=1$ is the boundary of $|z|<1$ and $|z|\le 1$.



(7) open set

A set contains none of its boundary points. Ex: $|z|<1$ is an open set.

(8) closed set

A set contains all of its boundary points. Ex: $|z| \le 1$ is a closed set.

(9) closure of a set *S*

The closed set consists of all the points in *S* and its boundary.

For example, $|z| \le 1$ is the closure of $|z| < 1$.

Example A punctured disk $0 < |z| \le 1$ is neither open nor closed.

The set of complex numbers is both open and closed since it has no boundary points.

(10) domain *S* or connected open set *S*

If each pair z_1 and z_2 in the open set *S* can be joined by a polygonal line, then it is a connected open set or called a domain.

Example $|z| < 1$ and $1 < |z| < 2$ are connected open sets or domains.

(11) region

A domain together with some, none, or all boundary points is referred to as a region.

(12) bounded and unbounded set

A set is bounded if every point of *S* lies inside some circle $|z| = R$;

otherwise, it is unbounded.

Example The region $Re\ z \ge 0$ is unbounded.

The regions $|z| < 1$ and $1 < |z| < 2$ are bounded.

P3-1 Determine which are domains, which are neither open nor closed, and which are bounded:

(a) $|z-2+i| \le 1$; (b) $|2z+3| > 4$; (c) $Im z > 1$

(d) $Im z = 1$; (e) $0 \le arg\ z \le \frac{\pi}{4}\ (z \ne 0)$; (f) $|z-4| \ge |z|$.

Ans: domains-(b),(c); neither open nor closed-(e), bounded-(a)