Calculus: Homework 1

February 25th, 2008

1. Determine whether the following sequence

$$a_n = \int_0^n e^{-nx} \mathrm{d}x$$

is convergent or not; if it converges, also find the limit.

2. A sequence a_n is recursively defined as $a_1 = 1$ and

$$a_n = \sqrt{6 + a_{n-1}} \qquad (n \geq 2).$$

Show that a_n is increasing and bounded above. Use this to conclude that a_n is convergent and find the limit.

3. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series. Define the remainder as

$$R_n = \sum_{k=n+1}^{\infty} a_k.$$

Show that $R_n \to 0$ as $n \to \infty$.

4. If d_n is an arbitrary sequence of numbers, is the following relation always true

$$\sum_{n=1}^{\infty} (d_n - d_{n+1}) = d_1?$$

If yes, give a proof; if no, find conditions under which the above relation holds.

5. Use the integral test to determine whether the following sequences

$$\sum_{n=1}^{\infty} \frac{n}{n^3 + 1} \qquad \text{and} \qquad \sum_{n=1}^{\infty} n^2 2^{-n^3}$$

are convergent or not.