Homework 2

Chapter 11.4. :37

37. Since $\frac{d_n}{10^n} \leq \frac{9}{10^n}$ for each n, and since $\sum_{n=1}^{\infty} \frac{9}{10^n}$ is a convergent geometric series ($|r| = \frac{1}{10} < 1$), $0.d_1 d_2 d_3 \ldots = \sum_{n=1}^{\infty} \frac{d_n}{10^n}$ will always converge by the Comparison Test.

Chapter 11.5. :28

28. $b_6 = \frac{6}{8^6} = \frac{6}{262{,}144} \approx 0.000\,023$, so

$\sum_{n=1}^{\infty} \frac{(-1)^n n}{8^n} \approx s_5 = \sum_{n=1}^{5} \frac{(-1)^n n}{8^n} = -\frac{1}{8} + \frac{2}{64} - \frac{3}{512} + \frac{4}{4096} - \frac{5}{32{,}768} \approx -0.098\,785$. Adding b_6 to s_5 does not change the fourth decimal place of s_5, so the sum of the series, correct to four decimal places, is -0.0988.

Chapter 11.6. : 28

28.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\dfrac{2^{n+1}(n+1)!}{5\cdot 8\cdot 11\cdot \cdots \cdot (3n+5)}}{\dfrac{2^n n!}{5\cdot 8\cdot 11\cdot \cdots \cdot (3n+2)}} \right| = \lim_{n\to\infty} \frac{2(n+1)}{3n+5} = \frac{2}{3} < 1$$

, so the series converges absolutely by the Ratio Test.