

# Calculus: Homework 6

April 17th, 2008

1. Describe the level surfaces of the function

$$f(x, y, z) = 9x^2 - 4y^2 + 36z^2.$$

2. Find the limit or show that it does not exist

$$\lim_{(x,y) \rightarrow (0,0)} \cos\left(\frac{x^3 - y^3}{x^2 + y^2}\right).$$

3. Use the  $\epsilon$ - $\delta$  definition of the limit to show that

$$\lim_{(x,y,z) \rightarrow (a,b,c)} (x + 2y - 3z) = a + 2b - 3c$$

for all  $(a, b, c) \in \mathbb{R}^3$ .

4. Find all first-order partial derivatives of

$$f(x, y, z) = \sin^{-1}(xyz), \quad g(x, y, z) = \sinh(xy - z^2).$$

5. Show that the following functions

$$f_1(x, y, z) = \ln \sqrt{x^2 + y^2}, \quad f_2(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$$

satisfy the Laplace equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$$