Calculus: Homework 11

May 22nd, 2008

1. For which p does the limit
$$\lim_{\rho \to 0+} \iint_{\rho \leq x^2+y^2 \leq 1} \frac{1}{(x^2+y^2)^p} \mathrm{d}A$$
exist?

2. (a) Show that
$$\int_0^x \int_0^y f(t) \mathrm{d}t \mathrm{d}y = \int_0^x (x-t) f(t) \mathrm{d}t.$$
(b) Show that
$$\int_0^x \int_0^y \int_0^z f(t) \mathrm{d}t \mathrm{d}z \mathrm{d}y = \frac{1}{2} \int_0^x (x-t)^2 f(t) \mathrm{d}t.$$

3. Evaluate
$$\iiint_E y \mathrm{d}V,$$
where E is the solid which lies in the first octant and is bounded by $x^2/4 + y^2/9 = 1$ and $x^2 + y^2 + z^2 = 16$.

4. Let E be the solid bounded by $y + z = 2, 2x = y, x = 0$, and $z = 0$. Evaluate
$$\iiint_E x e^z \mathrm{d}V$$
by integrating first over the projection of E onto the (a) xy-plane; (b) xz-plane; (c) yz-plane.

5. Evaluate
$$\iiint_{x^2+y^2+z^2 \leq 1} \cos z \mathrm{d}V.$$