Calculus: Homework 5

March 31st, 2008

1. Let $P = (m, n, r)$ be a point on the ellipsoid with equation $(x/a)^2 + (y/b)^2 + (z/c)^2 = 1$. Consider the plane with the equation
$$\frac{mx}{a^2} + \frac{ny}{b^2} + \frac{rz}{c^2} = 1.$$
Show that P is on the plane and that all tangents to the cross-sections at P are contained in the plane as well.

2. Which surface is given by the following equation in cylindrical coordinates
$$r^2(1 - 2\sin^2\theta) + z^2 = 1?$$

3. We say that two vector functions $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ *intersect* if the corresponding space curves intersect; we say they *collide* if $\mathbf{r}_1(t_0) = \mathbf{r}_2(t_0)$ for some t_0. Determine whether
$$\mathbf{r}_1(t) = \langle t^2 + 3, t + 1, 6t^{-1}\rangle, \quad \mathbf{r}_2(t) = \langle 4t, 2t - 2, t^2 - 7\rangle$$
intersect or collide.

4. The space curve traced out by the vector function $\mathbf{r}(t) = \langle t - \tanh t, \operatorname{sech} t\rangle$ is called a *tractrix*. Find the arc-length function of the tractrix and re-parametrize it by using the arc length as new parameter.

5. Find a parametrization of the osculating circle to $y = \sin x$ at $x = \pi/2$.