Calculus: Homework 3

March 13th, 2008

1. Find all x for which the following power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}} x^n$$

is convergent.

2. Let $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ be two power series with radius of convergence R and L, respectively. If $R \neq L$, what can be said about the radius of convergence of $\sum_{n=0}^{\infty}(a_n + b_n)x^n$? Is your claim true for $R = L$ as well?

3. Find a power series representation (centered at 0) for the function

$$f(x) = \frac{x}{1 - x - x^2}.$$

Compute the first 10 coefficients. What do you observe?

Hint: Use partial fraction expansion and the geometric series.

4. Find the Taylor series of $\sin(x)$ centered at $\pi/6$. Show that it converges and represents the function for all real numbers x.

5. Consider

$$J = \int_0^1 \sin(x^2)\mathrm{d}x.$$

(a) Express J as an infinity series.

(b) Determine J to within an error less than 10^{-4}.