Homework 12

Chapter 15.9. :24

24. Let $u=x+y$ and $v=y$, then $x=u-v$, $y=v$, $\frac{\partial(x,y)}{\partial(u,v)}=1$ and R is the image under T of the triangular region with vertices $(0,0)$, $(1,0)$ and $(1,1)$. Thus

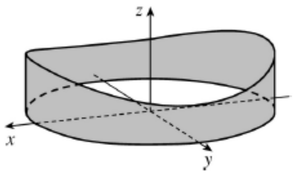
$$\iint_R f(x+y)\,dA=\int_0^1\int_0^u (1)\,f(u)\,dv\,du=\int_0^1 f(u)\,[\,v\,]_{v=0}^{v=u}du=\int_0^1 uf(u)\,du$$ as desired.

Chapter 16.2. :44

44. Consider the base of the fence in the xy –plane, centered at the origin, with the height given by $z=h(x,y)$. The fence can be graphed using the parametric equations

$x=10\cos u$, $y=10\sin u$,

$$\begin{aligned} z &= v\left[4+0.01((10\cos u)^2-(10\sin u)^2)\right] \\ &= v(4+\cos^2 u-\sin^2 u) \\ &= v(4+\cos 2u)\ ,\ 0\le u\le 2\pi\ ,\ 0\le v\le 1\ . \end{aligned}$$



The area of the fence is $\int_C h(x,y)\,ds$ where C , the base of the fence, is given by $x=10\cos t$, $y=10\sin t$, $0\le t\le 2\pi$. Then

$$\begin{aligned} \int_C h(x,y)\,ds &= \int_0^{2\pi}\left[4+0.01((10\cos t)^2-(10\sin t)^2)\right]\sqrt{(-10\sin t)^2+(10\cos t)^2}\,dt \\ &= \int_0^{2\pi}(4+\cos 2t)\sqrt{100}\,dt=10\left[\,4t+\frac{1}{2}\sin 2t\,\right]_0^{2\pi} \\ &= 10(8\pi)=80\pi\ \mathrm{m}^2 \end{aligned}$$

If we paint both sides of the fence, the total surface area to cover is $160\pi\ \mathrm{m}^2$, and since 1 L of paint covers $100\ \mathrm{m}^2$, we require $\frac{160\pi}{100}=1.6\pi\approx 5.03$ L of paint.

Chapter 16.3. :26

26. $\nabla f(x, y)=\cos(x-2y)\,\boldsymbol{i}-2\cos(x-2y)\,\boldsymbol{j}$

(a) We use Theorem 2: $\int_{C_1} \boldsymbol{F}\cdot d\boldsymbol{r}=\int_{C_1} \nabla f\cdot d\boldsymbol{r}=f(\boldsymbol{r}(b))-f(\boldsymbol{r}(a))$ where C_1 starts at $t=a$ and ends at $t=b$. So because $f(0, 0)=\sin 0=0$ and $f(\pi,\pi)=\sin(\pi-2\pi)=0$, one possible curve C_1 is the straight line from $(0, 0)$ to (π, π); that is, $\boldsymbol{r}(t)=\pi t\,\boldsymbol{i}+\pi t\,\boldsymbol{j}$, $0\leq t\leq 1$.

(b) From (a), $\int_{C_2} \boldsymbol{F}\cdot d\boldsymbol{r}=f(\boldsymbol{r}(b))-f(\boldsymbol{r}(a))$. So because $f(0, 0)=\sin 0=0$ and $f\left(\frac{\pi}{2},0\right)=1$, one possible curve C_2 is $\boldsymbol{r}(t)=\frac{\pi}{2}t\,\boldsymbol{i}$, $0\leq t\leq 1$, the straight line from $(0, 0)$ to $\left(\frac{\pi}{2},0\right)$.