

Homework 8

Chapter 14.5 :52

52. (a) $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}$. Then

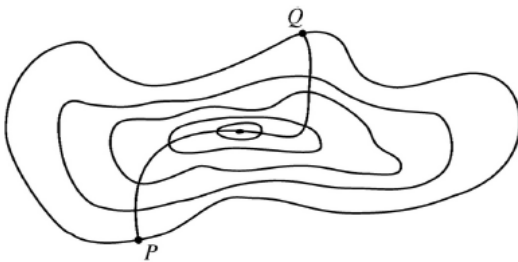
$$\begin{aligned} \frac{\partial^2 z}{\partial t^2} &= \frac{\partial}{\partial t} \left(\frac{\partial z}{\partial x} \frac{\partial x}{\partial t} \right) + \frac{\partial}{\partial t} \left(\frac{\partial z}{\partial y} \frac{\partial y}{\partial t} \right) \\ &= \frac{\partial}{\partial t} \left(\frac{\partial z}{\partial x} \right) \frac{\partial x}{\partial t} + \frac{\partial^2 x}{\partial t^2} \frac{\partial z}{\partial x} + \frac{\partial}{\partial t} \left(\frac{\partial z}{\partial y} \right) \frac{\partial y}{\partial t} + \frac{\partial^2 y}{\partial t^2} \frac{\partial z}{\partial y} \\ &= \frac{\partial^2 z}{\partial x^2} \left(\frac{\partial x}{\partial t} \right)^2 + \frac{\partial^2 z}{\partial y \partial x} \frac{\partial x}{\partial t} \frac{\partial y}{\partial t} + \frac{\partial^2 x}{\partial t^2} \frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial y^2} \left(\frac{\partial y}{\partial t} \right)^2 + \frac{\partial^2 z}{\partial x \partial y} \frac{\partial y}{\partial t} \frac{\partial x}{\partial t} + \frac{\partial^2 y}{\partial t^2} \frac{\partial z}{\partial y} \\ &= \frac{\partial^2 z}{\partial x^2} \left(\frac{\partial x}{\partial t} \right)^2 + 2 \frac{\partial^2 z}{\partial x \partial y} \frac{\partial x}{\partial t} \frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2} \left(\frac{\partial y}{\partial t} \right)^2 + \frac{\partial^2 x}{\partial t^2} \frac{\partial z}{\partial x} + \frac{\partial^2 y}{\partial t^2} \frac{\partial z}{\partial y} \end{aligned}$$

(b)

$$\begin{aligned} \frac{\partial^2 z}{\partial s \partial t} &= \frac{\partial}{\partial s} \left(\frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t} \right) \\ &= \left(\frac{\partial^2 z}{\partial x^2} \frac{\partial x}{\partial s} + \frac{\partial^2 z}{\partial y \partial x} \frac{\partial y}{\partial s} \right) \frac{\partial x}{\partial t} + \frac{\partial z}{\partial x} \frac{\partial^2 x}{\partial s \partial t} + \left(\frac{\partial^2 z}{\partial y^2} \frac{\partial y}{\partial s} + \frac{\partial^2 z}{\partial x \partial y} \frac{\partial x}{\partial s} \right) \frac{\partial y}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial^2 y}{\partial s \partial t} \\ &= \frac{\partial^2 z}{\partial x^2} \frac{\partial x}{\partial s} \frac{\partial x}{\partial t} + \frac{\partial^2 z}{\partial x \partial y} \left(\frac{\partial y}{\partial s} \frac{\partial x}{\partial t} + \frac{\partial y}{\partial t} \frac{\partial x}{\partial s} \right) + \frac{\partial z}{\partial x} \frac{\partial^2 x}{\partial s \partial t} + \frac{\partial z}{\partial y} \frac{\partial^2 y}{\partial s \partial t} + \frac{\partial^2 z}{\partial y^2} \frac{\partial y}{\partial s} \frac{\partial y}{\partial t} \end{aligned}$$

Chapter 14.6 :36

36. The curve of steepest ascent is perpendicular to all of the contour lines.



Chapter 14.6 :49

46. $F(x,y,z)=xyz$, $\nabla F(x,y,z)=\langle yz,xz,yx \rangle$, $\nabla F(1,2,3)=\langle 6,3,2 \rangle$, so an equation of the tangent plane is $6x+3y+2z=18$, and the normal line is given by $\frac{x-1}{6} = \frac{y-2}{3} = \frac{z-3}{2}$ or $x=1+6t$, $y=2+3t$, $z=3+2t$.

