Homework 11

Chapter 15.6. :23

23. If we project the surface onto the xz –plane, then the surface lies ''above'' the disk $x^2+z^2 \le 25$ in the xz –plane.

We have $y=f(x,z)=x^2+z^2$ and, adapting Formula 2, the area of the surface is

$$A(S)=\iint_{x^2+z^2\le 25} \sqrt{[f_x(x,z)]^2+[f_z(x,z)]^2+1}\;dA=\iint_{x^2+z^2\le 25} \sqrt{4x^2+4z^2+1}\;dA$$

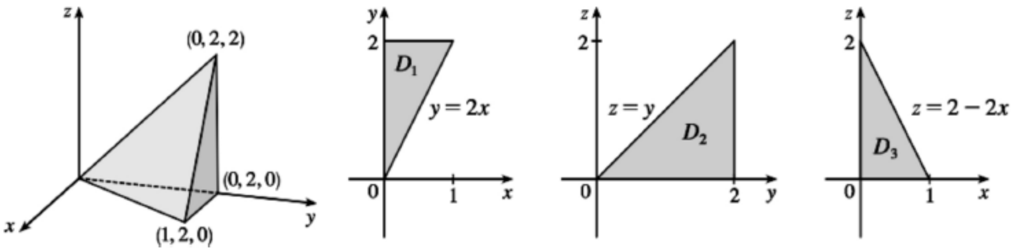
Converting to polar coordinates $x=r\cos\theta$, $z=r\sin\theta$ we have

$$A(S)=\int_0^{2\pi}\int_0^5 \sqrt{4r^2+1}\; r\,dr\,d\theta=\int_0^{2\pi} d\theta \int_0^5 r(4r^2+1)^{1/2}dr=[\theta]_0^{2\pi}\left[\frac{1}{12}(4r^2+1)^{3/2}\right]_0^5$$

$$=\frac{\pi}{6}\left(101\sqrt{101}-1\right)$$

Chapter 15.7. :28

28.



If D_1 , D_2 , and D_3 are the projections of E on the xy –, yz –, and xz –planes, then

$$D_1=\{(x,y)\,|\,0\le x\le 1, 2x\le y\le 2\}=\{(x,y)\,|\,0\le y\le 2, 0\le x\le y/2\},$$

$$D_2 = \{(y,z)\,|\,0\le y\le 2, 0\le z\le y\}=\{(y,z)\,|\,0\le z\le 2, z\le y\le 2\}\text{ ,and}$$

$$D_3 = \{(x,z)\,|\,0\le x\le 1, 0\le z\le 2-2x\}=\{(x,z)\,|\,0\le z\le 2, 0\le x\le (2-z)/2\}$$

Therefore

$$E = \{(x,y,z)\,|\,0\le x\le 1, 2x\le y\le 2, 0\le z\le y-2x\}$$
$$= \{(x,y,z)\,|\,0\le y\le 2, 0\le x\le y/2, 0\le z\le y-2x\}$$

$= \{(x,y,z) \mid 0 \le y \le 2, 0 \le z \le y, 0 \le x \le (y-z)/2\}$

$= \{(x,y,z) \mid 0 \le z \le 2, z \le y \le 2, 0 \le x \le (y-z)/2\}$

$= \{(x,y,z) \mid 0 \le x \le 1, 0 \le z \le 2-2x, z+2x \le y \le 2\}$

$= \{(x,y,z) \mid 0 \le z \le 2, 0 \le x \le (2-z)/2, z+2x \le y \le 2\}$

Then

$$\iiint_E f(x,y,z)\,dV = \int_0^1 \int_{2x}^2 \int_0^{y-2x} f(x,y,z)\,dz\,dy\,dx$$

$$= \int_0^2 \int_0^{y/2} \int_0^{y-2x} f(x,y,z)\,dz\,dx\,dy$$

$$= \int_0^2 \int_0^y \int_0^{(y-z)/2} f(x,y,z)\,dx\,dz\,dy$$

$$= \int_0^2 \int_z^2 \int_0^{(y-z)/2} f(x,y,z)\,dx\,dy\,dz$$

$$= \int_0^1 \int_0^{2-2x} \int_{z+2x}^2 f(x,y,z)\,dy\,dz\,dx$$

$$= \int_0^2 \int_0^{(2-z)/2} \int_{z+2x}^2 f(x,y,z)\,dy\,dx\,dz$$

Chapter 15.8. :21

21.

$$\iiint_E x^2\,dV = \int_0^\pi \int_0^\pi \int_3^4 (\rho \sin\phi \cos\theta)^2 \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$$

$$= \int_0^\pi \cos^2\theta \; d\theta \int_0^\pi \sin^3\phi \; d\phi \int_3^4 \rho^4 d\rho$$

$$= \left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta \right]_0^\pi \left[-\frac{1}{3}(2+\sin^2\phi)\cos\phi \right]_0^\pi \left[\frac{1}{5}\rho^5 \right]_3^4$$

$$= \left(\frac{\pi}{2} \right) \left(\frac{2}{3} + \frac{2}{3} \right) \frac{1}{5}(4^5 - 3^5) = \frac{1562}{15}\pi$$