Calculus: Homework 10

May 15th, 2008

1. Evaluate the following double integral
$$\iint_{[1,3]\times[0,1]} ye^{xy}\mathrm{d}A.$$

2. Let
$$\frac{\partial^2 F}{\partial x \partial y} = f(x,y).$$
Show that
$$\iint_{[a,b]\times[c,d]} f(x,y)\mathrm{d}A = F(b,d) - F(a,d) - F(b,c) + F(a,c).$$

3. Show that the integral
$$I(a) = \int_0^\infty \frac{e^{-x} - e^{-ax}}{x}\mathrm{d}x$$
is convergent for all $a > 0$ and evaluate it.

4. Change the order of integration of
$$\int_0^9 \int_0^{\sqrt{y}} \frac{x^3 \mathrm{d}x\mathrm{d}y}{\sqrt{3x^2+y}}$$
and evaluate the resulting iterated integral.

5. Find the volume of the solid cut from the first octant by the surface $z = 4 - x^2 - y$.