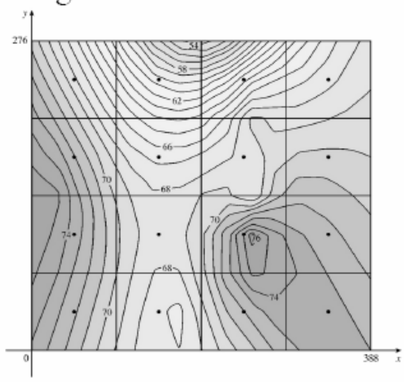
Homework 10

Chapter15.1. :10

10. As in Example 4, we place the origin at the southwest corner of the state. Then $R=[0,388]\times[0,276]$ (in miles) is the rectangle corresponding to Colorado and we define $f(x,y)$ to be the temperature at the location (x,y) . The average temperature is given by

$$f_{\text{ave}}=\frac{1}{A(R)}\iint_R f(x,y)\,dA=\frac{1}{388\cdot 276}\iint_R f(x,y)\,dA$$

We can use the Midpoint Rule with $m=n=4$ to give a reasonable estimate of the value of the double integral.



Thus, we divide R into 16 regions of equal size, as shown in the figure, with the center of each subrectangle indicated. The area of each subrectangle is $\Delta A=\frac{388}{4}\cdot\frac{276}{4}=6693$, so using the contour map to estimate the function values at each midpoint, we have

$$\iint_R f(x,y)\,dA\approx\sum_{i=1}^{4}\sum_{j=1}^{4} f\left(\bar{x}_i,\bar{y}_j\right)\Delta A$$

$$\approx\Delta A$$

$$+72.0+74.9+68.4+63.7+73.2+72.3+70.3+67.7]$$

$$=6693(1111.5)$$

Therefore,

$f_{\text{ave}}\approx\frac{6693\cdot 1111.5}{388\cdot 276}\approx 69.5$, so the average temperature in Colorado on May 1, 1996, was approximately 69.5° .

Alternatively, we can use the Midpoint Rule with $m=n=2$ which is easier computationally but will most likely be less accurate since we have fewer subrectangles. In this case, $\Delta A=\frac{388}{2}\cdot\frac{276}{2}=26,772$ and we can use the same grid to estimate the function values at the midpoints of the four subrectangles. Then

$$\iint_R f(x,y)\,dA\approx\sum_{i=1}^{2}\sum_{j=1}^{2} f\left(\bar{x}_i,\bar{y}_j\right)\Delta A\approx 26,772[70.0+66.5+74.3+68.5]$$

$$=26,772\cdot 279.3$$

and $f_{\text{ave}}\approx\frac{26,772\cdot 279.3}{388\cdot 276}\approx 69.8^\circ$.

Chapter15.2. :25

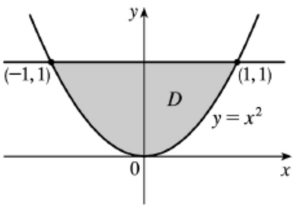
25.

$$V=\int_{-2}^{2}\int_{-1}^{1}\left(1-\frac{1}{4}x^{2}-\frac{1}{9}y^{2}\right)dx\,dy=4\int_{0}^{2}\int_{0}^{1}\left(1-\frac{1}{4}x^{2}-\frac{1}{9}y^{2}\right)dx\,dy$$

$$=4\int_{0}^{2}\left[x-\frac{1}{12}x^{3}-\frac{1}{9}y^{2}x\right]_{x=0}^{x=1}dy=4\int_{0}^{2}\left(\frac{11}{12}-\frac{1}{9}y^{2}\right)dy=4\left[\frac{11}{12}y-\frac{1}{27}y^{3}\right]_{0}^{2}=4\cdot\frac{83}{54}=\frac{166}{27}$$

Chapter15.3. :32

32. The two planes intersect in the line $y=1$, $z=3$, so the region of integration is the plane region enclosed by the parabola $y=x^{2}$ and the line $y=1$. We have $2+y\geq 3y$ for $0\leq y\leq 1$, so the solid region is bounded above by $z=2+y$ and bounded below by $z=3y$.



$$V=\int_{-1}^{1}\int_{x^{2}}^{1}(2+y)\,dy\,dx-\int_{-1}^{1}\int_{x^{2}}^{1}(3y)\,dy\,dx=\int_{-1}^{1}\int_{x^{2}}^{1}(2+y-3y)\,dy\,dx=\int_{-1}^{1}\int_{x^{2}}^{1}(2-2y)\,dy\,dx$$

$$=\int_{-1}^{1}\left[2y-y^{2}\right]_{y=x^{2}}^{y=1}dx=\int_{-1}^{1}(1-2x^{2}+x^{4})\,dx=\left[x-\frac{2}{3}x^{3}+\frac{1}{5}x^{5}\right]_{-1}^{1}=\frac{16}{15}$$