# Calculus: Homework 2

March 6th, 2008

1. Use the comparison test to determine whether the following sequences

$$\sum_{n=1}^{\infty} \frac{n}{n^3+1} \qquad \text{and} \qquad \sum_{n=1}^{\infty} n^2 2^{-n^3}$$

are convergent or not. Compared to the integral test which test do you prefer?

2. Consider the series $\sum_{n=2}^{\infty} a_n$ with

$$a_n = \frac{1}{\ln(\ln n)^{\ln n}}.$$

   (a) Show that $a_n = n^{-\ln(\ln(\ln n))}$.

   (b) Show that $\ln(\ln(\ln n)) \geq 2$ for all $n > e^{e^{e^2}}$.

   (c) Show that the series is convergent.

3. Determine wether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}} \qquad \text{and} \qquad \sum_{n=1}^{\infty} (-1)^n \frac{n}{e^n}$$

are absolutely convergent, conditionally convergent, or not convergent at all.

4. Let $a_n$ be a sequence with $a_n \neq 0$. Assume that

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 4.$$

Is this information enough to determine whether the series $\sum_{n=1}^{\infty} 1/a_n$ is convergent or not?

5. Use the ratio and the root test to determine whether the following series

$$\sum_{n=1}^{\infty} \left(2 + \frac{1}{n}\right)^{-n} \qquad \text{and} \qquad \sum_{n=1}^{\infty} \frac{2^n}{n^k} \quad (k \in \mathbb{N})$$

are convergent or not. Which test do you prefer?