Homework 4

Chapter11.12. :17(withoutpart(c))

17.

n	$f^{(n)}(x)$	$f^{(n)}(0)$
0	$\tan x$	0
1	$\sec^2 x$	1
2	$2\sec^2 x \tan x$	0
3	$4\sec^2 x \tan^2 x + 2\sec^4 x$	2
4	$8\sec^2 x \tan^3 x + 16\sec^4 x \tan x$	

(a) $f(x) = \tan x \approx T_3(x) = x + \frac{1}{3}x^3$

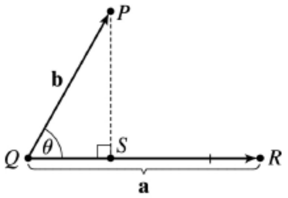
(b) $\left|R_3(x)\right| \leq \frac{M}{4!}|x|^4$, where $\left|f^{(4)}(x)\right| \leq M$. Now $0 \leq x \leq \frac{\pi}{6} \Rightarrow x^4 \leq \left(\frac{\pi}{6}\right)^4$, and letting $x = \frac{\pi}{6}$ $\left[\text{since } f^{(4)} \text{ is increasing on } \left(0, \frac{\pi}{6}\right)\right]$ gives

$$\left|R_3(x)\right| \leq \frac{8\left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{1}{\sqrt{3}}\right)^3 + 16\left(\frac{2}{\sqrt{3}}\right)^4\left(\frac{1}{\sqrt{3}}\right)}{4!}\left(\frac{\pi}{6}\right)^4$$

$$= \frac{4\sqrt{3}}{9}\left(\frac{\pi}{6}\right)^4 \approx 0.057859$$

Chapter12.4. :39

39. **(a)**



The distance between a point and a line is the length of the perpendicular from the point to the line, here $|\overrightarrow{PS}|=d$. But referring to triangle PQS , $d=|\overrightarrow{PS}|=|\overrightarrow{QP}|\sin\theta=|\boldsymbol{b}|\sin\theta$. But θ is the angle between $\overrightarrow{QP}=\boldsymbol{b}$ and $\overrightarrow{QR}=\boldsymbol{a}$. Thus by Theorem 6, $\sin\theta=\dfrac{|\boldsymbol{a}\times\boldsymbol{b}|}{|\boldsymbol{a}|\,|\boldsymbol{b}|}$ and so

$$d=|\boldsymbol{b}|\sin\theta=\frac{|\boldsymbol{b}|\,|\boldsymbol{a}\times\boldsymbol{b}|}{|\boldsymbol{a}|\,|\boldsymbol{b}|}=\frac{|\boldsymbol{a}\times\boldsymbol{b}|}{|\boldsymbol{a}|}\ .$$

(b) $\boldsymbol{a}=\overrightarrow{QR}=\langle -1,-2,-1\rangle$ and $\boldsymbol{b}=\overrightarrow{QP}=\langle 1,-5,-7\rangle$. Then
$\boldsymbol{a}\times\boldsymbol{b}=\langle(-2)(-7)-(-1)(-5),(-1)(1)-(-1)(-7),(-1)(-5)-(-2)(1)\rangle=\langle 9,-8,7\rangle$. Thus the distance is

$$d=\frac{|\boldsymbol{a}\times\boldsymbol{b}|}{|\boldsymbol{a}|}=\frac{1}{\sqrt{6}}\sqrt{81+64+49}=\sqrt{\frac{194}{6}}=\sqrt{\frac{97}{3}}\ .$$

Chapter 12.5. : 43

43. Setting $x=0$, we see that $(0,1,0)$ satisfies the equations of both planes, so that they do in fact have a line of intersection. $\boldsymbol{v}=\boldsymbol{n}_1\times\boldsymbol{n}_2=\langle 1,1,1\rangle\times\langle 1,0,1\rangle=\langle 1,0,-1\rangle$ is the direction of this line. Therefore, direction numbers of the intersecting line are 1 , 0 , -1 .