Homework 6

Chapter14.1. :30

30. All six graphs have different traces in the planes $x=0$ and $y=0$, so we investigate these for each function.

(a) $f(x,y)=|x|+|y|$. The trace in $x=0$ is $z=|y|$, and in $y=0$ is $z=|x|$, so it must be graph VI.

(b) $f(x,y)=|xy|$. The trace in $x=0$ is $z=0$, and in $y=0$ is $z=0$, so it must be graph V.

(c) $f(x,y)=\dfrac{1}{1+x^2+y^2}$. The trace in $x=0$ is $z=\dfrac{1}{1+y^2}$, and in $y=0$ is $z=\dfrac{1}{1+x^2}$. In addition, we can see that f is close to 0 for large values of x and y , so this is graph I.

(d) $f(x,y)=(x^2-y^2)^2$. The trace in $x=0$ is $z=y^4$, and in $y=0$ is $z=x^4$. Both graph II and graph IV seem plausible; notice the trace in $z=0$ is $0=(x^2-y^2)^2 \Rightarrow y=\pm x$, so it must be graph IV.

(e) $f(x,y)=(x-y)^2$. The trace in $x=0$ is $z=y^2$, and in $y=0$ is $z=x^2$. Both graph II and graph IV seem plausible; notice the trace in $z=0$ is $0=(x-y)^2 \Rightarrow y=x$, so it must be graph II.

(f) $f(x,y)=\sin(|x|+|y|)$. The trace in $x=0$ is $z=\sin|y|$, and in $y=0$ is $z=\sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.

Chapter14.2. :29

29. $F(x,y)=\arctan\left(x+\sqrt{y}\right)=g(f(x,y))$ where $f(x,y)=x+\sqrt{y}$, continuous on its domain $\{(x,y)\,|\,y\geq 0\}$, and $g(t)=\arctan t$ is continuous everywhere. Thus F is continuous on its domain $\{(x,y)\,|\,y\geq 0\}$.

Chapter 14.3. :66

66. **(a)** If we fix y and allow x to vary, the level curves indicate that the value of f decreases as we move through P in the positive x –direction, so f_x is negative at P .

(b) If we fix x and allow y to vary, the level curves indicate that the value of f increases as we move through P in the positive y –direction, so f_y is positive at P .

(c) $f_{xx} = \frac{\partial}{\partial x}\left(f_x\right)$, so if we fix y and allow x to vary, f_{xx} is the rate of change of f_x as x increases. Note that at points to the right of P the level curves are spaced farther apart (in the x –direction) than at points to the left of P , demonstrating that f decreases less quickly with respect to x to the right of P. So as we move through P in the positive x –direction the (negative) value of f_x increases, hence $\frac{\partial}{\partial x}\left(f_x\right) = f_{xx}$ is positive at P .

(d) $f_{xy} = \frac{\partial}{\partial y}\left(f_x\right)$, so if we fix x and allow y to vary, f_{xy} is the rate of change of f_x as y increases. The level curves are closer together (in the x –direction) at points above P than at those below P , demonstrating that f decreases more quickly with respect to x for y –values above P . So as we move through P in the positive y –direction, the (negative) value of f_x decreases, hence f_{xy} is negative.

(e) $f_{yy} = \frac{\partial}{\partial y}\left(f_y\right)$, so if we fix x and allow y to vary, f_{yy} is the rate of change of f_y as y increases. The level curves are closer together (in the y –direction) at points above P than at those below P , demonstrating that f increases more quickly with respect to y above P . So as we move through P in the positive y –direction the (positive) value of f_y increases, hence $\frac{\partial}{\partial y}\left(f_y\right) = f_{yy}$ is positive at P .