Calculus: Homework 8

May 1st, 2008

1. Let $f(u, v, w)$ be differentiable and $u = x - y, v = y - z$, and $w = z - x$. Show that
$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0.$$

2. Under mild continuity restriction, it is true that if
$$F(x) = \int_a^b g(t, x)\mathrm{d}t, \text{ then } F'(x) = \int_a^b g_x(t, x)\mathrm{d}t.$$
Use this fact to find the derivative of
$$G(x) = \int_a^{f(x)} g(t, x)\mathrm{d}t.$$

3. If a sound with frequency f_s is produced by a source travelling along a line with speed v_s and an observer is travelling with speed v_o along the same line from the opposite direction towards the source, then the frequency of the sound heard by the observer is
$$f_o = \left(\frac{c + v_o}{c - v_s}\right) f_s,$$
where c is the speed of sound, about $332\ m/s$. Suppose that, at a particular moment, you are in a train travelling at $34\ m/s$ and accelerating at $1.2\ m/s^2$. A train is approaching you from the opposite direction on the other track at $40\ m/s$, accelerating at $1.4\ m/s^2$, and sounds its whistle, which has a frequency of 460 Hz. At this moment, what is the perceived frequency that you hear and how fast is it changing (use the chain rule from Section 14.6 to compute the latter)?

4. Find the directional derivative of $f(x, y, z) = xe^y + ye^z + ze^x$ at $(0, 0, 0)$ in direction of $\langle 5, 1, -2\rangle$.

5. If the directional derivative of $f(x, y)$ in direction of $\mathbf{i} + \mathbf{j}$ is $2\sqrt{2}$ and in direction of $-2\mathbf{j}$ is -3, what is the directional derivative of f in direction of $-\mathbf{i} - 2\mathbf{j}$?