Calculus: Homework 12

May 5th, 2008

1. Use a change of variable to compute the double integral

$$\iint_R e^{x+y}\mathrm{d}A,$$

where

$$R = \{(x, y)|1 \le x + y \le 4, -4 \le y - 2x \le 1\}.$$

2. Use the transformation T with

$$x = \frac{\sin u}{\cos v}, \qquad y = \frac{\sin v}{\cos u}$$

to evaluate the integral

$$\int_0^1 \int_0^1 \frac{\mathrm{d}x\mathrm{d}y}{1 - x^2 y^2}.$$

3. Compute the work done by the force field

$$\mathbf{F}(x, y) = \left\langle e^{y-x}, e^{2x} \right\rangle$$

in moving a particle along the piecewise linear path from $(1, 1)$ to $(2, 2)$ to $(0, 2)$.

4. Consider the vector field

$$\mathbf{F} = \left\langle \frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2} \right\rangle.$$

on $D = \{(x, y) \ne (0, 0)\}$.

   (a) Show that $\mathbf{F}$ is conservative.
   (b) Show that $f(x, y) = \ln(x^2 + y^2)/2$ is a potential function of $\mathbf{F}$.
   (c) Is D simple connected.
   (d) Do these results contradict the results in Chapter 16.3?

5. Use a line integral to compute the area enclosed by one arc of the cycloid and the x-axis.