Homework 7

Chapter 14.3. :89

89. Let $g(x)=f(x,0)=x(x^2)^{-3/2}e^0=x|x|^{-3}$. But we are using the point $(1,0)$, so near $(1,0)$, $g(x)=x^{-2}$. Then $g'(x)=-2x^{-3}$ and $g'(1)=-2$, so using (1) we have $f_x(1,0)=g'(1)=-2$.

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41. To show that f is continuous at (a,b) we need to show that

$\lim_{(x,y)\to(a,b)} f(x,y)=f(a,b)$ or equivalently $\lim_{(\Delta x,\Delta y)\to(0,0)} f(a+\Delta x,b+\Delta y)=f(a,b)$. Since f is differentiable at (a,b) , $f(a+\Delta x,b+\Delta y)-f(a,b)=\Delta z=f_x(a,b)\Delta x+f_y(a,b)\Delta y+\varepsilon_1\Delta x+\varepsilon_2\Delta y$, where ε_1 and $\varepsilon_2\to 0$ as $(\Delta x,\Delta y)\to(0,0)$. Thus $f(a+\Delta x,b+\Delta y)=f(a,b)+f_x(a,b)\Delta x+f_y(a,b)\Delta y+\varepsilon_1\Delta x+\varepsilon_2\Delta y$. Taking the limit of both sides as $(\Delta x,\Delta y)\to(0,0)$ gives $\lim_{(\Delta x,\Delta y)\to(0,0)} f(a+\Delta x,b+\Delta y)=f(a,b)$. Thus f is continuous at (a,b) .

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42. **(a)** $\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{0-0}{h}=0$ and $\lim_{h\to 0}\frac{f(0,h)-f(0,0)}{h}=\lim_{h\to 0}\frac{0-0}{h}=0$. Thus $f_x(0,0)=f_y(0,0)=0$. To show that f isn't differentiable at (0,0) we need only show that f is not continuous at (0,0) and apply Exercise 41 . As $(x,y)\to(0,0)$ along the x –axis $f(x,y)=0/x^2=0$ for $x\neq 0$ so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along the x –axis. But as $(x,y)\to(0,0)$ along the line $y=x$, $f(x,x)=x^2/\left(2x^2\right)=\frac{1}{2}$ for $x\neq 0$ so $f(x,y)\to\frac{1}{2}$ as $(x,y)\to(0,0)$ along this line. Thus $\lim_{(x,y)\to(0,0)} f(x,y)$ doesn't exist, so f is discontinuous at (0,0) and thus not differentiable there.

(b) For $(x,y)\neq(0,0)$, $f_x(x,y)=\frac{\left(x^2+y^2\right)y-xy(2x)}{\left(x^2+y^2\right)^2}=\frac{y\left(y^2+x^2\right)}{\left(x^2+y^2\right)^2}$. If we approach (0,0) along the y –axis, then $f_x(x,y)=f_x(0,y)=\frac{y^3}{y^4}=\frac{1}{y}$, so $f_x(x,y)\to\pm\infty$ as $(x,y)\to(0,0)$. Thus $\lim_{(x,y)\to(0,0)} f_x(x,y)$ does not exist and $f_x(x,y)$ is not continuous at (0,0). Similarly, $f_y(x,y)=\frac{\left(x^2+y^2\right)x-xy(2y)}{\left(x^2+y^2\right)^2}=\frac{x(x^2-y^2)}{\left(x^2+y^2\right)^2}$ for $(x,y)\neq(0,0)$, and if we approach (0,0) along the x –axis, then $f_y(x,y)=f_x(x,0)=\frac{x^3}{x^4}=\frac{1}{x}$. Thus $\lim_{(x,y)\to(0,0)} f_y(x,y)$ does not exist and $f_y(x,y)$ is not continuous at (0,0).