Calculus: Homework 9

May 8th, 2008

1. Find all critical points of the function

$$f(x, y) = \ln x + 2\ln y - x - 4y.$$

   Then, use the second derivative test to decide at which points a local extreme value is achieved.

2. Find absolute extreme values of

$$f(x, y) = (4y^2 - x^2)e^{-x^2-y^2}$$

   on the disk $D = \{(x, y) | x^2 + y^2 \leq 2\}$.

3. Show that for $\alpha, \beta \geq 1$ with $1/\alpha + 1/\beta = 1$, we have

$$\frac{1}{\alpha}x^\alpha + \frac{1}{\beta}y^\beta \geq xy$$

   for all $x, y \geq 0$.

4. Use Lagrange multipliers to find the maximum area of a rectangle inscribed in the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

5. For constants $E_1, \ldots, E_n, E$ consider the maximum of

$$S = x_1 \ln x_1 + \cdots + x_n \ln x_n$$

   subject to the constraints

$$x_1 + \cdots + x_n = N, \qquad E_1x_1 + \cdots + E_nx_n = E.$$

   Show that there is some constant μ such that $x_i = A^{-1}e^{\mu E_i}$, where

$$A = N^{-1}\left(e^{\mu E_1} + \cdots + e^{\mu E_n}\right).$$